CHAPTER-1

INTRODUCTION

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1.1 Fluid Mechanics: Importance and Applications

The matter is divided into two categories, namely, fluids and solids. If under given thermodynamic conditions and in the absence of any external force, a matter does not change its shape, it is called a solid. On the contrary, if it changes its shape, it is called a fluid. Thus the different particles in solids have their definite relative positions and their positions do not change till some external force is applied. The position of different particles in fluids can change even without any external force; rather the particles change their positions themselves according to the shape of the container which contains the fluid.

Fluids are further classified into three categories, namely, liquids, gases and plasma. Liquids have definite volume which changes only slightly when an external force is applied or temperature is changed. Gases have no definite volume and their volume changes significantly when an external force is applied or the temperature is changed. Plasma is a state of matter where the molecules are ionized fully or partially.

The subject of fluid mechanics is concerned with the properties and behavior of fluids, whether at rest or in motion. The subject deals with pressures, velocities, and accelerations in the fluid, including fluid deformation and compression or expansion. Fluid mechanics has a wide range of applications in many engineering and technological areas involving aerospace, chemical, civil, environmental, mechanical, climatological, geological, meteorological and ocean engineering. More applications of fluid mechanics can be seen in the study of flows in natural rivers and artificial channels and the flow of ground water, the dispersion of pollutants in the atmosphere, lakes, rivers, and oceans and the flows in the pipelines of crude oil and natural gas from the petroleum fields.
1.2 Newtonian Fluids

Newtonian Hypothesis states that if the two neighbouring fluid layers, in a rectilinear motion, are moving with velocities \( u \) and \( u + \delta u \) and are at a distance \( \delta y \) apart, then the tangential or shearing force \( \tau \) is directly proportional to relatively velocity \( \delta u \) and inversely proportional to the distance between the layers, and is given by

\[
\tau \propto \frac{\delta u}{\delta y} \quad \text{as} \quad \delta y \to 0
\]  

\[
\therefore \quad \tau = \mu \frac{du}{dy} = \mu \dot{\gamma},
\]

where the constant of proportionality \( \mu \) is called the coefficient of viscosity and \( \dot{\gamma} \) is the strain-rate of the fluid. A fluid satisfying this hypothesis is called a Newtonian fluid. Thus, in Newtonian fluids the plot of the strain-rate of the fluid with the applied stress is a straight line (Fig. 1.1).

Fluids like water, air, light oils, ethanol, benzene, glycerin, aqueous solutions of sugar or salt, gasoline and mercury are the examples of Newtonian...
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fluids. Water is the best example of Newtonian fluid because it continues to follow Newtonian Hypothesis no matter how fast it is stirred or mixed.

1.3 Non-Newtonian Fluids

Hypothetically, non-Newtonian fluids are those fluids which do not obey the linear relationship between the stress tensor and the rate of strain tensor. In other words, the fluids which do not obey the Newtonian hypothesis are called non-Newtonian fluids. Thus in non-Newtonian fluids, the slope of the shear stress with respect to the strain rate will not be constant if the strain rate is changed. Thus, in non-Newtonian fluids, the viscosity changes with the applied shear force, i.e., the effective viscosity depends on the relative velocity. Thus, non-Newtonian fluids will not have a well defined viscosity.

The salt solutions and molten polymers like ketchup, custard, toothpaste, starch suspensions, paint, blood, shampoo, complex mixtures such as slurries, pastes, gels, polymer solutions are common examples of non-Newtonian fluids.

Non-Newtonian fluids are classified into the following three categories:

(i) Purely Viscous Fluids
(ii) Viscoplastic Fluids
(iii) Viscoelastic Fluids

1.3.1 Purely Viscous Fluids

According to available literature, the viscous fluid is also called a Newtonian fluid, where the strain rate is linearly related to the applied stress. The internal friction between the fluid layers is termed as “viscous”. Further, when the fluid is at rest, the stress tensor is isotropic and the result shows in a pressure term only. Thus, a purely viscous fluid, in which the stress tensor $T_{ij}$
is a given function of strain-rate, provides zero stress tensor in the absence of the strain-rate. This is stated mathematically as

$$T_{ij} = f(e_{ij}), f(0) = 0. \quad \ldots(1.3)$$

Different types of purely viscous fluids are defined by choosing different forms of the function $f$. Some examples of this kind of fluid are milk, blood, ketchup, fruit juice, water and shampoo.

### 1.3.2 Viscoplastic Fluids

In viscous fluids the continuous deformation takes place under a certain shearing stress, however small it may be. On the contrary, in elastic solids when the external applied stress is less than a significant value known as the yield stress, the fluid will not flow. Once the magnitude of this external stress exceeds the value of the yield stress, the fluid moves like Newtonian fluids. For example, in day-to-day life when we open a tube of shaving cream, we need to apply an adequate force in order to make the shaving cream start to flow. As a result, viscoplastic fluids behave like solids when the applied shear stress is less than the yield stress. Some examples of viscoplastic fluids are drilling mud, grease, toothpaste, tomato puree and nuclear fuel slurries.

### 1.3.3 Viscoelastic Fluids

Fluids that have a stress recovery when shear is relaxed are called viscoelastic. It is a well-known form of Non-Newtonian fluids. Such fluids exhibit both viscous and elastic behavior. In elastic materials, the stress is a definite function of the strain. Therefore, the elastic materials have some deformation, when we apply a definite stress, however, the material returns to its original position when the stress is removed. Thus the elastic materials possess a memory (rather perfect memory), i.e., it is competent of recognizing its original state. In fluids, the stress depends upon the rate of deformation; if
the stress is removed and the strain rate becomes zero, the deformation persists, i.e., it forgets its original shape. Thus, we can say that fluids have no memory. There are some fluids like soap solution, silicon putty, the thick part of egg and molten polymers, which have some elastic properties in addition to fluid properties. In such type of fluids, a suddenly applied and maintained state of uniform stress induces an instantaneous deformation, followed by a flow process which may or may not be limited to magnitude, as time grows, i.e., both an instantaneous elastic effect and creep characteristics are exhibited. This behavior of fluids represents the combined features of elasticity and viscosity. Such fluids are called viscoelastic fluids.

In the motion of viscoelastic fluids, some energy is lost due to viscous dissipation while a definite amount of energy is stored in materials as strain energy. The strain in such types of fluids can not be neglected, unlike the case of purely viscous fluids, however small. It is responsible for the recovery to the original state and for the possible reverse flow that may follow the removal of the stress. At the time of the flow, the natural state of fluid changes regularly and tries to attain the instant state of deformation but it never changes entirely. This lag is a measure of the elasticity or the so-called “memory” or elastic response of the fluid.

The relaxation time is a property of viscoelastic fluids. This is defined as the measure of the time required for elastic effects to decay. It is observed that the viscoelastic effects play an important role when the sudden changes take place in the rates of deformation.

The mathematical models have been developed for determining the stress or strain interactions in viscoelastic fluids and their temporal dependencies. The viscoelastic fluids are categorized into the following four models:
1.3.3.1 Maxwell Fluid

Maxwell (1867) proposed a model of viscoelastic fluids, now known as Maxwell fluids. Such fluids have the properties both of viscosity and elasticity, i.e., the fluid is Newtonian and the viscosity obeys Hooke’s law for the elastic behaviour. In a Maxwell fluid, stress, strain and their rates of change with respect to time t are governed by the equation of the form

\[ T_{ij} = -p \delta_{ij} + \tau_{ij}, \]

with

\[ \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \tau_{ij} = 2\mu \frac{\partial}{\partial t}(e_{ij}) \]

and

\[ e_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad \ldots(1.4) \]

where \( \lambda_1 \) is the relaxation time for the stress with \( G = \frac{\mu}{\lambda_1} \) as modulus of rigidity. \( T_{ij}, \tau_{ij}, e_{ij}, \delta_{ij}, u_i, x_i, p \) and \( \mu \) respectively denote the stress tensor, shear stress tensor, rate of strain tensor, Kronecker delta, velocity vector, position vector, isotropic pressure and viscosity.

For steady flow, fluid behaves like a Newtonian fluid. If motion is stopped, the stress decays (at constant strain) as \( \exp(-t/\lambda_1) \).

1.3.3.2 Oldroyd Fluid

To study the rheological behavior of dilute suspensions and emulsions to the idealized incompressible viscoelastic liquids at small variable shear
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stresses, a model was proposed by Jeffreys (1929). The following equations represent the equations of state

\[ T_{ij} = \tau_{ij} - p\delta_{ij}, \]

with

\[ (1 + \lambda_1 \frac{\partial}{\partial t}) \tau_{ij} = 2\mu \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) e_{ij} \]

and

\[ e_{ij} = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \]  

\[ \ldots (1.5) \]

where \( \lambda_1 \left( \frac{\mu}{G} \right) \) is the stress relaxation time and \( \lambda_2 \left( \frac{\mu'}{G} \right) \) is the strain retardation time. Thus, when the fluid comes to rest, any small shear decays as \( \exp \left( -\frac{t}{\lambda_1} \right) \) and after becoming free from stress, any small rate of strain decays as \( \exp \left( -\frac{t}{\lambda_2} \right) \).

Oldroyd (1953) generalized Jeffery’s model and provided a different equation of state for viscoelastic fluids. Such fluids, in the absence of retardation, behave like Maxwell fluids while in the absence of stress relaxation and retardation both, yield Newtonian fluids. Thus he proposed the following equation:

\[ \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \tau_{ij} = \mu \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) e_{ij}. \]  

\[ \ldots (1.6) \]

1.3.3.3 Rivlin-Ericksen Fluid

The constitutive equation of Rivlin-Ericksen (1955) fluid is given by

\[ T_{ij} = -p\delta_{ij} + \tau_{ij}, \]
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with \[ \tau_{ij} = \left( 2\mu + 2\mu' \frac{\partial}{\partial t} \right) e_{ij} \]

and \[ e_{ij} = \frac{1}{2} \left( \frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i} \right), \quad \ldots(1.7) \]

where \( \mu' \) is the coefficient of viscoelasticity.

Such fluids and other polymers are used in the manufacturing of parts of spacecrafts, airplanes, engineering equipments, seats, foams, tyres, belt conveyers, ropes, cushions, plastics, adhesives, contact lens etc. In recent times, polymers are also used in agriculture, communication appliances and in bio-medical applications.

1.3.3.4 Walters (Model B') Fluid

Walters (1962) proposed a model for a viscoelastic fluid having shear thinning and thickening characteristics. The constitutive equation for such kind of fluids is

\[ T_{ij} = -p\delta_{ij} + \tau_{ij}, \]

with \[ \tau_{ij} = \left( 2\mu - 2\mu' \frac{\partial}{\partial t} \right) e_{ij} \]

and \[ e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \ldots(1.8) \]

A viscoelastic fluid has relevance and importance in chemical technology and industry. The characteristics of such fluids reflect the cumulative effect of many blood parameters such as plasma viscosity, red blood cell deformability, aggregation and hematocrit.
1.4 Porous Medium

A porous medium is a material containing a large number of voids (also called pores). In other words, it is a rigid body with a large number of openings (pores). All rocks in the upper parts of the earth’s crust, irrespective of their type, age or origin, contain openings. However, it is difficult to provide an exact geometrical definition of the notion of a pore. Pores may be either linked (connected) or isolated (non-connected), distributed in the material more or less frequently either in a regular manner or in an indiscriminate manner. Interconnected pores are the effective pores, and by ineffective pores, we mean the pores though which fluid can not pass. This may be either due to surface-tension caused by fine holes or the holes may not be interconnected, so that they do not influence the flow behavior but may influence the compressibility of a medium.

In a porous medium, the voids are classified according to the behavior of the fluid within these spaces. The interstices or capillaries are the smallest void spaces in which molecular forces between the molecules of the solid and those of the fluid are significant. Caverns are the largest void spaces in which the fluids are partially influenced by the walls of the voids. Intermediary space in size between capillaries and caverns are referred to as pores. In all situations, the void spaces partially or completely influence the motion of the flowing fluids in these spaces.

The interconnectedness of the pores allows the fluids to flow through the material. The nature of the porous medium depends on the interconnected pores and their location, size and shape. A porous medium is homogeneous if its average properties are independent of location, and heterogeneous if they depend on location. A natural porous medium has an irregular distribution of pores with respect to shape and size. A porous medium is not necessarily
restricted to have the pores belonging to only one class and may be embedded with the pores of different sizes and shapes. According to this description, the term porous medium covers a very wide range of substances. Beach sand, sandstones, limestone, rye bread, wood, rocks, soil and human lungs are examples of the natural porous medium whereas concrete, cement, ceramics, bricks, paper cloth and filter paper are some examples of the man-made porous medium.

All kinds of porous media both natural and artificial have a random void structure. It is quite difficult to characterize a porous medium in terms of sizes, shapes, orientations and interconnections of voids in the medium, but it is rather characterized by its two properties namely porosity and permeability, which take care of all the complexity of a porous medium.

1.4.1 Porosity

In a porous medium the presence of voids scattered within the solid matrix is characterized by porosity. It is defined as the fraction of the voids to the total volume. Thus

$$\phi = \frac{\text{Volume of the voids}}{\text{Bulk volume}}, \quad 0 < \phi < 1.$$  

This is a dimensionless quantity expressed either as a fraction of one or in percent. In a homogeneous, isotropic material, the porosity is a pure constant and for a non-homogeneous material it depends upon its position.

Porosity is effectively a static property and has tremendous importance in the evaluation of the economic value of the porous media bearing oil or gas, as it determines the total fluid content of such reservoirs. Porosity can be classified as absolute or total porosity and effective porosity. The ratio of the total void volume including the volume of all the pores whether or not they are
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effective to the bulk volume is said to be total porosity whereas the porosity defined on the basis of effective pores only is named as effective porosity. Throughout the present analysis, the word ‘porosity’ means effective porosity.

For a natural porous medium, porosity does not normally exceed 0.6, for beds of solid spheres with uniform diameter it can vary between 0.2525 and 0.4764 and for very fluffy foam materials porosity is nearly one. Fancher et al. (1933), Muskat (1937), Scheidegger (1960) and Bear (1972) have provided a detailed discussion of porosity and porous materials.

1.4.2 Permeability

Permeability is a combination of the size of the pores and the degree to which the pores are interconnected. It controls the ease with which the fluid can flow through the solid. The permeability is independent of the nature of the fluid although it depends on the geometry of the medium. This permeability constant was first introduced by Darcy (1856) with the help of his famous experiment known as Darcy’s law which introduced permeability as a measurable quantity.

In a horizontal flow of an incompressible fluid through a porous material of length \( l \) in the direction of flow and the cross-sectional area \( A \), the permeability \( K \) of the material is defined as

\[
K = \frac{Q \mu}{A \left( \frac{\Delta p}{l} \right)}.
\]  

...(1.9)

Here \( Q \) is the volumetric flow rate, \( \mu \) is the viscosity of the fluid and \( \Delta p \) is the applied pressure difference across the length of the porous bed.
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<table>
<thead>
<tr>
<th>S. No.</th>
<th>Material</th>
<th>Porosity</th>
<th>Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Black state powder</td>
<td>0.57-0.66</td>
<td>$4.9 \times 10^{-10} - 1.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>2</td>
<td>Brick</td>
<td>0.12-0.34</td>
<td>$4.8 \times 10^{-11} - 2.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>3</td>
<td>Concrete</td>
<td>0.02-0.07</td>
<td>$1.0 \times 10^{-9} - 2.3 \times 10^{-7}$</td>
</tr>
<tr>
<td>4</td>
<td>Copper powder</td>
<td>0.09-0.34</td>
<td>$3.3 \times 10^{-6} - 1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>5</td>
<td>Leather</td>
<td>0.56-0.59</td>
<td>$9.5 \times 10^{-10} - 1.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>6</td>
<td>Limestone (Dolomite)</td>
<td>0.04-0.10</td>
<td>$2.0 \times 10^{-11} - 4.5 \times 10^{-10}$</td>
</tr>
<tr>
<td>7</td>
<td>Sand</td>
<td>0.37-0.50</td>
<td>$2.0 \times 10^{-7} - 1.8 \times 10^{-6}$</td>
</tr>
<tr>
<td>8</td>
<td>Sand stone</td>
<td>0.08-0.38</td>
<td>$5.0 \times 10^{-12} - 3.0 \times 10^{-8}$</td>
</tr>
<tr>
<td>9</td>
<td>Silica powder</td>
<td>0.37-0.49</td>
<td>$1.3 \times 10^{-10} - 5.1 \times 10^{-10}$</td>
</tr>
<tr>
<td>10</td>
<td>Soil</td>
<td>0.43-0.54</td>
<td>$2.9 \times 10^{-9} - 1.4 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Table 1.1: Porosity and Permeability of different materials
The dimension of permeability is length squared so that in c. g. s. system permeability unit should be cm\(^2\). In different branches of applied science some other units have been adopted for it, however, the unit most commonly used for permeability is the ‘darcy’ (d) with

\[ 1 \text{ darcy} = 9.87 \times 10^{-9} \text{ cm}^2. \]

### 1.4.3 Isotropic and Anisotropic Porous Media

A macroscopic description of a porous medium is the depiction that can be related to the average or bulk properties at sizes much larger than a single pore. A given porous medium is ‘ideal’, if it is homogeneous, uniform and isotropic. However, due to the irregularities in size and shape of the natural pores, there may be variations either in horizontal or in vertical permeability depending upon the structure of the porous medium. A single value of permeability is used for a homogeneous porous fluid reservoir.

If the permeability at a given point in a porous medium is independent of direction, the medium is said to be isotropic. The assumption of isotropy of a porous medium can be justified and is strictly correct when the flow systems are two-dimensional with the planes of flow parallel to the binding plane. The solution of the problem having flow components along more than one direction with different permeabilities at different points is the solution of the problem for isotropic system with some modified geometry. Therefore, a complete discussion of the latter will implicitly include the solution for similar problems for anisotropic system. Hence in many situations, it is sufficient to consider the porous medium to be isotropic.

A porous medium is said to be anisotropic if permeability depends upon the directions within the medium. In case of problems such as those of partially penetrating wells or seepage under dams, in which an appreciable part or all of
the flow takes place in the direction of large permeable variations, the latter may of course seriously have an effect on the fundamental features of the flow. The precise results can only be obtained by taking into account the variations in permeability.

Hence as long as horizontal variations in permeability are not of appreciable magnitude, those in the vertical direction may be entirely neglected in the theoretical treatment of such problems which are essentially two dimensional and whose planes of flow are horizontal. In most of the problems of qualitative nature, the effect of the inhomogeneity will be rather easy to deduce from physical argument, and the results derived on the assumption of uniformity in permeability will represent limiting cases with respect to those where the permeability is variable.

1.5 Darcy’s Law

Darcy (1856) performed a classical experiment and obtained a simple and interesting result, which relates the movement of fluid to the acting pressure gradients. He examined experimentally the flow of water through sand filter and observed that the rate of percolation of water through that filter bed is directly proportional to the cross sectional area of the filter bed and the total force impressed on it and inversely proportional to the thickness of the bed. A rough sketch of Darcy’s experiment performed on a sand bed is shown in Figure 1.2. The fluid enters through a homogeneous filter bed of height L which is bounded by horizontal plane areas between the points 1 and 2, at the top of the bed the volumetric flow rate is Q; it percolates through the bed and comes out of the bottom of the bed at the same volumetric flow rate Q. An open manometer tube is attached at the upper and the lower boundaries of the filter bed, the difference between the pressure heads or the hydraulic head can be measured from the same datum point z = 0. Clearly \((h_1 - h_2)\) indicates the
difference between the fluid heads of the inlet and outlet faces of the bed. Therefore, Darcy’s law provides

\[ Q = -\frac{CA(h_2 - h_1)}{L}, \]  

...(1.10)

where the constant of proportionality \( C \) depends upon the properties of the fluid and of the porous medium and the minus sign shows that the flow is in the opposite direction of \( h \) increasing.

In order to examine the influence of fluid and porous medium separately, \( C \) can be defined as

\[ C = \frac{K}{\mu}, \]  

...(1.11)

so that

\[ \frac{Q}{A} = q = -\frac{K(h_2 - h_1)}{\mu L}, \]  

...(1.12)

where \( \mu \) is the viscosity of the fluid, \( K \) is the specific permeability of the medium and \( q \) is called the seepage velocity. The study of seepage is based on a statistical method so that the real flow of fluid in the pores is replaced by an imaginary seepage flow of that fluid.

In terms of pressure gradient, say \( \frac{dp}{dx} \), Darcy’s law can be expressed as

\[ q = -\frac{K}{\mu} \frac{dp}{dx}. \]  

...(1.13)

Darcy’s law is for the porous media in which the flow is necessarily a one-dimensional flow. Generalized Darcy’s law for a three-dimensional flow, when gravity force is acting on the fluid and affecting the velocity just as the pressure gradient, defined in vector notation, is given by
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Fig. 1.2: Darcy's experiment.
\[
\frac{\mu q}{K} = \nabla p - \rho g \lambda, \quad \text{...(1.14)}
\]

where \( q = (u, v, w) \) and \( \lambda = (0, 0, 1) \).

De Wiest (1969) showed the empirical equivalence of Darcy’s law to the Navier Stokes equation, in which the term \( \frac{\mu q}{K} \) calculated from Darcy’s law, known as Darcy’s drag force or Darcy resistance, replaces the usual viscous term \( \mu \nabla^2 q \) in the equations of motion.

### 1.5.1 Limitations and Validity of Darcy’s Law

Darcy’s law is used when the flow is streamlined or laminar in which the movement of individual fluid particles in a pore flow along paths nearly parallel to the walls of the pores. The laminar flow region is at low flow rates, where inertial effects are negligible and it breaks down at adequately high flow rates. If the velocities in the pores become significantly high, the inertial forces may become commensurable with the frictional forces. The Darcy’s law loses its applicability when the linear relationship between the velocity and the forces does not hold. The range of flow rates for which laminar flow exists is defined in terms of the Reynolds number \( R \) and at a certain value of it Darcy’s law breaks down. In fact, it is established that Darcy’s law applies only for the values of \( R \) less than one. For a high Reynolds number, the situation when inertia forces rather than the viscous forces become high, Darcy’s law loses its applicability. Another restriction on the use of Darcy’s law applies to the flow of gases at low pressures.

### 1.6 Stability Theory: Concept and Techniques

The stability theory can be explained as follows:
Consider an equilibrium state or basic state (steady state) of a physical problem whose stability is to be examined. We consider a state near this basic state and examine whether the system will tend to the considered basic state as time passes.

In fact, a mathematical model, equivalent to a given physical situation, is constructed. In this process, we seek to make our model physically realistic and as close to the given physical situation as possible. For example, in constructing a mathematical model of a situation in the atmospheric studies, since some factors, such as, the compressibility of air and its thermally conducting nature introduce much complication, therefore, during the mathematical formulation, we assume certain approximations and assumptions close to the physical requirements of the problem. The results so obtained are theoretical results and must be in accordance with the functioning of the physical system and have to confirm the experiments.

To examine the stability or instability of a given physical system, we examine as to how the equilibrium configuration reacts to small fluctuations (disturbances) which are either inherent in the system or man made. The system is said to be stable with respect to a particular disturbance (also called perturbation) if the perturbation dies down gradually and the system returns to its original position. On the other hand, if even a single perturbation grows with time and the system never returns to its original position, the system is said to be unstable. A system is said to be stable if it is stable with respect to all perturbations to which it could be subjected. If the system is unstable even for one special mode of perturbation, it is said to be unstable.

There are two main techniques for analyzing the stability of a flow:

(i) Energy Technique

(ii) Normal Mode Technique
(i) Energy Technique

This technique deals with the kinetic energy of the perturbations. According to this technique, if there is a transfer of energy from basic flow to the perturbations, the kinetic energy of these perturbations increases with time and the flow becomes unstable. On the other hand, if there is a transfer of kinetic energy from the perturbations into the basic flow, the kinetic energy of these perturbations decays with time, the flow will be stable. This method is global in nature as we calculate the kinetic energy of the whole system and because of this, the applications are restricted. While this provides a surest limit for the stability of the flow, it is crude in giving the unstable limit. Moreover, it gives very little information about the local behavior of the perturbations. For investigating the stability of a flow, once the fluid is confined within rigid boundaries, sometimes we consider the vorticity of the perturbations in place of kinetic energy, so we calculate

\[ w = \int \left( \xi^2 + \eta^2 + \zeta^2 \right) \, dV, \]  

...(1.15)

where \((\xi, \eta, \zeta)\) are the components of the vorticity of the perturbations, and the integration is taken over the whole of the flow region. Then the basic flow is stable or unstable according to whether \(\frac{dw}{dt}\) is negative or positive.

(ii) Normal Mode Technique

In this technique, to determine the stability of a system, the linearized perturbation equations are set up, within the linear framework, from the equations of conservation of mass, momentum and energy, retaining only the linear terms in perturbed quantities. These equations are then solved either analytically or with the help of variational method or through an integral equation under a set of appropriate boundary conditions. The stability of the
system leads to the dispersion relation in the parameters. Sometimes this dispersion relation is quite complex and an analytical interpretation may not be possible. Therefore, to determine the effect of a particular physical parameter on the growth rate of the perturbations, we analyze the change by varying that parameter while keeping the other parameters fixed. An increase in the growth rate implies the destabilizing influence of that particular parameter and a decrease in the growth rate shows the stabilizing influence of the parameter.

For providing a complete stability analysis of a given system, it is assumed that the perturbations can be resolved into dynamically independent wave-like components, each component satisfying the linearized equations of motion and the boundary conditions separately. Thus if $f'(x,y,z,t)$ is a typical wave component describing a disturbance, we expand it as

$$f'(x,y,z,t) = f(z) \exp \left\{ i \left( a_x x + a_y y \right) + nt \right\}, \quad \ldots(1.16)$$

where $k = \left( a_x^2 + a_y^2 \right)^{1/2}$ is the real wave number of the disturbance and $n$ is a constant to be determined which, in general, is complex so that $n = n_r + i n_i$. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter $n$. $n_r < 0$ means the stability of the system and $n_r > 0$, even, for a single perturbation, indicates that the system is unstable. When $n_r = 0 \Rightarrow n_i = 0$, the system is marginally stable under the principle of exchange of stabilities while $n_r = 0$ but $n_i \neq 0$ represents overstability of periodic oscillatory motion.

In particular, the stability of the system will depend on its stability to the disturbances of all wave numbers, and its instability will emanate from the instability with respect to even one wave number.
1.7 Mono Diffusive Convection

Bénard (1900) examined experimentally the problem of the onset of thermal instability in horizontal layers of fluid heated from underside. Since the fluids typically have positive thermal expansion coefficient, the fluid in the lower part of the layer expands as it becomes hotter. Thus, due to thermal convection, the fluid at the bottom will be lighter than the fluid at the top; so it becomes a top-heavy arrangement, which is potentially unstable, and when the temperature gradient or layer depth is sufficiently large to overcome the effect of gravity, the fluid rises and a pattern of cellular motion may be seen. This is famous as Bénard convection.

Bénard took, in his experiment, a very thin layer of non-volatile fluid, with 1mm depth, placed on a carefully leveled metallic plate which he kept at a constant temperature. The upper surface of layer was kept in contact with the free air. He observed that the layer resolved itself into a number of cells, known as Bénard cells. The principal facts established by the experiment of Bénard are as follows:

(i) A certain critical adverse temperature gradient must be exceeded before the instability can set in.

(ii) The motions that ensue on suppressing the critical adverse temperature gradient have a stationary cellular convection.

Carefully performed experiments show that the layer of fluid resolves itself into a number of cells at the onset of instability and the cells become equal and take the form of regular hexagon. On earlier occasions, Rumford (1870) and Thomson (1882) also acknowledged the phenomenon of thermal convection. After sixteen years from Bénard, Rayleigh (1916), for the first time in his pioneer paper, could succeed in laying down the theoretical foundations...
of the facts for a correct interpretation. He showed that there is a non-dimensional parameter, which represents physical factors entering the problem, known as Rayleigh number and is given by

\[ R = \frac{g\alpha\beta d^4}{\nu\kappa}, \]  

...(1.17)

which decides the stability of a layer of fluid heated from below. Here \( g \) is the magnitude of the acceleration due to gravity, \( \alpha \) is the coefficient of volume expansion, \( \beta = \left( \frac{dT}{dz} \right) \) is the uniform adverse temperature gradient, \( d \) is the depth of the fluid layer, \( \kappa \) is the thermal diffusivity and \( \nu \) is the kinematics viscosity. Rayleigh also showed that (i) the instability must set in when \( R \) exceeds a certain critical value \( R_c \) and (ii) the stationary pattern of motions must prevail when \( R \) just exceeds \( R_c \). The theoretical aspects of the Bénard problem have also been discussed by Jeffreys (1926, 1928). He modified the Rayleigh criterion for different sets of boundary conditions. Pellew and Southwell (1940) have confirmed that oscillatory motions are damped, while non-oscillatory motions are always manifested. Chandrasekhar (1954) considered the Bénard problem as a characteristic value problem and determined the critical Rayleigh number given by

\[ R_c = \frac{g\alpha\beta_c d^4}{\nu\kappa}, \]  

...(1.18)

which yields the critical adverse temperature gradient at which the thermal instability sets in. Here \( \beta_c \) is the critical adverse temperature gradient.

Chandrasekhar (1958) re-examined the Bénard problem in the presence of vertical magnetic field and obtained the critical Rayleigh number at
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the marginal stability in the three cases, for free-free boundaries, for rigid-rigid boundaries and for rigid-free boundaries.

![Geometrical configuration of Bénard convection](image)

Fig. 1.3: Geometrical configuration of Bénard convection

1.8 Double-diffusive Convection

Double-diffusive convection arises due to combined diffusion of heat and mass, driven by buoyancy effect. The density gradients that provide the driving buoyancy force are induced by the combined effects of temperature and concentration in the fluid non-uniformly. If heat and salt are involved in fluid saturated medium, the heat diffuses more rapidly than a dissolved substance and convection sets in. A two component fluid layer is displaced vertically and loses excess heat more rapidly than excess solute. To compare with monodiffusion, a single component fluid layer is stable if the density decreases in the vertically upward direction whereas a fluid layer consisting of two components can diffuse relative to each other and may be dynamically unstable. The resulting buoyancy may act either to increase the displacement of
particle, cause of a monotonic instability, or reverse the direction of
displacement of particles, cause of a oscillatory instability, depending on
whether the solute gradient is destabilizing and the temperature gradient is
stabilizing or vice-versa.

In terms of temperature $T$ and concentration $C$, the density of mixture is
expressed as

$$\rho_i = \rho_0 \left[ 1 - \beta (T - T_0) - \beta_c (C - C_0) \right], \quad \ldots(1.19)$$

where $\beta = -\frac{1}{\rho_i} \frac{\partial \rho_i}{\partial T}$ and $\beta_c = -\frac{1}{\rho_i} \frac{\partial \rho_i}{\partial C}$ are respectively the thermal expansion
coefficient and the concentration expansion coefficients.

### 1.9 Soret Phenomenon

The Soret effect is a mechanism by which applied temperature gradient
can establish a concentration gradient in a binary fluid mixture. The instability
of such a mixture in the Benard configuration has much in common with the
problem of thermosolutal convection. In this process, the flow of a component
depends not only on its own gradient, but also on the gradient of the other
component. Thus in the heat-salt pair, the flux of salt is a function of the
temperature gradient besides being a function of the salinity gradient. This
effect of the temperature gradient on salt flux is termed as Soret effect
introduced by Soret (1879) in fluids.

The phenomenological equations relating the fluxes of heat ($J_Q$) and
matter ($J_c$) to the thermal and solutal gradient present in a binary fluid mixture,
in terms of the thermal conductivity $\kappa$, diffusion constant $D$ and density $\rho$
of the mixture, may be written as
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\[ J_Q = -\kappa \nabla T - \rho T \mathbf{C} \left( \frac{\partial \mu}{\partial \mathbf{c}} \right) \nabla \mathbf{C} \] \quad \text{...(1.20)}

and \[ J_c = -\rho D \left[ S_T C (1 - C) \nabla T + \nabla C \right], \quad \text{...(1.21)} \]

where \( \mu \) is the chemical potential of the solute, \( T \) and \( C \) are temperature and concentration, \( S_T \) is Soret coefficient.

Luikov and Berkovsky (1970) showed for the first time that if an unbounded fluid at rest in undisturbed state is heated from below, the two pure diffusion waves propagating in the horizontal direction become coupled and give rise to two modes of modified waves.

1.10 Dufour Phenomenon

The analogous effect arises when a concentration gradient is imposed on originally isothermal fluid mixture and it happens that a temperature gradient is developed as diffusion of salt occurs. This phenomenon is termed as the Dufour effect, named after, Dufour (1873). It can also be known as diffusion-thermo effect to indicate that it is the inverse of thermal diffusion.

The Dufour effect is important in liquids and a little work has been done in this direction. The difference in temperatures is retained if a concentration gradient is maintained. The temperature variations could cause complications and the Dufour effect can be used to verify the heat-matter Onsagar reciprocal relation.

The existence of these effects was further established during the World War-II when Clusius and Waldmann (1942) for the first time, determined the thermal diffusion factor for the gases.
1.11 Hydromagnetic Stability

The recent scientific and technological advances have developed a great interest in the phenomena encountered in magnetohydrodynamics. The study of hydromagnetics deals with the situation where the motion of a fluid is influenced by Lorentz force, the force produced by a magnetic field on a current carrying conductor. The motion of an electrically conducting fluid like liquid sodium, mercury etc. in the presence of a magnetic field, in general, gives rise to induced currents on which the mechanical forces are exerted by the magnetic field. This induced magnetic field produced by induced electric currents modified the original magnetic field. Thus there is a two-way interaction between the flow field and the magnetic field, the magnetic field exerts force on the fluid by producing induced currents, and secondly, the induced currents change the original magnetic field. Therefore, the hydromagnetic flows, i.e., the flows of electrically conducting fluids in the presence of a magnetic field, are more complex than the ordinary hydrodynamic flows.

Faraday (1832) predicted induced currents in the ocean due to the earth’s magnetic field. The important contribution in this direction was made by Alfven (1942) who established transverse waves in electrically conducting fluids and explained many astrophysical phenomena with it. After these studies, the subject of hydromagnetics has received much attention of researchers in past decades.

Generally, the effect of a magnetic field is to inhibit the onset of instability. Kent (1966) showed that the effect of magnetic field with variations in vertical direction is destabilizing under certain conditions whereas in the absence of magnetic field the system was stable. Further, Gilman (1970) showed that magnetic buoyancy gives rise to normal mode instability in fluids.
in magnetohydrostatic balance whose diffusivity of heat is large as compared to the viscous and magnetic diffusivities as in stellar interiors. This dual effect of magnetic field was also confirmed by Jain (1983). This nature of magnetic field makes the study of hydromagnetics, in general, and hydromagnetics stability, in particular, interesting and more challenging.