Chapter 01

1.0 INTRODUCTION

Dams are generally constructed for flood control and conservation. Various purposes for conservation are irrigation, power generation, navigation, domestic, recreational and industrial purposes. The selection of dam type depends on the purpose for which it is built, site topography, storage capacity, hydrology, availability of local construction materials etc.[1,2] Consideration of the tri-dimensional arch action to hold water load in a narrow gorge brought the idea of arch dam. Of all the dam types creating water storage facilities, arch dams curving across the breadth of the valley with one mighty vault are doubtlessly the most economic and elegant, exacting the needs of concrete and rock.[3,4] The conventional methods adopted for the analysis of all types of arch dams are cylinder theory, method of independent arches, trial load and model analysis which are found to be of limitations for multiple radius arch dams of height greater than 100 m.[5,6] Later, accurate methods are necessitated by eliminating many assumptions made in the traditional methods for ensuring safety and economy which led to numerical methods such as finite difference, finite element and boundary element for arch dams. Of these, finite element is the most effective method for handling a continuum like arch dam since, it gives a more realistic stress distribution and more flexibility with regard to geometry and boundary conditions than other methods.[6-9] Hence, a critical study on how the finite element method resolves the complexity in the case of an arch dam of varying geometry is presented in this thesis.
1.1 THE NEED

Earlier, finite element method with two dimensional analysis using plane stress and plane strain as well as shell theory that actually approximates three-dimensional problem by two-dimensional one were used. Though it gives good results for a thin arch dam, thick arch dam requires a rigorous three dimensional analysis.[9-12] Studies as well as software are seen developed so far using isoparametric brick elements; 8-21 nodes, as well as shell elements; 16 nodes, for modeling the geometry which will not accurately define the complex geometry of a thick arch dam of variable curvature especially the extrados curve.[12-20] Hence, there is an urgent need for considering the effect of variable curvature by approximating the geometry with higher order polynomials incorporating more nodal points at element level itself while modeling.

The available literature and software show that the hydrostatic pressure on the curved surface is seen approximated as normal to the surface by means of certain global coefficients to the horizontal pressure on vertical surface.[14-19] In fact, the magnitude as well as direction will be varying at each point, i.e. water pressure will be normal to the curved surface, horizontal and vertical extrados, with components in the three directions. In the finite element method, water pressure needs to be considered more accurately as actual distributed surface forces on each element by direction cosines and numerical integration.[20,21] Similarly the silt pressure, uplift and dynamic effect of the reservoir water also will have to be considered at element level itself.

In the available general purpose programs, the self-weight of each element is assumed to act at the centre of gravity and provided as lumped masses at the nodes.
which is also approximate.[15-20] The load vector due to gravity, if taken as a body force by numerical integration throughout the volume of each element, can give more dependable results. The seismic inertial effects by way of equivalent static load components as horizontal and vertical peak ground acceleration then can also be incorporated in the body force like this.

Trial load analysis gives comparable results with 3D Finite element method only for the simple cylindrical shape and for variable curvature arch dams of greater height the trial load assumptions are dubious. Recent studies considering the various aspects of design, variable curvature arch dams are found to be more practical for valleys of heights exceeding 100m.[4,5,22,23] Hence, it is felt that there is a great need for rigorous three dimensional finite element analysis to study the actual performance of a practical arch dam structure by approximating the actual geometry with higher order polynomials considering the surface pressure components in all the three directions and body force components in the respective direction by developing a software using advanced programming technique equipped with effective pre-post processing facilities.[20-27]

1.2 TYPES OF ARCH DAMS

The definition for an arch dam by ICOLD includes all curved dams, where the base-thickness is less than 0.6 times the height.[2] Mainly arch dams are grouped into:

(i) Constant radius

(ii) Variable radius

(iii) Constant angle
(iv) Multiple arch
(v) Cupola (shell)
(vi) Arch gravity
(vii) Mixed type

1.3 METHODS OF ANALYSIS OF ARCH DAMS

This is decided by the shape optimization studies.[4] Main methods of design of arch dams according to Varshney are categorized into:

(i) **Preliminary methods**
   a. Thin cylinder theory
   b. Thick cylinder theory
   c. Elastic theory
   d. Active arch method
   e. Cain’s method
   f. U.S.B.R. criteria
   g. Institution of Engineers, London
   h. R. S. Varshney’s equations

(ii) **Elaborate methods**
   a. Inclined arch method
   b. Tolke method

(iii) **Trial load analysis**
    USBR

(iii) **More elaborate methods**
   a. Finite element method
b. Shell analysis method

c. Three-dimensional elastic solution

d. Finite difference method

e. Three-dimensional electric analogue

f. Dynamic relaxation of three-dimensional elastic solution

(v) Experimental method

Model studies

According to CBIP publication the methods of analysis commonly adopted are discussed below:[5]

(i) Cylinder Theory

The simplest and the earliest of the methods available for the design of an arch dam is the cylinder theory. In this theory, the stress in an arch dam is assumed to be the same as in a cylindrical ring of equal external radius. The arch thickness is calculated by the thin cylinder formula. The cylinder theory does not allow for the discontinuity of the arch at the abutment and is, therefore, highly approximate. The use of cylinder theory has been restricted to dams less than 30 m in height located in narrow valleys. A low value of permissible stress in concrete, usually about 60 per cent of the permissible stress, issued to allow for the highly approximate nature of the formula. The cylinder theory is only of historical importance now.

(ii) Method of Independent Arches

This method considers the dam to be made up of a series of arches with no interaction between them. It is assumed that all horizontal water loads are carried horizontally to the arch abutments by arch action and that only the dead load weights
plus the vertical water loads in the case of sloping upstream face are carried vertically to the foundation by cantilever action. If the canyon is relatively regular and narrow and the dam is of low height so that a symmetrical thin structure with large central angle can be adopted this method may give reasonably satisfactory results.

Practically the water load is transferred to the foundation and abutments, both by horizontal arch action and vertical cantilever action. The vertical cantilevers are restrained at the foundation and must bend under their share of water load until their deflected positions coincide with the deflected positions of horizontal arch elements. The theory that the entire water load is carried horizontally to the abutments is therefore, incorrect and the design that ignores vertical cantilever action can seldom be considered as wholly satisfactory.

(iii) Arch Cantilever (Trial Load) Method

The most commonly accepted method of analysing arch dams assumes that the horizontal water load is divided between the arches and cantilevers so that the calculated arch and cantilever deflections are equal at all conjugate points in all parts of the structure. Because the required agreement of all deformations is obtained by estimating various load distributions and computing the resulting movements until the specified conditions are fulfilled, the procedure is logically called trial-load method. Trial load analyses may be classified into the following types depending on their relative accuracy and corresponding complexity.

a. Crown Cantilever Analysis

Crown-cantilever analysis consists of an adjustment of radial deflections at the crown cantilever with the corresponding deflections at the crowns of arches. This type
of analysis assumes a uniform distribution of radial load from the crowns of arches to their abutments and neglects the effects of tangential shear and twist. While the results obtained from this analysis are rather crude, it has the advantage of very short time to complete the analysis. If used with judgment, it is an effective tool for appraisal studies.

b. Radial Deflection Analysis

A radial deflection analysis is one in which radial deflection agreement is obtained at arch quarter points with several representative cantilevers by an adjustment of radial loads between these structural elements. With the use of this type of analysis, loads may be varied between the crowns and abutments of arches, thus producing a more realistic distribution of load in the dam. A radial deflection analysis may be used for a feasibility study.

c. Complete Trial Load Analysis

A complete trial-load analysis is carried out by properly dividing the radial, tangential and twists loads between the arch and cantilever elements until-agreement is reached for arch of the three axial and three rotational movements for each arch cantilever node point. The accuracy of this analysis is limited only by the exactness of the basic assumptions, the number of vertical and horizontal elements chosen, and the magnitude of the error permitted in the slope and deflection adjustments. In view of the comprehensive and involved nature of the complete trial-load analysis, it is desirable that preliminary studies of tentative dams are first carried out by simplified methods; crown cantilever analysis and radial deflection analysis, to obtain a dam; proposed for complete trial load analysis, which is most suitable for the given site and whose dimensions are as close to the final as practicable.
(iv) **3D Finite Element Analysis**

The deformations and stresses in an arch dam can alternatively be determined by three-dimensional finite element analysis which provides a more accurate solution of the problem and is being increasingly used. The finite elements can be extended to include the foundation and appropriate moduli values can be used whether the foundation is homogeneous or not, which avoids the use of Vogt's approximate assumptions on contact area and distribution of loading. According to Zienkewicz, the trial load method gives comparable results with 3D finite element analysis only for the simple cylindrical shapes. In doubly curved dams of modern type, the trial load assumptions are dubious and recent comparisons show that, in fact, considerable differences exist between its results and those of full 3D treatment.

### 1.4 Finite Element Method

Although the detailed analysis of an arch dam can be done by various methods as explained in Section 1.3, the most reliable and accurate method used is the finite element method. A theoretical prediction by using a mathematical model which contains a set of differential equations will be more reliable than a physical model; *experimental investigation*, because the small scale model may not always simulate all the features of the actual continua. For this purpose, it is appropriate to go for a mathematical description of the physical model; each ordinary as well as partial differential equations expresses a certain conservative principle. Here, for an arch dam, a three dimensional continua is to be analysed. Each variable inside the continua as well as its dependent variable must be in balance with various factors that influence the variable. Thus, for the quantitative description of physical phenomena an analyst go
for a system of partial and ordinary differential equations valid for a certain domain and impose on the system suitable boundary and initial conditions. Various forms of discretisations exist in which the infinite set of unknown functions is replaced by a finite number of unknown parameters.

Various numerical methods for continuum discretisations are finite difference, finite element and boundary element methods. Although finite element and finite difference methods discretise the continuum, both generate algebraic simultaneous equations to be solved for nodal degrees of freedom and of about the same accuracy. Finite element method is very much appropriate for the analysis of continua with irregular geometry and non homogeneous materials. Finite element method requires more computation time. This most accurate method, with most powerful modern computers, is used to analyse structural mechanics problems.

The merits of a computer aided finite element analysis of an arch dams are:

- The cost of a computer run will be far economical than a corresponding experimental investigation.
- Computational investigations will be of remarkable speed and designer can study the implications of different configurations faster and can choose the optimum design from among several possible designs.
- Computer solution gives detailed and complete information for all the relevant variables throughout the domain of interest.
- Realistic conditions can be simulated in the theoretical calculations and convergence achieved faster.
1.5 LITERATURE REVIEW

The researches done for the arch dam design, analysis, construction and functioning are summarized as under:

The search for the best shape for arch dams and the optimization of the shapes with relevant analysis methods taking into account the effect of curvature and constraints of construction evolved various solutions like constant/variable angle, constant/variable curvature, single/double/multiple curvature arches, membrane shapes etc. These aspects have been very critically studied, papers consolidated and published.[4] A numerical model for membrane shape is seen developed to find the equilibrium position of a membrane shell under external loadings and boundary conditions. The automatic checking of the stresses were then done by the general finite element programs like ADAP, SAP etc or dependable methods of analysis like shell theory and complete adjustment methods. US Commission on Large Dams has concluded the next iteration of reporting on the numerical analysis of dams in June 1999, indicates that computer simulation of dams continues to be a topic of pressing national interest.[8]

In the shape optimization studies, the main methods adopted for finite element mesh formulation and refinement were movements of selected boundary nodes, boundary shapes and spline functions which eliminates higher order polynomials.[4] Since conventional methods were used for analysis, the most common trial load method in which the dam as an assemblage of arches and cantilevers, the finite element mesh was also developed along vertical and horizontal directions in resemblance with the arches and cantilevers.[26,27] Criteria for analysis and design of arch dams have been dealt with by many engineers and methods evolved as explained in 1.3.
Extensive researches are seen in the area of rock mechanics, since the characteristics of the foundation strata has of vital importance in the stability, using boundary and block element methods. Recently block theory has been widely used for the elastic visco plastic and stochastic analysis of discontinuous rock masses.[28-32] Arch dams were also analyzed by block elements and coupled trial load and block element method where the arch dam is considered as an arch cantilever system and foundation as block element system.[33] Stability analysis of dam abutments by 3D elasto plastic finite element method is seen in a case study with Hough gravity arch dam in China.[34] Sliding and stability analysis of dam foundation as well as slope stability analysis with finite elements was attempted for the stability of rock slopes for foundation and abutments.[35-39] Simulation and stability of cracks and fractures have been experimentally as well as with crack model studied, stability checked.[40-44] Fracture analysis of concrete with discrete crack model was done by the boundary element method seepage analysis of crack and cracking analysis with a three dimensional finite element model.[45] In many of the conferences it has came a matter of importance the numerical analysis of arch dams and the ultimate strength evaluation numerically.[4-9] Shell elements were widely used for the finite element analysis of thin curved dams and for which software have been developed.[14,46,47]

Boundary element method has been found very effective in reservoir discretisation and modeling for seismic analysis. Fluid domain may be more conveniently handled by the boundary element method and dam water foundation interaction effects.[48-51] Dynamic analysis of arch dam including hydro dynamic effect and seismic analysis with boundary element method are also attempted.[52,53] The finite element and boundary element methods are seen combined for the fluid
structure interaction studies in time domain.[54,55] Shaking table test for fragments of gravity and arch dams were also carried out.[56,57] Dynamic response of arch dams in earthquake is experimentally and theoretically studied.[58-62] Researches are carried out for the seismic analysis which include dam water interaction and dam foundation interaction with instrumented structures case studies California and Kobe dam foundation rock interaction effects in frequency domain response functions.[63-66] Three dimensional analysis of spatially varying ground motions around a uniform canyon and impedance functions for foundation in a homogeneous half space.[67,68] The seismic response of a dam in frequency domain by boundary element, modal approach, (FE-HE)-BE model which allows for the rigorous representation of the dynamic interaction between the dam, foundation rock and water are studied.[69-74] Three dimensional fluid hyper element for dynamic analysis of concrete arch dams and impedance functions for three dimensional foundation supported on an infinitely long canyon of uniform cross-section in a homogeneous half-space are also studied.[75-77] Analysis of arch dam including seismic effect is attempted by continuum damage concrete model for gravity dam reservoir systems.[78-81] Distributed memory parallel element by element scheme based on Jacobi conditioned conjugate gradient for 3D finite element analysis is seen developed.[82]

Most investigators recently use Finite Element Method, a procedure by which a three dimensional continuum is approximated by an assemblage of discrete elements interconnected only at a finite number of nodal points having a finite number of unknowns, for the numerical simulation of the continuum. Detailed formulations of the FEM are given using isoparametric elements and three dimensional mapping techniques.[6,9,20,21,24] Zienkiewicz has developed an automatic mesh generator
scheme for plane and curved surfaces using isoparametric coordinates in which a complex region is divided into eight noded quadrilaterals which are viewed in the form of a rectangular pattern. [10] Saini in 1991, has attempted finite element static analysis for the Kasari dam. As two-dimensional analysis is an approximation in the transition zone of the dam where rapid changes occur in the thickness of the dam, in order to obtain a realistic picture of the stress distribution and the region of stress concentration, a three-dimensional finite element analysis was carried out using twenty noded elements. But this analysis is applicable for straight gravity dams only. Dynamic analysis was also done for the effect of seismic forces.

S. S Rao carried out a two-dimensional stress analysis for selected concrete dams by the finite element method and this was also intended for straight concrete gravity dams. [11] Bathe, Wilson, and Peterson in 1974 and Ghanaat in 1993 has developed two widely used software for finite element analysis of arch dams namely SAP-IV and GDAP. [14] SAP-IV is a general purpose finite element computer program for the static and dynamic analysis of linearly elastic structures and continua. This program has been designed for the analysis of large structural systems. Its element library for dam analysis includes eight node and variable number node, 3D solid elements. [16] The program can handle various static loads including hydrostatic pressures, temperature, gravity due to weight of the material, and concentrated loads applied at the nodal points. However, the program lacks pre and post processing capabilities. Thus, finite element meshes of the dam and foundation must be constructed manually from the input nodal coordinates and element connectivities.
GDAP has been specifically designed for the analysis of arch dams. It uses the basic program organization and numerical techniques of SAP-IV but has pre and post processing capabilities. It uses thick shell elements, which is represented by its mid surface nodes. The 16 noded shell element in the GDAP and general 3D solid elements of 8 and 21 nodes are found used for modeling the geometry. An appropriate finite element mesh for an arch dam can only be achieved by careful consideration of the dam geometry and the type of analysis for which the dam is modeled. For example, the finite element model of a double curvature thin shell structure differs from the model of a thick gravity arch section.[15] Other general purpose FEM programs such as ABAQUS (1988), NISA (2001), STAD PRO 2005 etc., can also be used in the analysis of three dimensional continuums by shell or tri linear quadratic elements, but not specific to the point of thick curved elements. One of the most important requirements in arch dam analysis is to develop accurate models, representative of the actual 3D behavior of the system.

The necessary Geometric Data for constructing a finite element mesh of an arch dam is obtained from drawings containing information defining the geometry of the dam shape.[7,9,15] These include the plan view and section along the reference plane. In practice, arch dams are geometrically described as multi centered arches with their centers varied by elevation in addition to the arch opening angles and radii varying for each side with elevation. Preparation of finite element mesh data from these geometric data is very time consuming especially for multi layer meshes because most general purpose finite element programs cannot directly handle these data which is another draw back.
1.6 LIMITATIONS OF THE RESEARCH

A specialized arch dam analysis program is necessary which can automatically generate coordinates of all nodal points, element data, element distributed loads and the nodal boundary conditions from the available limited geometric data. Though studies have been done for the analysis of arch dams by various numerical methods, the main ambiguity is whether the model represents the three dimensional geometry effectively; which then accompanies with a model study. Moderately thick arch dams are modeled essentially similar to the thin arch dams, except that 3D solid elements should be used near the base and the abutment regions where the shell behavior assumption becomes invalid due to excessive thickness of the arch whereas gravity arch dams should be modeled by two or more layers of solid elements in the thickness direction depending on their section thickness. It is very important to note that multilayer element meshes are essential to determine a detailed stress distribution across the thickness of a thick arch dam. A three dimensional automatic mesh generator is essential for the pre processing stage of such a multilayer discretization. A sketch showing half of an arch dam on Rock canyon is shown in Fig 1.1 and the plan and elevations in Fig 1.2 and 1.3 respectively.
The arch dam discretisation are seen as an assemblage of horizontal arches and vertical cantilevers. The 16 node shell and general 3D solid elements of 8 and 21 nodes provided for modeling the geometry are not sufficient to accurately define the shape of a thick variable curvature arch dam. *This limitation in shape approximation as explained above is to be resolved by the help of higher order polynomial shape functions including more number of nodes at element level relevant to the curve in each direction to take care of the curvature effect.*[24,26,83,84] The discretisation using tri quadratic brick element and vertical arch-cantilever system is shown in Fig 1.4.
In most of the studies and programs intended for concrete gravity dams as explained above, the hydrostatic pressure is considered to act normal to the surface and thus acts in the same direction for all the elements of that face. Even in the finite element method analysis studies for an arch dam, the load vectors assumed with certain coefficients for horizontal hydrostatic pressure. For an arch dam of complex geometry, the water pressure on each point acts in a direction normal to the surface; the surface being curved in both directions the magnitude as well as direction of water pressure varies from point to point; element to element depending on the height of water column. This normal hydrostatic pressure to the surface will have components in all the three directions which have to be dealt accurately in the finite element method rather using coefficients.[20] Similarly the pressure due to the accumulated silt is also to be considered. This aspect is to be effectively solved taking into consideration, the direction cosines of the pressure at each point and by numerical integration in the finite elements itself for which a program which resolves the pressure to all the three directions at a particular point is required. Fig1.5 gives a schematic representation of the distribution of loads and radial deflection of cantilevers in the Trial load method.
In addition, hydrodynamic pressure is also generated due to successive lateral movements of the upstream face of the dam against reservoir water occurring under earthquake. This pressure is found to be the same as that would occur if a body of water confined between a certain parabola and the face of the dam were forced to move with the dam while the rest of the reservoir remained inactive. Depending on the direction of peak ground acceleration, this hydrodynamic effect is to be incorporated as surface pressures at element level by numerical integration and direction cosines as well as inertial forces acting on the dam and water body. Arch dams constructed in the narrowest gorges in the river contour for forming the reservoir causes chances of generation of water waves due to gravity as well as wind effects. These formed waves on impinging causes energy dissipation or reflected standing waves. This dynamic effect of water pressure incorporated in the finite element program by suitable methods will improve the efficiency.

The other possible load vectors due to change in chemical properties with aging, temperature effects, silt pressure, uplift, fluctuation of reservoir level, gravity and body forces, seismic effect etc which affects deflection and stress values will have to be
considered in the finite element program.\[5,9,85,86\] This will be required for obtaining a reliable computer simulation.

It is felt from the above limitations that, apart from the general purpose software, a problem specific software for the 3D analysis of an arch gravity dam structure using quadratic hexahedral elements, defining the geometry accurately with the help of advanced computer programming technique with verification by plotting capable of handling various load vectors and applicable to a variety of structural mechanics problems is essential to be developed.

Hence the scope of the present work is to find out suitable methods for resolving the above limitations using finite element method and the development of a finite element analysis software for the elastostatic analysis of an arch dam having efficient pre and post processing capabilities equipped with the above load vectors which is capable of handling a variety of structural mechanic problems using the advanced object oriented programming technique: Visual C++, a simultaneous finite-element plotting program in Matlab corresponding to the Visual C++ program arrived with various options of plotting the original shape, deflected shape, sectional plan.\[87\] Established works of similar nature/case studies are intended for verification and comparison of results along with parametric studies.

1.7 OBJECTIVES

Objectives are formulated based on the above limitations as follows:
• To develop an accurate basic *Finite Element Software* having pre and post processing capabilities for three dimensional elasto static analysis of a solid continuum using linear and quadratic hexahedral elements capable of accommodating an arch dam geometry using the Object Oriented Programming Technique; *Visual C++*.  

• To develop a more reliable method for approximating an arch dam geometry of complexity; *variable curvature*, using three dimensional mapping with higher order polynomials of the required degree in the respective directions and to supplement with a problem specific *Automatic Mesh Generator* which gives two and three dimensional plots of the original and deflected profile directly in *Matlab*.  

• To develop a method to handle the varying *hydrostatic pressure* for each reservoir levels acting on the curved boundary of an arch dam in magnitude and direction from element to element so as to improve the result of analysis and further verification of the actual instrumentation values. Similarly, *silt pressure* as well as *hydrodynamic pressure* due to reservoir water during earthquake, *self weight* as body force due to gravity at element level by numerical integration and *inertial forces* of dam due to horizontal and vertical earthquake accelerations.  

• Validation of the Mesh Generator by plotting in *Matlab* and analysis software developed with linear quadratic elements and various load vectors by comparing the results with basic structural mechanics solutions and established similar works.  

• To study the applicability of the developed program in the effect of curvature, different loading conditions, varying reservoir levels; *parametric study*, with coarse and fine discretisation and comparison of the results such as deflections at nodal points, element and nodal stresses with established works.
1.8 METHODOLOGY

A flow diagram to arrive the above objectives of the research is shown below.

Finite element program is developed for a three dimensional solid continuum in *Visual C++* and plots using *Matlab*.

DEVELOPMENT OF FEA PROGRAM FOR 3D SOLID CONTINUA BY

08 NODED HEXAHEDRAL ELEMENTS
20 NODED HEXAHEDRAL ELEMENTS

PROGRAM VALIDATION
✦ USING THREE DIMENSIONAL EXAMPLES
✦ NODAL LOADS AND BOUNDARY CONDITIONS

MESH GENERATION USING 8, 20 & 27 NODED ELEMENTS
✦ LxTxH PARALLELEPIPED
✦ 2x2x2 CUBE WITH CENTRE (1,1,1)
✦ 2x2x2 CUBE WITH CENTRE (0,0,0)

VALIDATION OF MESH GENERATOR BY PLOTTING USING “MATLAB”

THREE DIMENSIONAL MAPPING OF THE GENERATED MESH TO AN ARBITRARY SHAPE USING:
✦ SHAPE FUNCTIONS
✦ LANGRANGE SHAPE FUNCTION OF ANY ORDER

VALIDATION OF THE MAPPED CONTINUUM BY PLOTTING

MODIFICATION OF THE PROGRAM TO ACCOMMODATE BODY LOAD VECTORS, SURFACE LOAD VECTORS AND VALIDATION

APPLICATION OF THE DEVELOPED PROGRAM AND VERIFICATION WITH ESTABLISHED RESULTS