Chapter 6

Summary of the Thesis

In this thesis, we have derived a class of exact solutions of Einstein’s field equations in Chapter 2, 3, 4, and 5 including dark energy solutions (stationary and non-stationary), Schwarzschild-dark energy, Reissner-Nordstrom-dark energy and Vaidya-de Sitter with variable $\Lambda(u)$. The differences of the solutions can be summarized in the forms of metric coefficient $g_{uu}$ as follows:

- Chapter 2. $\Delta^{(DE)} = 1 - 2mr$,
- Chapter 3. $\Delta^{(SchDE)} = 1 - \frac{2}{r}(M + mr^2)$,
- Chapter 4. $\Delta^{(RNDE)} = 1 - \frac{1}{r^2}\{2r(M + mr^2) - e^2\}$,
- Chapter 5. $\Delta^{(VDS)} = 1 - \frac{2}{r}M(u) - \frac{1}{3}r^2\Lambda(u)$.

The basic difference between the Reissner-Nordstrom-dark energy metric and Schwarzschild-dark energy metric is only in the definition of the “metric functions” $\Delta$s, that Schwarzschild dark energy has two roots of $\Delta^{(SchDE)} = 0$ in (3.2.28), whereas Reissner-Nordstrom-dark energy has three roots for $\Delta^{(RNDE)} = 0$ in (4.2.30). These roots provide the horizons of the embedded black holes. Accordingly the surface gravities and areas of embedded black holes on the horizons are also discussed in the respective chapters. We have
also shown the fact that every embedded black hole discussed in the thesis is expressible in Kerr-Schild ansatz. That the embedded black holes are solutions of Einstein’s field equations. Their physical properties are also investigated that the time-like vector fields of the matter distributions are expanding, accelerating, shearing and non-zero twist.

It is interesting to note that once the Reissner-Nordstrom black hole is embedded into dark energy space time, it will continue to embed forever without disturbing the nature of the dark energy background space. Thus, it follows the proof of the Theorem 14 stated in Chapter 4 “If an electrically radiating Reissner-Nordstrom black hole is embedded into dark energy space, it will continue to embed into the same space forever.” If we accepts the Hawking continuous evaporation of charged black holes introduced in Chapter 4, the loss of mass and creation of new mass are the process of the continuous radiation of Reissner-Nordstrom black hole. So, it may also be concluded that once electrical radiation starts, it will continue to radiate forever describing the various stages of the life of radiating black hole (Ibohal, 2005b).

We have introduced briefly the non-stationary de Sitter solution with variable $\Lambda(u)$ (Ibohal, 2009) in Section 5.2 of Chapter 5 for embedding into the Vaidya black hole. This enables to show the possibility of Guth’s modification (Guth, 1981) of energy-momentum tensor $T_{ab} \to T_{ab} + \Lambda g_{ab}$ for early inflation of the Vaidya black hole in non-stationary de Sitter space in equation (5.3.14).

In the concluding remark, it is worth to mention that the exact solutions discussed in chapter 2, 3, 4 and 5 are non-rotating solutions. They may be extended to the rotating ones, just like the non-rotating Schwarzschild solution to the rotating Kerr, non-rotating Reissner-Nordstrom to rotating Kerr-Newman, etc. The extended versions of the solutions may be seen elsewhere in future.

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