CHAPTER 5

GRW LOCALLY CLOSED SETS AND GRW LOCALLY CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

5.1 INTRODUCTION

The first step of locally closedness was done by Bourbaki (1966). He defined a set $A$ to be locally closed if it is the intersection of an open set and a closed set. In literature many general topologists introduced the studies of locally closed sets. Extensive research on locally closedness and generalizing locally closedness were done in recent years. Stone (1980) used the term FG for a locally closed set. Ganster and Reilly used locally closed sets in (1989) defined LC-continuity and LC-irresoluteness. Balachandran et al (1996) introduced the concept of generalized locally closed sets. Locally closed sets called $\approx g$-locally closed sets, $g$-lc* sets and $\approx g$-lc** sets are introduced and the properties of these concepts are studied as well as their relations to the other classes of locally closed sets are investigated by Ravi et al (2012).

In this chapter a new decomposition called GRW-locally closed sets, semi-GRW- closed set and functions called GRW-lc-continuous function and semi-GRW-continuous function in topological spaces are introduced and some of their properties are investigated.
5.2 GRWLC, GRWCL* AND GRWCL** IN TOPOLOGICAL SPACES

In this section new decomposition namely GRW-locally closed (GRWC) sets, semi-GRW-closed sets, GRWC* and GRWC** are defined and some of their properties are studied.

**Definition 5.2.1:** A subset S of a topological space \((X, \tau)\) is called GRW-locally closed set (briefly GRWlc set) if \(S = A \cap B\) where A is a GRW-open set and B is a GRW-closed set in \((X, \tau)\).

**Theorem 5.2.2:** If a subset S of a topological space \((X, \tau)\) is (i) locally closed ii) GLC then it is GRW-lc in X.

**Proof :** (i) Let S be a locally closed sub set of the topological space \((X, \tau)\), then it can be written as \(S = A \cap B\) where A is an open set and B is a closed set in \((X, \tau)\). Since an open set is a GRW-open and a closed set is a GRW-closed in X, A is GRW-open and B is GRW-closed in X. Hence S is GRW-locally closed.

ii) Let S be a GLC sub set of the topological space \((X, \tau)\), then it can be written as \(S = A \cap B\) where A is a g-open set and B is a g-closed set in \((X, \tau)\). Since a g-open set is a GRW-open set and a g-closed set is a GRW-closed in X, A is GRW-open and B is GRW-closed in X. Hence S is GRW-locally closed in X.

**Remark 5.2.3:** The converse of the theorem need not be true, from the following example.

**Example 5.2.4:** Let \(X = \{a, b, c, d\}\) be with a topology \(\tau = \{\emptyset, \{a\}, \{a, b\}, X\}\). Then \(\{a, b\}\) is GRW-lc but it is not locally closed and not GLC in X.
Theorem 5.2.5: A subset $S$ of a topological space $(X, \tau)$ is (i) LC if and only if it is GRWlc, provided $X$ is a $T_{GRW}$-space. (ii) A subset $S$ of a topological space $(X, \tau)$ is GLC if and only if it is GRWlc, provided $X$ is a $T_{GRW}$-space.

Proof: (i) Assume that $S$ is LC. Let $S = A \cap B$ where $A$ is open and $B$ is closed in $X$. But open set $A$ is GRW-open and closed set $B$ is GRW-closed in $X$. Thus, $S$ is GRWlc. Conversely assume that $S$ is GRWlc. Let $S = A \cap B$ where $A$ is GRW-open and $B$ is GRW-closed in $X$. Since $X$ is a $T_{GRW}$-space, $A$ is open and $B$ is closed in $X$. Hence $S$ is LC.

(ii) Assume that $S$ is GLC. Let $S = A \cap B$ where $A$ is g-open and $B$ is g-closed in $X$. But g-open set $A$ is GRW-open and g-closed set $B$ is GRW-closed in $X$. Thus, $S$ is GRWlc. Conversely assume that $S$ is GRWlc. Let $S = A \cap B$ where $A$ is GRW-open and $B$ is GRW-closed in $X$. Since $X$ is a $T_{GRW}$-space, $A$ is open and $B$ is closed in $X$, which implies that $A$ is g-open and $B$ is g-closed in $X$. Hence $S$ is GLC.

Theorem 5.2.6: If $A$ is GRWlc in $X$ and $B$ is GRW-open in $X$ then $A \cap B$ is GRWlc in $X$.

Proof: Since $A$ is GRWlc, $A = P \cap Q$, where $P$ is GRW-open and $Q$ is GRW-closed in $X$. Now $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B) = P \cap (B \cap Q) = (P \cap B) \cap Q$. Since $P$ and $B$ are GRW-open, $P \cap B$ is also GRW-open and $Q$ is GRW-closed. Hence $A \cap B$ is GRWlc in $X$.

Definition 5.2.7: A subset $S$ of a topological space $X$ is called GRW lc* if $S = P \cap Q$ where $P$ is GRW-open and $Q$ is closed in $X$.

Definition 5.2.7: A subset $S$ of a topological space $X$ is called GRW lc** if $S = P \cap Q$ where $P$ is open and $Q$ is GRW-closed in $X$. 
Theorem 5.2.8: In a topological space \((X, \tau)\), \(\text{GRWlc}^*\)-sets and \(\text{GRWlc}^{**}\)-sets are \(\text{GRWlc}\)-sets in \(X\).

**Proof:** If \(A\) is \(\text{GRWlc}^*\), then there exists a GRW-open set \(P\) and a closed set \(Q\) in \(X\) such that \(A = P \cap Q\), but closed sets are GRW-closed sets hence \(A\) is GRWlc-set. If a sub set \(B\) is \(\text{GRWlc}^{**}\) then, \(B = P \cap Q\), \(P\) is open and \(Q\) is GRW-closed, but open sets are GRW-open hence \(B\) is GRWlc set in \(X\).

Theorem 5.2.9: If \(A\) is \(\text{GRWlc}^*\) in \(X\) and \(B\) is a GRW-open set in \(X\), then \(A \cap B\) is \(\text{GRWlc}^*\) in \(X\).

**Proof:** Since \(A\) is \(\text{GRWlc}^*\), there exists a GRW-open set \(P\) and a closed set \(Q\) in \(X\) such that \(A = P \cap Q\); Now \(A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q\). Since \(P\) and \(B\) are GRW-open, \(P \cap B\) is also GRW-open and \(Q\) is closed. Hence \(A \cap B\) is GRWlc*.

Theorem 5.2.10: If \(A\) is \(\text{GRWlc}^{**}\) in \(X\) and \(B\) is an open set in \(X\) then \(A \cap B\) is \(\text{GRWlc}^{**}\) in \(X\).

**Proof:** Since \(A\) is \(\text{GRWlc}^{**}\), \(A = P \cap Q\), where \(P\) is a open set and \(Q\) is a GRW-closed set in \(X\). Now \(A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q\). It is given that \(B\) is open. Therefore \(P \cap B\) is open set in \(X\). Hence \(A \cap B\) is \(\text{GRWlc}^{**}\) in \(X\).

Theorem 5.2.11: If \(A\) is \(\text{GRWlc}^{**}\) in \(X\) and \(B\) is a GRW-open set in \(X\) then \(A \cap B\) is \(\text{GRWlc}\) in \(X\).

**Proof:** Since \(A\) is \(\text{GRWlc}^{**}\), \(A = P \cap Q\), where \(P\) is an open set and \(Q\) is a GRW-closed set in \(X\). Now \(A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q\). It is given that \(B\) is GRW-open. Therefore \(P \cap B\) is GRW-open set in \(X\). Hence \(A \cap B\) is \(\text{GRWlc}\) in \(X\).
**Theorem 5.2.12:** A subset $A$ of a topological space $X$ is GRWlc*, then there exists a GRW-open set $P$ such that $A = P \cap \text{cl}^*(A)$.

**Proof:** Let $A$ be a GRWlc* set. Then there exists a GRW-open set $P$ and a closed set $Q$ such that $A = P \cap Q$. Since $A \subseteq Q$ and $Q$ is closed, $A \subseteq \text{cl}^*(A) \subseteq Q$. Also, $A \subseteq P$ and $A \subseteq \text{cl}^*(A)$ together implies $A \subseteq P \cap \text{cl}^*(A)$. On the other hand, take $x \in P \cap \text{cl}^*(A)$. Then $x \in P$ and $x \in \text{cl}^*(A) \subseteq Q$. So, $x \in P \cap Q = A$. Hence $P \cap \text{cl}^*(A) \subseteq A$. Therefore $A = P \cap \text{cl}^*(A)$.

**Remark 5.2.13:** Converse of the above theorem need not be true from the following example.

**Example 5.2.14:** Let $X = \{a, b, c, d\}$ be with the topology $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\} = X \cap \text{cl}^*(\{b, c, d\})$, but $\{b, c, d\}$ is not GRWcl*.

**Theorem 5.2.15:** A subset $A$ of a topological space $X$ is GRWlc* if and only if there exists a GRW-open set $P$ such that $A = P \cap \text{cl}(A)$.

**Proof:** Let $A$ be a GRWlc* set. Then there exists a GRW-open set $P$ and a closed set $Q$ such that $A = P \cap Q$. Since $A \subseteq Q$ and $Q$ is closed, $A \subseteq \text{cl}(A) \subseteq Q$. Also, $A \subseteq P$ and $A \subseteq \text{cl}(A)$ together implies $A \subseteq P \cap \text{cl}(A)$.

On the other hand, take $x \in P \cap \text{cl}(A)$. Then $x \in P$ and $x \in \text{cl}(A) \subseteq Q$. So, $x \in P \cap Q = A$. Hence $P \cap \text{cl}(A) \subseteq A$. Therefore $A = P \cap \text{cl}(A)$. Conversely assume that $A = P \cap \text{cl}(A)$, where $P$ is GRW-open and $A$ is a subset of a topological space $X$. Here $\text{cl}(A)$ is a closed set, therefore $A$ is GRWlc*.

**Theorem 5.2.16:** If a subset $A$ of a topological space $X$ is GRWlc**, then there exists an open set $P$ such that $A = P \cap \text{cl}^{GRW}(A)$. 
**Proof:** If A is a GRWlc**, then there exist an open set P and a GRW-closed set Q such that $A = P \cap Q$, which implies that $A \subseteq \text{cl}^{GRW}(A) \subseteq Q$ and $A \subseteq P \cap \text{cl}^{GRW}(A)$. Also, if $x \in P \cap \text{cl}^{GRW}(A)$, then $x \notin Q$ and $x \notin P$. Therefore $x \notin P \cap Q = A$. Hence $P \cap \text{cl}^{GRW}(A) \subseteq A$. Therefore $A = P \cap \text{cl}^{GRW}(A)$.

**Remark 5.2.17:** Converse of the above theorem need not be true from the following example

**Example 5.2.18:** Let $X = \{a, b, c, d\}$ be with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $\{a, c\} = X \cap \text{cl}^{GRW}(\{a, c\})$, but $\{a, c\}$ is not a GRWccl** in X.

**Theorem 5.2.19:** If A and B are any two GRWlc*-sets in a topological space X, then $A \cap B$ is GRWlc* in X.

**Proof:** Since A and B are GRWlc*-sets, by the theorem 5.2.15, there exists GRW-open sets $P$ and $Q$ such that $A = P \cap \text{cl}(A)$ and $B = Q \cap \text{cl}(B)$. Therefore $A \cap B = (P \cap Q) \cap (\text{cl}(A) \cap \text{cl}(B))$. Since $P \cap Q$ is GRW-open and $\text{cl}(A) \cap \text{cl}(B)$ is closed, $A \cap B$ is GRWlc*.

**5.3 GRWLC-CONTINUOUS FUNCTIONS AND GRWLC – IRRESOLUITE FUNCTIONS**

In this section GRWlc-continuous functions and GRWlc-irresolute functions are defined and some of their properties are studied.

**Definition 5.3.1:** A function $f: X \rightarrow Y$ from a topological space X in to a topological space Y is called GRWlc-continuous if for each open set $V$ in Y, $f^{-1}(V)$ is GRWlc.
Definition 5.3.2: A function \( f: X \to Y \) from a topological space \( X \) in to a topological space \( Y \) is called GRWlc-irresolute if for each \( V \) in \( \text{GRWlc}(Y) \), \( f^{-1}(V) \subseteq \text{GRWlc}(X) \), where \( \text{GRWlc}(X) \) is set of all GRWlc sets in \( X \).

Theorem 5.3.3: If a function \( f: X \to Y \) is LC-continuous, then it is GRWlc-continuous but not conversely.

Proof: Assume that \( f \) is LC-continuous. Let \( V \) be a closed set in \( Y \). Then \( f^{-1}(V) \) is locally closed in \( X \). But, every locally closed set is GRWlc. Thus \( f^{-1}(V) \) is GRWlc in \( X \). Therefore \( f \) is GRWlc-continuous.

Remark 5.3.4: The converse of the above theorem need not be true as seen from the following example.

Example 5.3.5: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, X\} \) and \( \sigma = \{\emptyset, \{b\}, \{a, b\}, \{a, b, c\}, Y\} \) Let \( f: X \to Y \) be an identity function, \( f \) is GRWlc-continuous. But it is not LC-continuous, since for the closed set \( V = \{a, c\} \) in \( Y \), \( f^{-1}(V) \) is not locally closed in \( X \).

Theorem 5.3.6: If a function \( f: X \to Y \) from a topological space \( X \) into a topological space \( Y \) is GLC-continuous then it is GRWlc-continuous.

Proof: If \( f: X \to Y \) is GLC-continuous and \( V \) is a closed set in \( Y \), then \( f^{-1}(V) \) is GLC set in \( X \). Therefore by definition \( f^{-1}(V) = A \cap B \), where \( A \) is g-open and \( B \) is g-closed in \( X \). Since every g-open set is GRW-open, \( A \) is GRW-open and since every g-closed set is GRW-closed, \( B \) is GRW-closed. Hence \( f^{-1}(V) \) is GRWlc in \( X \), so \( f \) is GRWlc-continuous.

Theorem 5.3.7: If a function \( f: X \to Y \) from a topological space \( X \) into a topological space \( Y \) is GRWlc-irresolute, then it is GRWlc-continuous.
**Proof:** Let $V$ be an open set in $Y$, so $V \in \text{GRWlc}(Y)$. Since $f$ is GRWlc-irresolute, $f^{-1}(V) \in \text{GRWlc}(X)$. Therefore $f$ is GRWlc-continuous.

**Remark 5.3.8:** The converse of the above theorem need not be true from the following example.

**Example 5.3.9:** Let $X = \{a, b, c, d\}$ be with the topology $\tau = \{\emptyset, \{a\}, \{b\},\{a, b\},\{a, b, c\}, X\}$. Let $Y = \{a, b, c, d\}$ be with the topology $\sigma = \{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, Y\}$. Let $f$ be the function $f: X \to Y$ defined as $f(a) = a$, $f(b) = b$, $f(c) = d$, $f(d) = c$. $f$ is GRWlc-continuous but not GRWlc-irresolute. Since $f^{-1}(\{a, d\}) = \{a, c\}$, the set $\{a, d\}$ is GRWlc but $\{a, c\}$ is not GRWlc in $X$.

**Remark 5.3.10:** From the above discussion following observation is made as it is the Figure 5.1.

\[
\begin{array}{ccc}
\text{LC-continuous} & \rightarrow & \text{GRWlc-continuous} \\
\text{GRWlc-irresolute} & \leftarrow & \text{GLC-continous}
\end{array}
\]

A $\rightarrow$ B means A implies B $\leftarrow$ B means A does not imply B.

**Figure 5.1 Relationships of GRWlc-continuity**

**Theorem 5.3.11:** If a function $f: X \to Y$ is GRWlc-continuous and $A$ is a GRW-open subset of $X$, then the restriction $f|_A : A \to Y$ is GRWlc-continuous.

**Proof:** Let $V$ be an open set in $Y$. Since $f$ is GRWlc-continuous, $f^{-1}(V)$ is GRWlc in $X$. $f^{-1}(V) = P \cap Q$, where $P$ is open and $Q$ is GRW-closed in $X$. Now $(f|_A)^{-1}(V) = f^{-1}(V) \cap A = (P \cap A) \cap Q$. But $P \cap A$ is GRW-open in $X$ and therefore the restriction $f|_A$ is GRWlc-continuous.
Theorem 5.3.12: (i) Let $f: X \to Y$ from a topological space $X$ into a topological space $Y$ is GRWlc-continuous and $B$ be a open subset of $Y$ containing $f(X)$, then $f: X \to B$ is GRWlc-continuous.(ii) If $f: X \to Y$ and $g: Y \to Z$ are GRWlc-irresolute, then the composition $g \circ f: X \to Z$ is GRWlc-irresolute.(iii) If $f: X \to Y$ is GRWlc-continuous and $g: Y \to Z$ is continuous, then the composition $g \circ f: X \to Z$ is GRWlc-continuous.

Proof: (i) Let $V$ is an open set in $B$. Since $B$ is an open subset of $Y$, the set $V$ is open in $Y$ and $f$ is GRWlc-continuous, $f^{-1}(V)$ is GRWlc in $X$. Therefore $f: X \to B$ is GRWlc-continuous.(ii) Let $V$ be a GRW-open set in $Z$. Since $g$ is GRWlc-irresolute, $g^{-1}(V)$ is GRW-open in $Y$. Also, since $f$ is GRWlc-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is GRW-open hence $g \circ f$ is GRWlc-irresolute. (iii) Let $V$ be an open set in $Z$, then $g^{-1}(V)$ is open in $Y$, since $g$ is continuous, but $f$ is GRWlc-continuous, so $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is GRWlc(X), so $g \circ f$ is GRWlc-continuous.

5.4 SEMI-GRW-CLOSED SETS AND SEMI-GRW-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

In this section semi-GRW-closed sets and semi GRW-continuous functions and semi-GRW-irresolute functions in Topological Spaces are introduced. Further semi GRW-locally closed set, semi-GRWlc-continuous function and semi-GRW-irresolute function are introduced and some of their properties are studied.

Definition 5.4.1: A subset $A$ of $X$ is called semi-GRW-closed set (briefly sGRW-closed) if $\text{cl}^*(\text{int}(A)) \subseteq U$ where $A \subseteq U$ is RSO in $X$. Complement of sGRW-closed set is sGRW-open in $X$.

Remark 5.4.2: Since $\text{int}(A) \subseteq A$, GRW-closed sets are sGRW-closed but the converse need not be true.
**Remark 5.4.3:** Since $\text{int}(A) = A$ for an open set, sGRW-closed sets are GRW-closed in $X$ when it is open.

**Example 5.4.4:** Let $X = \{a, b, c, d\}$ be with the topology $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. The set $\{a\}$ is sGRW-closed but not GRW-closed.

**Definition 5.4.5:** A subset $A$ of $X$ is called semi GRW locally closed set (briefly sGRWlc) if $A = P \cap Q$, where $P$ is sGRW-open and $Q$ is sGRW-closed.

**Theorem 5.4.6:** If a subset $S$ of $X$ is locally closed in $X$ then it is sGRWlc in $X$ but not conversely.

**Proof:** Assume that $S$ is locally closed in $X$. Then $S = A \cap B$ where $A$ is open and $B$ is closed in $X$. Since $A$ is open implies sGRW open and $B$ is closed implies $B$ is GRW closed, hence it is sGRW-closed in $X$. Therefore $S$ is sGRWlc in $X$.

**Remark 5.4.7:** The converse of the above theorem need not be true as seen from the following example.

**Example 5.4.8:** Let $X = \{a, b, c, d\}$ be with the topology $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. The set $\{c, d\}$ is sGRW-locally closed but not locally closed.

**Definition 5.4.9:** A function $f: X \to Y$ is said to be sGRW-continuous if the inverse image of every open set in $Y$ is sGRW-open in $X$.

**Definition 5.4.10:** A function $f: X \to Y$ is said to be sGRW-irresolute if the inverse image of every sGRW-closed set in $Y$ is sGRW-closed in $X$.

**Definition 5.4.11:** A function $f: X \to Y$ is said to be sGRWlc-continuous if the inverse image of every open set in $Y$ is sGRW-locally closed in $X$.  

**Definition 5.4.12:** A function \( f: X \to Y \) is said to be sGRWl- irresolute if the inverse image of every sGRWlc set in \( Y \) is sGRWlc in \( X \).

**Theorem 5.4.13:** If a function \( f: X \to Y \) is continuous, then it is sGRW-continuous but not conversely.

**Proof:** Assume that \( f: X \to Y \) is continuous. Then by definition, \( f^{-1}(V) \) is closed in \( X \), whenever \( V \) is closed in \( Y \). Hence \( f^{-1}(V) \) is closed in \( X \), but closed sets are GRW-closed and hence it is sGRW closed. Therefore \( f \) is sGRW-continuous.

**Remark 5.4.14:** The converse of the above theorem need not be true as seen from the following example.

**Example 5.4.15:** Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\} \) and \( \sigma = \{Y, \phi, \{c\}, \{a, c\}, \{a, b, c\}\} \) If \( f: X \to Y \) is an identity function then, \( f \) is sGRW-continuous. But it is not continuous, since for the open set \( V = \{a, c\} \) in \( Y \), \( f^{-1}(V) \) is not open in \( X \).

**Theorem 5.4.16:** If a function \( f: X \to Y \) is GRW-continuous, then it is sGRW-continuous.

**Proof:** Let \( f: X \to Y \) be GRW-continuous. Let \( V \) be a closed set in \( Y \). Since \( f \) is GRW-continuous, \( f^{-1}(V) \) GRW-closed in \( X \), but GRW-closed sets are sGRW closed, thus \( f^{-1}(V) \) is sGRW closed. Hence \( f \) is sGRW-continuous.

**Theorem 5.4.17:** If a function \( f: X \to Y \) is g-continuous, then it is semi-GRW-continuous but not conversely.

**Proof:** Let \( f: X \to Y \) be g-continuous. Let \( V \) be a closed set in \( Y \). Then \( f^{-1}(V) \) is g- closed in \( X \). So \( f^{-1}(V) \) is GRW-closed and hence sGRW-closed. Therefore \( f \) is sGRW-continuous.
**Remark 5.4.18:** The converse of the above theorem need not be true as seen from the following example.

**Example 5.4.19:** Let \( X = Y = \{a, b, c\}, \quad \tau = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, X\} \) and \( \sigma = \{\phi, \{c\}, \{a, c\}, \{a, b, c\}, Y\} \). If \( f: X \to Y \) is an identity function then, \( f \) is sGRW-continuous. But it is not g-continuous, since for the open set \( V = \{a, c\} \) in \( Y \), \( f^{-1}(V) \) is not g-open in \( X \).

**Remark 5.4.20:** The following examples prove that semi-continuous functions are independent of sGRW-continuous function.

**Example 5.4.21:** Let \( X = Y = \{a, b, c, d\}, \quad \tau = \{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\} \) and \( \sigma = \{Y, \phi, \{c\}, \{a, c\}, \{a, b, c\}\} \). If \( f: X \to Y \) is an identity function then it is not sGRW-continuous, but it is semi-continuous.

**Example 5.4.22:** Let \( X = Y = \{a, b, c\}, \quad \tau = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\} \) and \( \sigma = \{Y, \phi, \{c\}, \{a, c\}, \{a, b, c\}\} \). Let \( f: X \to Y \) be an identity function. The function \( f \) is sGRW-continuous function which is not semi-continuous, since for the open set \( V = \{a, c\} \) in \( Y \), \( f^{-1}(V) \) is not semi open in \( X \). The set \( \{a, c\} \) is open in \( Y \) where as \( f^{-1}(\{a, c\}) \) is not semi open.

**Theorem 5.4.23:** If a function \( f: X \to Y \) is semi-GRW-irresolute, then it is semi-GRW-continuous but not conversely.

**Proof:** Let \( f: X \to Y \) be sGRW-irresolute. Let \( V \) be a closed set in \( Y \).

Then \( f^{-1}(V) \) is sGRW closed in \( X \), since \( V \) is sGRW closed and \( f \) is sGRW-irresolute. Therefore \( f \) is sGRW-continuous.

**Remark 5.4.24:** The following example gives proves that the converse of the above theorem need not be true.
Example 5.4.25: Let $X = Y = \{a, b, c, d\}$ be with the topology $\tau = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, c\}, \{a, b, c\}\}$, let $f: X \rightarrow Y$ be an identity function. $f$ is sGRW-continuous, which is not a sGRW-irresolute, since $\{c\}$ is sGRW-closed but $f^{-1}(\{c\}) = \{c\}$ is not sGRW-closed.

Remark 5.4.26: From the above discussion, the following implication is obtained as it is in Figure 5.2.

\[ \text{continuous} \rightarrow \text{g-continuous} \rightarrow \text{sGRW-irresolute} \]
\[ \text{sGRW-continuous} \]
\[ \text{semi-continuous} \rightarrow \text{GRW-continuous} \]

A $\rightarrow$ B means A implies AB $\rightarrow$ means A does not imply B.

Figure 5.2 Relationship of sGRW-continuity

5.5 SEMI-GRW LOCALLY CLOSED SETS AND SEMI GRW LC-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

In this section semi-GRW locally closed sets, semi-GRWlc-continuous functions and semi GRWlc- irresolute functions are introduced and some of their properties and relations with other functions are studied.

Definition 5.5.1: A subset $S$ of a topological space $X$ is called sGRWlc if $S = A \cap B$ where $A$ is sGRW-open and $B$ is sGRW-closed in $X$. The set of all sGRWlc sets of $X$ is denoted by sGRWlc($X$).
**Definition 5.5.2:** A subset $S$ of a topological space $X$ is called sGRWlc* if $S = A \cap B$ where $A$ is sGRW-open and $B$ is closed in $X$. The set of all sGRWlc* sets of $X$ is denoted by $sGRWlc*(X)$.

**Definition 5.5.3:** A subset $S$ of a topological space $X$ is called sGRWlc** if $S = A \cap B$ where $A$ is open and $B$ is sGRW-closed in $X$. The set of all sGRWlc** sets of $X$ is denoted by $sGRWlc**(X)$.

**Definition 5.5.4:** A function $f: X \to Y$ from a topological space $X$ into a topological space $Y$ is called:

(i) sGRWlc-continuous if $f^{-1}(V) \subseteq sGRWlc(X)$ for each open set $V$ in $Y$.

(ii) sGRWlc*-continuous if $f^{-1}(V) \subseteq sGRWlc^*(X)$ for each open set $V$ in $Y$.

(iii) sGRWlc**-continuous iff $f^{-1}(V) \subseteq sGRWlc**(X)$ for each open set $V$ in $Y$.

(iv) sGRWlc*-irresolute if $f^{-1}(V) \subseteq sGRWlc^*(X)$ for each $V$ in $sGRWlc^*(Y)$.

(v) sGRWlc**-irresolute if $f^{-1}(V) \subseteq sGRWlc**(X)$ for each $V$ in $sGRWlc**(Y)$.

**Theorem 5.5.5:** Let $f: (X, \tau) \to (Y, \sigma)$ from a topological space $X$ into a topological space $Y$. (i) If $f$ is locally continuous then it is sGRWlc-continuous, sGRWlc*-continuous and sGRWlc**-continuous. (ii) If $f$ is sGRWlc*-continuous or sGRWlc**-continuous then it is sGRWlc-continuous. (iii) If $f$ is GLC-continuous then it is sGRWlc-continuous. (iv) If $f$ is GRWlc-continuous then it is sGRWlc-continuous.
Proof: (i) Let V be an open set in Y, since f is locally continuous, \( f^{-1}(V) \) is a locally closed set in X, that is \( f^{-1}(V) \subseteq \text{sGRWlc}(X) \), hence f is sGRWlc continuous. (ii) Let V be an open set in Y, since f is sGRWlc*-continuous or sGRWlc**-continuous, \( f^{-1}(V) \subseteq \text{sGRWlc}^*(X) \) or \( f^{-1}(V) \subseteq \text{sGRWlc}^{**}(X) \) that is \( f^{-1}(V) \subseteq \text{sGRWlc}(X) \) in both the cases, hence f is sGRWlc continuous. (iii) Let V be an open set in Y, since f is GLC-continuous, \( f^{-1}(V) \subseteq \text{GLC}(X) \subseteq \text{sGRWlc}(X) \), hence f is sGRWlc continuous. (iv) Let V be an open set in Y, since f is GRWlc-continuous, \( f^{-1}(V) \subseteq \text{GRWlc}(X) \subseteq \text{sGRWlc}(X) \), hence f is sGRWlc continuous.

Example 5.5.6: Let \( X = Y = \{a, b, c\} \), \( \tau = \{ \phi , \{a\} , \{a, b\}, \{a, b, c\} \} \) and \( \sigma = \{ \phi , \{c\} , \{a, c\} , \{a, b, c\} \} \). Let f: \( X \to Y \) be the identity function. It is sGRWlc-continuous, which is not LC-continuous, since for the open set \( V = \{a,c\} \) in Y, \( f^{-1}(V) \) is not locally closed in X.

Example 5.5.7: Let \( X = Y = \{a, b, c, d\} \) be with the topology \( \tau = \{ X , \phi , \{b\} , \{c\} , \{a, b\} , \{b,c\} , \{a, b, c\} \} \) and \( \sigma = \{ X , \phi , \{a\} , \{a,c\} \} \). Let f: \( X \to Y \) be an identity function, it is sGRWlc-continuous, which is not GLC-continuous.

Theorem 5.5.8: Let \( f: (X, \tau) \to (Y, \sigma) \) be a function from a topological space X into a topological space Y. (i) If f is sGRWlc*-irresolute, then it is sGRWlc*-continuous (ii) If f is sGRWlc**-irresolute, then it is sGRWlc**-continuous. (iii) If f and g are two sGRWlc*-irresolute functions or sGRWlc**-irresolute functions then their compositions are also sGRWlc*-irresolute or sGRWlc**-irresolute function.

Proof: Let f: \( (X, \tau) \to (Y, \sigma) \) and g: \( (Y, \sigma) \to (Z, \delta) \) be two functions which are sGRWlc*-irresolute. If \( V \subseteq \text{sGRWlc}^*(Z) \), then \( g^{-1}(V) \subseteq \text{sGRWlc}^*(Y) \), f is sGRWlc*-irresolute then \( f^{-1}(g^{-1}(V)) = (gf)^{-1}(V) \subseteq \text{sGRWlc}^*(X) \). Hence g \( \sqsubseteq f \) is a sGRWlc**-irresolute function from X into Z. Similarly if f and g are
sGRWlc**-irresolute functions, \( f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \subseteq s\text{GRWlc}**(X) \), whenever \( V \subseteq s\text{GRWlc}**(Z) \), hence \( \text{gof} \) is also \( s\text{GRWlc}**\)-irresolute function from \( X \) into \( Z \).

**Remark 5.5.8:**

Now the following conclusion is made as it is in Figure 5.3.

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![Diagram](attachment:image.png)

**Figure 5.3 Relationships of sGRWlc*-continuity**

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### 5.6 CONCLUSION

In this chapter generalization of GRW-closed sets, semi GRW-closed sets, GRWlc*-sets, GRWlc*-sets and GRWlc**-sets are introduced and studied. Further sGRW-continuity, sGRWlc-continuity, sGRWlc* -continuity and sGRWlc**-continuity have been defined and few of their relationship with lc-continuity and GLC-continuity were observed. In the next chapter, this study can be used in GRW-closed sets and GRW-continuity in bitopological spaces.