3.1 Introduction

The literature review section of this thesis is divided into several sub-sections starting from broader to narrow one as per the pattern of collection of literature related to the title. The researcher has opted to use Detective Funnel method to collect the literature for this work. On the basis of this method the researcher narrowed down this study towards Black-Scholes Option Pricing Model. The study has been further focussed towards Black-Scholes Option Pricing Model Assumptions. After conducting in-depth study of Black-Scholes Option Pricing Model Assumptions, the researcher found certain gaps and discrepancies in the calculation of Risk Free Interest Rate valuation because no perfect method has so far been used to calculate risk free interest rate. In present scenario most of the derivative market stock exchanges follow different methods to calculate risk free interest rate because it varies from country to country due to different economic levels and situations. In the Indian derivative market different experts follow different risk free interest rate to calculate the value of call and put option according to the formula of Black-Scholes Option Pricing Model. After giving basic idea related to the structural framework of Literature Review, the researcher developed literature review structure model, which is given below in figure 3.1:

![Literature Review Collection Process](image)

**Figure 3.1**
Literature Review Collection Process

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On the basis of the above figure 3.1 the researchers can easily understand the structural blueprint of literature review section of this thesis, in which the researcher began from Derivative Basic and in the next step the researcher has attempted to define derivative product such as Forward, Futures, Options and Swaps and try to define its working mechanism and valuation process in chapter one. After that the researcher again narrowed down this study towards option pricing model under which various options pricing model such as Finite Differences Method, Monte-Carlo Simulation, Binomial Method, Black-Scholes Model, The GARCH Option Pricing Model, Jump Diffusion Model and Artificial Neural Network (ANN) model have been detailed. While studying all these option pricing model the researcher found research gap in Black-Scholes Option Pricing Model in risk free interest rate variable because this model was based on various assumptions and risk free interest rate is also one of the major assumption of this model.

3.2 Derivative Literature

A derivative is defined by the BIS (1995) as “a contract whose value depends on the price of underlying assets, but which does not require any investment of principal in those assets. As a contract between two counterparts to exchange payments based on underlying prices or yields, any transfer of ownership of the underlying asset and cash flows becomes unnecessary”. This definition is rigorously associated to the capability of derivatives of replicating financial instruments [See S. Neftci (2000) for mathematical details]. The exchange-traded derivatives’ markets assure all requirements of transparency, liquidity and risk monitoring and are looked at and prescribed by the Exchange Trade Authority and the Clearing House (BIS, 1995). Derivatives are financial instruments which are extensively used by all economic agents to invest, speculate and hedge in financial market (Hull, 2002). These functions are rigorously associated with the financial and mathematical explanation of instruments and do not reflect on the economic stuffing of financial assets. Savona (2003) also highlighted that derivative market is also worked as economic function in the segment of reducing the risk factor related to the investor point of view. The exchange-traded derivatives market fulfilled all requirements of intelligibility, liquidity and risk monitoring and are looked at and controlled by the Exchange Trade Authority and the clearing House (BIS 1995). The BIS and IMF set out the rules for protected and resonance market structure. Various Central Banks have also made it obligatory to
Chapter: 3

include capital ratios for bank and financial instruments to include derivatives. At the present time these markets do not pretend any particular well being problem most of all after 1987, when a crash of exchange traded options gave rise to the control and monitoring activity. In addition, derivatives are exceptional or excellent substitute of complex investment strategies at a low cost which help investor for investment (Haugh & Lo, 2001). Those firms which were actively using derivative instrument showed different risk portfolio as against the non- using firms (Hentschel and Kothari, 2001). Those banks which were using interest rates derivative instrument experienced a greater growth in their commercial and industrial loan portfolio in contrast of those banks which were not using such kind of derivative instruments (Brewer, Minton and Moser, 2000), The trading size of OTC derivative has been stressed by Hogan and Malmquist (1999), which are on the other hand, consistent with profit-maximization. Peek and Rosengreen (1996) analysed uncertainties in the derivative trading activities in those banks which were facing financial trouble. This result was due to risk loving behaviour which manifested into unmonitored moral hazard. Herrera and Schroth (2002) found that the un-interrupted creating of the different types of derivatives by financial institutions will be done in the absence of any exclusive rights shelter, this position confirmed that the returns on this investment is higher for the creator of the innovation, and also pays back the research and developments expenditures as well. According to Barrieu and Karoui (2002) study the available statistics and analyses of the exchange traded derivatives on the basis of divergent and they found the growth of Over The Counter (OTC) derivatives to have an exponential velocity and might cause some logical problems as it would be almost impossible to quantify and control the risks related measures. According to Hunter and Marshall (1999), the introduction of derivatives might affect the risk of financial markets from a macroeconomic approach. Risk can be divided in systematic and non systematic risk and on the basis of this parameter the risk can be diversified and consequently lowered, and it is also not affected by portfolio diversification. On the basis of macro-prudential approach, the risk related with international exchange traded derivative is settled by BIS regulation, where as risk, associated with OTC derivative is settled basically by code of conduct and self regulation (Kroszner, 1999). Donmez and Yilmaz (1999), state that a mature derivatives market on an organised exchange leads to an improved risk management and better allocation of resources in the economy. This finding was also confirmed by Hunter and Marshall (1999) who
verified that derivatives trading may increase informational efficiency of financial markets and make available instruments for more valuable risk management.

In the current literature, there seems to be no clear confirmation on the subject of increase of risk, either systematic or non-systematic, in the non-attendance of shocks; in presence of exogenous shocks, they tend to intensify the effects, according to their different risk propensity. Hunter and Marshall (1999) and Hunter and Smith (2002) emphasize the important association between systematic risk and derivatives. The central bank can act as a liquidity supplier for financial markets in the case of systematic risk. By means of the preface of derivatives, markets are more ideal as a result it influences monetary policy actions (Vrolijk, 1997); the surprise cause is no longer a way to influence markets for the reason of the unfeasibility to compensate their enormous liquidity (Von Hagen and Fender, 1998). The credit can be substituted by derivatives, as shown by Fender (2000) and Gorton and Rosen (1995). They also state that credit channel relies its power on market imperfections, whichever on the information side or the money side; with derivative it slowly but surely loses its importance. Donmez and Yilmaz (1999) analysed many dramatic incidents involving derivatives markets concluding, “They do not seem to create new risk but only change the type structure and nature of the existing”. Morales (2001) looking at emerging market, noted that there seems to be no confident evidence about the real danger coming from derivative markets and in addition they also found that derivatives tend to incorporate faster than the spot market, and that the beginning of restrictions on emerging financial markets increase risk, by increasing the cost of investment and moving capital abroad. Tinsley (1998), Rossetti (1998) and others explained the advantages for central bank in using derivatives to manage the exchange and interest rates, most of all in the absence of liquid primary market.

The definition of money base used by monetary authority influences directly the symphony of money aggregation; using the analytic definition of money base (Fratianni and Savona, 1972) and it also gives the econometric result on derivatives and their property of reacting with interest rates, the inclusion of derivatives into money aggregation should be clear-cut. Savona and Maccario (1988), Savona, Maccario and Oldani (2000) and Oldani (2002) have tested various derivative market products and its reaction property, and concluded that certain derivative product give meaningful result related to investment risk and hedging.
The exercise of derivatives by the government or any other unrestricted agencies to manage debt (or balance due) and lower its cost is a very essential field of study, especially in Europe. The data about this trading or hedging action are not available, and the study of Piga (2001) on the use of Swap by some government is only available research at present. His conclusions are quite encouraging, because the uses of derivatives decrease the cost of the debt service and lower the need for further debt overturn. According to Hull (2002), Wong (2000) and Anson (2001) the compensation of using derivatives for tax savings are difficult to illustrate in general. Different banks and firms in numerous countries undertake to take care of another way of derivatives losses or gains if they come from hedging or speculation. The tax saving is better for hedging, and the firms and banks might have a tendency to declare losses and gain in this form to a certain extent than speculation. In broad-spectrum, the problem of derivatives being off-balance sheet items poses numerous barriers to an absolute analysis of fiscal effects for all economic agents. Breuer (2000) has calculated off-balance sheet leverage for financial institutions in order to provide absolute information about risks’ exposure and capital adequacy, even though at a combined level of market or institutions.

Now a day’s derivative market are increasing tremendously, thousands of derivative contract are traded in the market place. According to Whaley (2006), 16% derivative contracts are traded in worldwide stock exchange and remaining 84% are private contract arranged by banks and other financial institution or houses. The Wall Street Journal also examined that derivative contract traded in US are approximately or roughly equal to the derivative contract traded in worldwide. Fouque, Papanicolaou & Sircar (2000) found that, trading in derivative market is more risky because by using futures hedging, risk segment does not include potential loss of capital amount, but also include potential loss of opportunity. Taylor (2007) stated that if two contracts have strong correlation between them, arbitrage possibilities, were higher.

3.3 Forward Literature
Whaley (2006), defined that forward contract is particularly a simple derivative product, it is an agreement or contract to buy or sell underlying assets at a certain futures for a certain price, no money changes hands until the expiration date, at which price the buyer pays the amount of cash specified in the contract and seller delivers the underlying assets. Mc Donald and Robert (2006), defined that forward contract is the systematic process in the risk of loss for one mirrors the
According to Quirk et al. (1988), in forward currency markets risk is shifted to agents, who are willing to assume it and required return on any transaction is positively related to its level of risk. Ethier (1973) introduced the separation theorem and the full-hedge theorem under exchange rate uncertainty, showing that demand for forward contracts absolutely compensates uncertainty. Benninga et al. (1985) and Kawai and Zilcha (1986) furthermore discussed price level uncertainty in forward contract and also obtained the same results which was discuss by Ethier but this small result has been subject to some speculations. Viaene and Zilcha (1998) also consider additionally output and cost uncertainty and it was also found that in forward contract set-up full double hedge and separation theorem fail to hold. Adam-Muller (2000) introduces inflation risk which cannot be hedged away and it also found that full hedged and separation break down if two sources of risk in the model are not statistically independent. Carse et al. (1980) and other researchers have shown that roughly only one-third of the value of international trade is covered by forward contract. According to Hau and Rey (2003), only eight percent of US equity holding out of the country is hedged beside exchange rate risk. In addition, there exists an ongoing debate in the empirical literature as to whether exchange rate volatility depresses trade levels or not. This debate is related to the issue of demand for forward contract in that often the disagreement is made that as long as agents have access to strapping forward market. Basically this uncertainty should not matter. This result is also justified by the empirical evidence provided by Cote (1994) and Wei (1998).

Allaz and Villa (1993) have done the first theoretical examination of the competition and its enhancing outcome on forward trading. On the basis of his theoretical work Green (1999) analyses the case of two leading producers of electricity who can lift up spot price well above marginal cost which is profitable for them in the nonexistence of forward contracts. In the process of fully hedge concept of forward market, the producers lose their incentive to raise prices above marginal costs. Brandts, Pezans-Christou and Schram (2003) use experiments to study the efficiency effects of adding the possibility of forward contract to a pre-existing spot market. Basically these people deal independently with the cases where spot market competition is in quantities and it also worked on supply function of forward contract. Le Coq and Orzen (2006) also studied ‘Allaz and Villa’ prediction in a controlled laboratory background and their findings support the main comparative-static predictions of the model but also recommend the
competition enhancing effect of a forward market, which is more weaker than predicted. There is some controversy about ‘Allaz and Villa’s’ result as per Hughes and Kao (1997). They found that, if the contract position is not perfectly observed by the other players, such that one’s contract choice has no influence on the others’ strategy, then the firm has no more incentive to sell forward contracts. Harvey and Hogan (2000) and Kamat and Oren (2004) doubt that the competition enhancing effect holds if firms play the game over and over again, as is unquestionably the case in most real markets. These people argue that a dynamic setting may facilitate firms to commit to keeping their forward contract position to minimum. Ferreira (2003) studies an oligopolistic industry where firm are able to sell in futures market at considerably many moments prior to the spot market.

3.4 Futures Literature

Taylor (2006), claimed that futures contract were traded around 100 years ago and were originally based on agriculture commodities but now a days, it’s of many kinds. Whaley (2006), said that futures contract are generated from forward contract, only basic difference is that gains or losses on futures contract are posted each day. According to Schwager (1984), trading in financial derivative involves commodities and financial instruments for a futures delivery date, as opposed to present time. It also examines that futures market are expended to other market such as currencies, interest rates instruments and stock indexes. As per the research of Hirani (2007), the futures contract being traded on organised exchanges report liquidity to the operation. In this process clearing houses, being their counter party to both side of transaction, provide a mechanism that guarantees the honouring of the contract and ensuring very low level of non-payment default.

Futures derivative contract include five type of derivative product such as

- Currency Futures;
- Foreign Exchange Futures;
- Stock Index Futures;
- Commodity Futures; and
- Interest Rate Futures;

In all of the above futures derivative market product have similar kind of working mechanism which was followed by forward derivative market contracts but the only difference is that the
futures derivative market products are organised in nature and are managed by registered stock exchange, as also shown in the statement of above futures derivative market literature. Most of the basic literature of futures derivative market is similar to forward derivative market. Only in case of default payment, futures derivative market play positive role due to the nature of organised trading platform which was not provided by forward derivative market contract.

3.5 Options Literature
Whaley (2006), defined that option contract is also a part of forward contract but the only difference is that, the buyer of option has the right but not the obligation to buy or sell the underlying assets at option expiration. Option has been measured to be the most dynamic subdivision of the security market since the commencement of the Chicago Board Option Exchange (CBOE) in April 1973, with more than one million contracts per day, CBOE is the chief and Business option exchange in the world. After that, more than a few other option exchanges such as London international financial futures and Option Exchange (LIFFE Euro-Next) had been set up for options trading.

3.5.1 Review of Options Pricing Literature
Prior to 1970’s, many researchers were interested in the calculation of the value of call or put options but till 1970’s there was no perfect method to calculate value of call and put option. In 1973, Black and Scholes published their influential article on option pricing. After that there has been an immeasurable explosion of theoretical and empirical examination on option pricing. Despite the fact, that Black and Scholes have maintained the assumption of Geometric Brownian Motion in most of their papers, various researchers have been interested in checking the validity of this Black-Scholes formula in various empirical testing. During the testing process of Black-Scholes Model formula various researchers have identified and suggested minor modification in their formula but no one can deny the powerful outcome of this formula which represents the main domain of the calculation process of call or put valuation.

Black and Scholes (1972) themselves tested the result of their formula by using the data of over-the-counter market (OTC) and they found that the result of their formula gives lower value than actual market value. Merton (1973) extended the Black formula and showed that the basic form of model was the same if the payment structure was increased or lifted, the interest rate is
stochastic and the option is exercisable prior to maturity. Black and Scholes (1973) used Itos’s lemma mathematical tools, which are used to calculate particular type of stochastic process. It also helps in the derivation of B-S formula. Rosenberg (1973) proposed that it follows an autoregressive scheme. Thorp (1973) has shown that the Black-Scholes formula still holds if there are limitations on the use of the proceeds of short sales. Blattberg and Gonedes (1974) recommend stochastic and random volatility of the underlying stock. Merton (1975), Cox and Ross (1975) have shown that if the return on ordinary stock do not follow a stochastic process with a continuous path, this basically showed a convinced form of Jump process, the hedge mechanism that was used by Black-Scholes will not be suitable. Ingersoll (1975) alluded to the tax question but did not develop the effect of taxes on listed call option. Cox, Ross and Rubenstein (1976) derived the tree method of pricing option, based on risk-neutral valuation, the binomial option pricing model pricing European option prices under various alternatives, including the absolute diffusion, pure-jump, and square root constant of variance model. Galai (1977) used the data of Chicago Board Option Exchange and found that the excess daily returns on the hedged portfolio are significantly different from zero, and addition of one percent transaction cost eliminates the positive excess return. Latane and Rendleman (1976), Mac-Beth and Merville (1979) solved the B-S formula in the form of implied variance rates by taking a sample of period 1975-76 and found a result that strike price is biased but this bias was exactly opposite of what has been reported by Black.

According to the existing literature, Black-Scholes Option Pricing Model was also tested on Australian Option Market (AOM) by Brown (1978), thereafter two other Australian papers, the first by Chiarella and Hughes (1978) and the second, by Brown and Shevlin (1983a) were also published. Brown and Shevlin found that empirical testing of Black-Scholes option pricing model on Australian Option Market (AOM) shows positive result in case of lower volatility segment of underlying assets in long run of continuously compounded rate of returns. Similar kind of results were also found by Frino et al. (1991) after testing option pricing model on Australian Option Market.

Bhattacharaya (1980) tested BSM under ideal conditions. He initiated the hedge by buying or selling the call at the model price and thus eliminated the efficiency of the option market and used the actual variance from the investigation period and thus isolated from the measurement of formula inputs, and reported the over valuation of model in case of at-the-money option under a
maturity of less than three weeks. On the other hand, near-the-money option is undervalued by the model, and this valuation error decreases as the time of maturity increases. Beckers (1980) experienced and tested the Black-Scholes assumption that the historical instantaneous volatility of the underlying stock is a function of the stock price, using S&P 500 index options 1972-1977 and he also found that the underlying stock is an inverse function of stock price. Geske, Roll and Shastri (1983) found that these differences are created due to imperfect protection of dividend in the OTC market.

The GARCH (Generalised Autoregressive Conditional Heteroskedastic) process of Bollerslev (1986) and its variants have gained greater than ever distinction for modelling financial time series in recent years and GARCH process is a generalized version of the ARCH by Engle (1982), and this model was also independently proposed by Taylor (1986). The GARCH option pricing model has three unique descriptions.

**First:** The GARCH option pricing is a function of the risk premium surrounded in the underlying assets.

**Second:** The GARCH option pricing model is non-Markovian (It is well known that a non-Markovian univariate process can be converted into a Markovian vector process through a change in dimension).

**Third:** The GARCH option pricing model can potentially make clear in a few well documented papers that systematic biases associated with the Black-Scholes model. These biases take account of under option pricing of out-of-the-money option, under-pricing of option on squat volatility securities, under-pricing of short-maturity option and U-shaped understood volatility curve in relation to exercise price (see Black 1975, Gultekin et al. 1982, Black and Scholes 1972, Rubinstein 1985 and Sheikh 1991 for more details). The GARCH process was used by Engle and Mustafa (1992) to study options and their implied condition volatility effect on option price. Duan (1990) also attempted to provide a careful theoretical foundation for option pricing in the GARCH frame work to easily understand GARCH process of option valuation. Afterwards Satchell and Timmermann (1992) and Amin and Nag (1993) projected option pricing models in the GARCH outline, invalidating the risk-neutral valuation relationship. The GARCH process is complex and complicated in nature due to that reason a generalised version of risk neutralization, referred to as the Locally Risk-Neutral Valuation.
Relationship (LRNVR) has been proposed. This version is a little bit easier than GARCH process.


Shimko (1993) argues with the result of S&P 500 index distribution and was of the view that it is negatively skewed and more leptokurtic than a lognormal distribution. Trautmann and Beinert (1994) calculate and measure the parameters of a jump-diffusion process on German Capital Market, in opposition to Black-Scholes Model. They have come across to a result that option prices generated through a jump-diffusion model are not equivalent to those obtained from the Black-Scholes Model. Jackwerth and Rubinstein (1996) came to the conclusion that the S&P 500 index futures distribution before the collide or break down of 1987 cash of Deutsche Market which resembles the lognormal distribution, despite the fact that the post crash distribution exhibits leptokurtosis and negative skewness (For additional confirmation, see Bates 1991 &1996 and Dumas, Fleming and Whaley 1998). Bates (1996), study the Deutsche Market options from 1984 to 1991 and found that the stochastic volatility model is not able to explain the volatility smile. After that Bakshi, Cao and Chen (1997) also used the data of S&P 500 index option from 1998 to 1991 and found that the magnitude of the volatility smile is negatively associated to the maturity. The maturity period was less than 60 days for this research and they observed in plain sight smiles for the three alternative models: Stochastic volatility, Stochastic volatility with jumps and Stochastic volatility with stochastic interest rate.

As we know from the existing literature that the Black-Scholes formula for theoretical pricing of options show signs of certain organized biases, as experimental prices in the market differ from the formula. Due to this problem a number of studies attempted to reduce these biases by incorporating a modification in the existing Black-Scholes Model or try to change the working mechanism in the input data. On the other hand ‘Artificial Neural Networks’ are found to be a capable alternative in this area of research. Hutchinson et. al. (1994) used three Artificial Neural Network (ANN) Model and also study their performance in American Style call or put options.
and they found that the Artificial Neural Network (ANN) models give better result in comparison to Black-Scholes Option Pricing Model. Geigle and Aronson (1999) have studied the routine of Artificial Neural Network models in American Style option on S&P 500 futures and they also advocate that ANN models give more positive result compared to Black-Scholes Option Pricing Model. Sachenberger (1993) also passed through some process but found that Black-Scholes Model was superior for in-the-money options and the Artificial Neural Network models perform quite well for out-of-the-money options. Ghaziri et. al. (2000), Saito and Jun (2000) and many others compared the performance of ANN models in European style call and put option, and came out with the result that the ANN models can give more positive result as compared to Black-Scholes Option Pricing Model. Lajbcygier et. al. (1996) used three Artificial Neural Network models in pricing American type call or put option similar to Hutchinson approach on Australian Share Price Index futures and found that the ANN models were less capable to the presumption based models in broad-spectrum but in case of near-the-money of short maturity period the ANN models were more effective. Similarly Anders et. al. (1998) also found the performance of ANN models better than the performance of Black-Scholes Model on European type DAX call or put options. Yao et. al. (2000) used ANN models on American type call or put options on Nikkei 225 futures of Japanese Market and he also came to same conclusion that ANN models give better result as compared to Black-Scholes Model in various number of places however, it does not mean that Black-Scholes formula is not good enough to calculate the value of call and put option. Saxena (2008) worked on CNX Nifty option in the Indian stock market and found that the ANN model was able to generate non-linear relationship underlying the Black-Scholes Model. On this basis more specific value of options can be calculated which help investors to generate more profit in derivative market segment.

In this option pricing literature the researcher has tried to cover few references on barrier option pricing. Basically barrier option pricing is of two types, which have recently appeared in the over-the-counter market. Both barrier options are the advanced version of European type up-and-out call or put options. The barrier options concept was first seen in the published work of Snyder (1969) and the first valuation of a barrier option appears with the valuation of down-and-out call option in the influential paper by Merton (1973). Bergman (1983) developed the framework of barrier options in pricing path-contingent claims on the valuation of options. The research papers by Benson and Daniel (1991), Hudson (1991) and Reiner and Rubinstein (1991)
focus on those cases which were not measured in Merton research, such as up-and-out call options or down put options. The research of Bowie and Carr (1994) looks at static hedging of barrier options and similar research was also conducted by Derman, Ergener and Kani (1993). Ultimately, more specific paper was published by Heynen and Kat (1994). They covered ‘outside’ barrier options i.e. barrier options which are knocked in or out when a second asset crosses a barrier. On the basis of this result barrier options are also called ‘contingent options’ which was also mentioned by Rich (1993), despite the fact that they are termed as ‘rainbow barrier options’ as various research papers belong to this area. Boyle and Lau (1994), and Kat and Verdonk (1994) covered the valuation of barrier options in the binomial model. The articles by Boyle and Lee (1993), Carr and Ellis (1994), and Duanmu (1994) covered the valuation of barrier options on that point when volatility jump at the barrier. The valuation of double barrier options is covered in Beaglehole (1992), Kunitomo and Ikeda (1992), and in Bhagavatula and Carr (1995). Chesney and Gibson (1994), and Rich and Leipus (1994) covered the application of barrier option valuation to corporate finance. Heynen and Kat (1994) covered those cases in which the valuation of barrier options when the barrier is not vigorous and the option’s entire life is covered.

Duan (1996) uses a GARCH model to price call or put options on FTSE 100 index and he also demonstrates that the Black-Scholes implied volatilities equivalent to the prices generated by the GARCH model similar to the Black-Scholes Model for most options. On the other hand Das and Sundaram (1999) point out that if we incorporate this feature it mitigates but does not eliminate the problem. They draw attention toward the skewness and Kurtosis which was not generated by the jump-diffusion and stochastic volatility model. Eberlein, Keller and Prause (1998) use a complex hyperbolic function for the distribution of underlying returns and find that the smile and the time-to-maturity effects are concentrated in assessment to the Black-Scholes model but it was not completely eliminated. They propose that options that are not many face extra risk for instance as liquidity and accordingly are high-priced. Further studies conducted by Longstaff (1995), Dumas et al. (1998), and Pena et al. (1999) reported that transaction costs and liquidity contributed to find volatility smile but it was not able to completely explain the volatility smile. Constantinides (1998) finds the transaction costs but it cannot explain the volatility smile. Leland (1985) also argues that transaction cost should depress the values of near-the-money option more than those of away from-the-money options. One intuitive explanation for this proposition is that
at-the-money option have higher gamma, and as a result it would need to be dynamically hedged more frequently than in- and out-of-the-money options.

Buraschi and Jackwerth (2001) carried out statistical tests based on instantaneous model and stochastic models using S&P 500 index option data from 1986 to 1995. On the basis of their analysis on given data they concluded that the data is more consistent with models that include supplementary risk factor such as stochastic volatility and jump-diffusion. According to the study of Yang (2006) the inferred volatilities used to assessment exchange traded call option on the AXS 200 index are impartial and better to historical on the spot volatility in forecasting futures realised volatility. Liu (2007) found an uncertainty theory that is a branch of mathematics based on normality, monotonicity, self-duality and countable subadditivity axioms. Khan et al., (2012) suggested modification in Black-Scholes Option Pricing Model on the basis of risk-free interest rate assumption without its practical implementation and developed new method to calculate risk free interest rate. After that Khan et al., (2013) tested their modified formula of Black-Scholes Option pricing model with respect to original Black-Scholes Option Pricing Model formula and found that modified Black-Scholes formula give little bit positive result with respect to original Black-Scholes formula by changing the value of risk free interest rate on the basis of new technique of risk free interest rate calculation. One of the meaningful outcomes of Black-Scholes equation is the fact that the expected return on the underlying assets, and the expected return on the derivative itself, does not appear in the equation.

### 3.5.2 Review of Option Pricing Models.

Option pricing calculation contains various models which suggest various types of option calculation pattern and style but common factor is that every model was based on the foundation of Black-Scholes Option Pricing Model. On the basis of our understanding and research the researcher has given below various option pricing models and has also tried to discuss its important features and formula structures. These models are:

- Black-Scholes Model;
- Jump Diffusion Model;
- Binomial Model;
- Monte-Carlo Simulation Model;
3.5.2.1 Black-Scholes Model

Financial derivative is a core area of financial mathematics, under financial derivative Black-Scholes Option Pricing Model shows suitable use of financial mathematics to derive the formula of valuation of Call and Put option but the derivation of Black-Scholes formula is more problematic in nature due to the use of Partial Differential Equation (PDE), Partial Differentiation, Definite Integration, Logarithmic Properties, Exponential, Maxima & Minima and other mathematical function. On the basis of formula complication, researchers and other scholars who are interested in understanding the valuation formula of BSM derive different types of meanings and conclusions. Black-Scholes formal is a joint work of two persons Fischer Black and Myron Scholes, maximum contribution to B-S model is by Fischer Black but due to less knowledge of Partial Differential Equation (PDE) he failed to derive final result. On the other hand Myron Scholes’s work on PDE on the movement of stock price helped him to contribute to the field. Finally in 1973, both of them worked together and shared their ideas, and developed Black-Scholes Option Pricing formula for Call and Put option valuation.

Structure of Black-Scholes Option Pricing Formula for Call and Put Option Given as:

\[
v_{\text{cop}} = \text{PN} \left( \frac{\log_{e} P_{E} + (R_{t} + \frac{1}{2} \sigma^{2})t}{\sigma \sqrt{t}} \right) - e^{-R_{t}t} \text{PE} \left( \frac{\log_{e} P_{E} + (R_{t} - \frac{1}{2} \sigma^{2})t}{\sigma \sqrt{t}} \right)
\]

and

\[
v_{\text{pop}} = e^{-R_{t}t} \text{PE} \left( - \left( \frac{\log_{e} P_{E} + (R_{t} - \frac{1}{2} \sigma^{2})t}{\sigma \sqrt{t}} \right) \right) - \text{PN} \left( - \left( \frac{\log_{e} P_{E} + (R_{t} + \frac{1}{2} \sigma^{2})t}{\sigma \sqrt{t}} \right) \right)
\]

where:

- \( P \) = Current or Spot price of underlying stock at time zero;
- \( P_{E} \) = The exercise or execute or strike price;
- \( R_{t} \) = Risk free interest rate or Risk free rate of returns;
$N = \text{The value of the cumulative normal distribution at } \left( \frac{\log \left( \frac{p_{F/E} + (R_f + \frac{1}{2} \sigma^2)t}{e^{-\frac{1}{2} \sigma^2 t}} \right) - \frac{1}{2} \sigma^2 t}{\sigma \sqrt{t}} \right) \text{ and } \left( \frac{\log \left( \frac{p_{F/E} + (R_f + \frac{1}{2} \sigma^2)t}{e^{-\frac{1}{2} \sigma^2 t}} \right) + \frac{1}{2} \sigma^2 t}{\sigma \sqrt{t}} \right)$;

$t = \text{The time remaining to expiration on an annual basis;}

\exp(-R_f t) = 1 + (-R_f t) + \frac{(-R_f t)^2}{2!} + \frac{(-R_f t)^3}{3!} + \frac{(-R_f t)^4}{4!} \ldots \ldots \ldots \ldots \approx 2.7183; \text{ and}

\sigma = \text{The standard deviation of the continuously compounded annual rate of return or long-
returns of the share and commonly referred to as volatility;}

Above BSM formula is based on various assumptions details of which are given below and it further more summarised by Wilmott (1999)

- Trading in stock take place continuously and market are always open;
- Interest rate risk is well-known and constant over the life of option;
- The stock pays no dividend on any type of underlying assets or security;
- There is zero transaction cost on buying and selling of assets in option contract;
- There is no provision of any type of tax;
- The stock price follows a Geometric Brownian motion process with $\mu$ and $\sigma$ as constant;
- The assets are completely divisible in nature;
- There are no penalties on short selling of shares and investor also get full use of short-
sell procedure; and
- Stock price option follows an explicit type of stochastic process called diffusion
process;

It is a logical approach to pricing options. The Black-Scholes Model captures most of the property but some restrictions with practical function have to be taken into consideration. The hypothesis of being able to set-up a risk-less hedge by rebalancing endlessly and immediately is not reasonable in real trading because contract or transaction cost barricade continuous process and scatter investment returns. Price changes in different circumstances can be quite considerable and hypothesis of risk-free rate is idealistic.

3.5.2.1.1 Factor Effecting Option Pricing
On the basis of existing literature, it was found that there are six parameters which influence the
value of option pricing. All these factors were also highlighted by different models of options
pricing formula. These six parameters were also identified by the Black-Scholes formula in
1973. Black-Scholes formula was a foundation stone for the revaluation of option pricing
calculation formula. After that various keen researchers upgraded this formula or also provided
various new methods to calculate the value of options pricing but in every section of various
options pricing model which the researcher discussed above, the researcher found that most of
the research parameter are always closely related or associated with these six parameters. The
only difference is that style of using these parameters may change in symbolic representation or
different methods or technique were used to calculate the parameter value.

These six major parameters affecting the stock option pricing as per Hull (2002) are:

1. The Current Stock Price (P);
2. The Strike Price (P_E);
3. The Time to Expiration (t);
4. The Volatility of The Stock Price (\sigma);
5. The Risk-Free Interest Rate (R_f); and
6. The Dividend Expected During The Life of Option;

The current stock price and strike price are two extremely noticeable factors affecting the options
pricing. In case of live trading call is worth zero, while the stock trades below the strike price and
is equal to the stock price minus the strike price when it is traded above. The current price is
obtained from stock exchange’s current underlying stock trade price and the strike price is
obtained from option exchange quotes. Next deep-seated factor is the time to maturity or expiry
of options. This is basically a time on which the options period has expired. In NSE derivative
market time of expiry of options derivative product starts from one, two and maximum three
month and it is calculated on yearly basis. This can also be obtained from option exchange
quotes.

Fourth important factor is the volatility (\sigma) of the stock because if we see the structure formula
of every model of option, we find that the volatility symbol (\sigma) is always available with the
format. The volatility of the stock shows uncertainty in the pattern of movement of stock price.
With the help of volatility parameter investor easily understands the price movement of the stock
or it also helps to forecast futures price. Conversely, this is limited down risk in the occurrence
of price decrease because most of the proprietors can get loses is the price of option. Correspondingly the proprietor of a put benefits from price decrease but has limited downside in the occurrence of price increase. The value of both calls and puts as a result increase as volatility increases. (See: Gross and Dai, 2006)

The fifth parameter is related to Risk-Free interest rate. This will also affect the pricing of options value by means of the time value of money in which the payoff is received in futures. The present value of any cash flow operation is mostly affected by risk free rate. In practical point of view related to the economic concept, if the interest rate in the economy decreases, then the expected return required by investor from the stock also decreases. For that reason the present value of futures cash flow received by the holder of the option increase. The researcher has discussed this parameter separately in the next section because this parameter is the main domain of this research work, in which with the help of this parameter the researcher has tried to modify Black-Scholes Option Pricing Formula.

The sixth and the last parameter that affect the option pricing is dividend. This is given during the life of the options. In post dividend date the stock price goes up and conversely, it reduces the stock price on ex-dividend date. The value of call options are negatively related to the size of any expected dividend while the value of a put option is positively related to the size of any expected dividend for more understanding read Hull (2002).

### 3.5.2.1.2 Risk Free Interest Rate Literature

According to the title of this research thesis ‘The Analysis of Black-Scholes Option Pricing Model and Its Effectiveness in The Valuation of Options’, The researcher has to study the Black-Scholes Option Pricing Model. In this model, the researcher has to give more focus on all assumptions on which the Black-Scholes Model was developed. In the previous section, the researcher discussed the factors affecting the option pricing. In these factors there is a risk-free interest rate \((R_f)\), which was also the major assumption in Black-Scholes Model. While reviewing the literature related to the option pricing along with the working pattern formula and structure of various models of options, the researcher did not find any specific method to calculate the value of risk-free interest rate \((R_f)\). As per the existing literature it is different in different economies as the US market the US Treasury Bill rate is recommended as risk-free interest rate and in European market government bond rate is taken as risk-free interest rate. In
Indian scenario most of the investors and brokers, who are trading in Indian derivative market take risk-free interest rate closely related to the bank’s interest rates. According to the views of various financial experts who are trading in NSE derivative market they have taken risk-free interest rate approximately or closely equal to 7.75% for the month of June in the financial year 2013-2014. Now a days when anybody is interested in calculating the value of call or put option, there are several online free option calculators available, in which anybody has to put the value of all these parameters which are given in previous section and get result within a minute but in these online calculator software’s, risk-free interest rate will be determined by the individual himself. As per the research most of the people have used different levels of risk free interest rates ranging from 7.2 to 8.5 percent during the period ranging from 1st April 2013 to 30th September 2013. It depends on the personal perception of the individual. More specific information related to the risk-free interest rate is given below in detail as per the past literature.

**Risk-Free Interest Rate** is the hypothetical or theoretical rate of return of an investment with no risk of financial or monetary loss. One explanation is that the risk-free rate represents the interest that an investor would expect from an absolutely risk-free investment over a given period of time. One more explanation is that the risk free interest rate is the reimbursement with the intention that it would be demanded by a representative investor holding a representative market portfolio. In calculation of risk free interest rate you have to add all the assets in the economy, it is also the recommendation for systemic risk which cannot be eliminated by holding a diversified portfolio. In another elucidation which was applied in the Capital Asset Pricing Model given by Campbell, Lo and MacKinlay (1996) the risk free interest rate can be obtained with no risk, it is understood that any supplementary risk taken by an investor has to be compensated with an interest rate higher than the risk-free interest rate.

According to the perception and research experience of Malcolm Kemp (2009), the risk-free interest rate means different things to different people in his or her point of view and there is no compromise on how to go about a direct measurement of risk free interest rate. On the other hand, the most common interpretation is aligned to Fisher's concept of inflationary expectations (which was perfect or not), described in his dissertation 'The Theory of Interest' (1930) which is based on the hypothetical or theoretical costs and settlement of holding currency. In Fisher’s model risk free interest was described by two potentially offsetting arrangements, first: expected increases in the money deliver rate have to be result in investors preferring present consumption
to futures income, second: expected increases in efficiency have to result in investors preferring futures income to present utilization.

The straightforward explanation is that the risk free interest rate might be either positive or negative and in tradition the sign of the expected risk free interest rate is an institutional principle; this is the resultant to the disagreement that was explained by Tobin and Golub (1998) he says it is a system with endogenous money foundation and where constructive decisions and outcomes are decentralized and potentially inflexible to forecasting, this investigation provides support to the perception that the risk free interest rate may not be directly observable or calculated. Conversely, it normally examines, that for people applying this rationalization, the measurement of holding currency is more often than not hypothetical as being positive. It is not clear what the proper starting point for this perception. Although it possibly relates to the convenient prerequisite of a few structure of currency to support the specialization of calculation of risk free interest rate which was further explained by Smith (2005). On the other hand it should be observed that ‘Smith’ did not provide an 'upper limit' to the desirable level of the risk free interest rate and he did not fully explain the issues related to the calculation of risk free interest rate at national or international level in various economic condition.

A substitute (less well developed) explanation is that the risk free interest rate represents the time prediction of a representative worker for a representative basket of utilization. Once more, there are reasons to consider with the purpose of the given circumstances that the risk free interest rate may not be directly observable or calculated.

Agreed on the hypothetical confusion around this matter, in tradition most industry practitioners rely on some form of substitute for the risk free interest rate, or use other forms of standard rate which are presupposed to have as features similar or equivalent to the risk free interest rate plus some risk of default as per Kemp’s (2009), this approach also include some issues, which are discussed in the next section of this research. The additional discussions on this concept of a 'risk free interest rate' are presented in the research work of Campbell, Lo and Mackinley (1996).

The return on nationally held short term government bonds is usually supposed as a good substitute for the risk free interest rate. However, theoretically this is no more than correct if there is no perceived risk of default associated with the bond. Government bonds are traditionally measured to be comparatively risk-free to a household proprietor of a government bond, for the reason that there is by explanation no risk of non-payment. The bond is a form of
government responsibility which is being discharged in the course of the payment of another form of government responsibility which was related to the domestic currency (see Tobin and Golub, 1998). This implies the risk of the government 'printing additional money' to convene the commitment is perceived as a form of tax, moderately than a form of default. For example the ‘inflation tax’, is often discussed as similar concept to that of ‘seigniorage’ (Seigniorage meaning according to Plender (2012) in Financial Times: The difference between the value of money and the cost to produce it - in other words, the economic cost of producing a currency within a given economy or country. If the seigniorage is positive, then the government will make an economic profit; a negative seigniorage will result in an economic loss).

The similar contemplation may not essentially be relevant to a foreign proprietor of a government bond, because a foreign holder also requires reimbursement for probable foreign exchange arrangements in addition to the reimbursement mandatory by a domestic holder. For that reason it is usually the case that currency degradation is viewed as an appearance of non-payment for foreign holders. In view of the fact that the Risk Free interest rate has to theoretically eliminate any risk of default, it implies that the yields on foreign owned government debt cannot be used as the basis for calculating the risk free interest rate. From the time while the compulsory return on government bonds for domestic and foreign holders cannot be illustrious in an international market for government debt, this is supposed to mean that yields on government debt are not a good alternative for risk free rate.

One more opportunity used to estimate the risk free interest rate is the inter-bank lending rate. Again it appears to be premised on the basis that these institutions advantage from an implied assurance, underpinned through the position of the financial authorities as 'the lending of last resort.' It has to be acceptable that in a system with endogenous currency supply the financial or monetary authorities' perhaps private agents plus the Central Bank. Again, the similar examination applies to banks as a substitute for the risk free interest rate. Conditionally if there is any apparent risk of default contained in the interbank lending rate, it is not suitable to this rate as a substitute for the risk free interest rate. Similar conclusions can be drawn from other possible standard rates, including short rated triple-A rated corporate bonds of institutions deemed 'too big to fail.' Unhappily it has not been possible to place a well comprehensive conversation on the foundation of a variety of conventions for estimating the risk free interest rate through substitute rates, which appears to be a foremost 'hole' in the theoretical literature.
Single explanation that has been anticipated for solving the issue of not having a good substitute for the risk free interest rate, to give a noticeable risk free interest rate is to have developed some form of internationally guaranteed asset which would afford a definite return over an indefinite time period. After doing research in this sector we found some assets in survival which was capable to replicate some of the hypothetical properties of this concept. Such as, a single potential nominee is the 'consul' bonds which were issued by the British Government in the 18th century.

The risk-free interest rate is extremely important in the situation of the universal application of ‘Modern Portfolio Theory’ which is based on the ‘Capital Asset Pricing Model’. According to the research there are abundant issues with this model, the most fundamental issue is the lack of the explanation of usefulness of stock investment to the expected mean and variance of the returns of the portfolio. In practice, there might be some more usefulness of stock investment, as described by Shiller (1984).

The risk free interest rate also plays important role in financial calculations, such as the Black–Scholes formula for pricing stock options and the Sharpe Ratio. Remember that some finance and economic theories pre-suppose that market participants can have access to the risk free interest rate, but in practice obviously, a small number of borrowers have access to finance at the risk free interest rate.

### 3.5.2.2 Jump Diffusion Model

In his study Merton (1976) has suggested the Jump diffusion model in which the stock price follow a Geometric Brownian movement and a chain of jumps which pre-suppose are Poisson Determined. In his suggestion he also tried to distinguish two types of changes in the stock prices. First: The Normal Vibration in Price and Second: Abnormal Vibration in price. Both these changes are taking place due to a number of economic factors, industry factor, and various number of national and international company factors. According to his research the first type of Vibration can be modelled by stochastic process with continuous sampling path, for example: Winner Process. Second type of jump vibration can be modelled with the process which openly allows for Jumps, for example: A Poisson Process.

Merton (1976) defines various variables in his research paper which is also based on the foundation of Black-Scholes Model.
\[ \mu = \text{Expected returns from asset or net dividend yield}; \]
\[ \omega = \text{Average number of jump per year}; \text{ and} \]
\[ \rho = \text{Average jump size measured as a percentage of the assets price}. \]
In his research, the percentage jump size is supposed to be drawn in probability distribution model. The probability of a jump in time \( \partial t \) is \( \omega \partial t \). The regular growth rate in the assets price from jumps is consequently \( \omega \rho \). The process of the asset is given on the basis of derivative dependent on a single market variable ‘S’. Then equation is given below.

\[ \frac{\partial s}{s} = (\mu - \omega \rho) \partial t + \sigma \partial z + \partial p \]  \hspace{1cm} (3.3)

Where
\[ \partial z = \text{Winner process}; \]
\[ \partial p = \text{Poisson process generating the jumps}; \text{ and} \]
\[ \sigma = \text{Volatility of the Geometric Brownian Motion}; \]

*Note:* In the above equation the process \( \partial z \) and \( \partial p \) are supposed to be independent.

Research of Merton is based on supposition that the jump factor of assets returns represents non-systematic risk. He also supposed that standard deviation (\( \sigma \)) of the normal distribution is \( S \).

Merton European option pricing formula:

\[ \sum_{n=0}^{n=\infty} e^{-\omega T} (\omega T)^n \frac{f_n}{n!} \]

Above formula of Merton gives increasable lift and also provides somewhat better results than the Black-Scholes formula and the result of this formula give more specific result in the case of currency option, as per Hull (2002).

### 3.5.2.3 Binomial Model

According to the study of various researchers, the Black-Scholes Model seems dominated by the option pricing; however other models also perform equal or somewhat better than this existing powerful model, such as ‘Binomial’ Model which was developed by Cox-Ross-Rubinstein.
Binomial Tree Model was presented in paper “Option Pricing: A Simplified Approach” in 1979. This model was somewhat simple and easy to understand, although it was exceptionally influential tool for pricing a wide range of options. This Binomial Tree Model show better association other than the Geometric Brown motion model which was applied by Black-Scholes Model. This model was tried to generate all possible stock prices and the related probability to achieve those prices.

Binomial Tree Model is divided in to three parts:

a) A One-Step Binomial Tree Model;

b) Two-Step Binomial Tree Model; and

c) Multi-Step Binomial Tree model;

a) A One-Step Binomial Tree Model

In the case of One-Step Binomial Tree Model we can oversimplify the fall out immediately by taking into consideration a stock whose price is \( P_s \) and an option on the stock whose current price is \( P \). In this process option ends at point \( T \) and throughout this procedure the price of the option moves up and down. We consider that up movement is \( P_s u \) and down movement is \( P_s d \), where \( u > 1 \) and \( d < 1 \). According to the probability increase in the stock price if there is an up movement is \( u - 1 \) and similarly down movement it is \( d - 1 \). We also suppose that if stock price moves up then payoff of the option is \( P_u \) and when goes down then payoff of the option is \( P_d \). In the probability section of binomial tree related to up and down, it creates long and short position of option. This portfolio position is calculated on the basis of \( \delta \) which represent change in option position.

If there is an up movement in the stock price, then the value of the portfolio at the end of the life of the option is

\[ P_s u \delta - P_u \]

If there is a down movement in the stock price, the value become

\[ P_s d \delta - P_d \]

According to the pattern of binomial method both the above relation are equal then

\[ P_s u \delta - P_u = P_s d \delta - P_d \]

Or

\[ P_s u \delta - P_s d \delta = P_u - P_d \]
\[ \delta (P_u - P_d) = P_u - P_d \]

Or

\[ \delta = \frac{P_u - P_d}{P_u - P_d} \quad (3.4) \]

If we consider risk-free interest rate (r) then above relation becomes

\[ (P_u \delta - P_u) e^{-rT} \]

Cost of initial portfolio given on the basis of above discussion is

\[ P_s \delta - p \]

Similarly, since both the above relations are equal then

\[ P_u \delta - p = (P_u \delta - P_u) e^{-rT} \]

Or

\[ p = P_u \delta - (P_u \delta - P_u) e^{-rT} \]

Substitute the value of \( \delta \) from equation (3.1)

\[ p = P_s \left( \frac{P_u - P_d}{P_u - P_d} \right) - \left[ P_s \left( \frac{P_u - P_d}{P_u - P_d} \right) - P_u \right] e^{-rT} \quad (3.5) \]

\[ p = e^{-rT} \left[ \left( \frac{e^{rt} - d}{u - d} \right) P_u + \left( 1 - \frac{e^{rt} - d}{u - d} \right) P_d \right] \]

After simplifying the above relation the researcher found

\[ P = e^{-rT} [P_b P_u + (1 - P_b) P_d] \quad (3.6) \]

Where

\[ P_b = \frac{e^{rt} - d}{u - d} = \text{Probability factor} \]

For understanding see figure 3.2

**Figure 3.2**

*Stock and Option Prices in a General One-Step Tree*
b) Two-Step Binomial Tree Model

In the case of Two-Step Binomial Tree, Initial movement of option is same as in One-Step Binomial tree. On the basis of that reason we used final equation of one-step binomial tree because probability factor is also the same in this case and risk free interest rate also same but the length of time step in changed, it become $T$ to $(T + \Delta t)$, where $\Delta t$ represent change in time.

With these changes equation (3.6) may be re-written as

$$P = e^{-r(T+\Delta t)}[P_u P_u + (1 - P_b)P_d]$$

(3.7)

$$P_u = e^{-r(T+\Delta t)}[P_b P_{uu} + (1 - P_b)P_{ud}]$$

(3.8)

And

$$P_d = e^{-r(T+\Delta t)}[P_b P_{ud} + (1 - P_b)P_{dd}]$$

(3.9)

Substituting the value of equation (3.8) and (3.9) in equation (3.7) we get

$$P = e^{-r(T+\Delta t)}[P_b (e^{-r(T+\Delta t)}[P_u P_{uu} + (1 - P_b)P_{ud}]) + (1 - P_b)(e^{-r(T+\Delta t)}[P_b P_{ud} + (1 - P_b)P_{dd}])]

Or

$$P = e^{-2r(T+\Delta t)}[P_b^2 P_{uu} + 2P_b (1 - P_b)P_{ud} + (1 - P_b)^2 P_{dd}]$$

(3.10)

For understanding see figure 3.3

Figure 3.3

Stock and Option Prices in a General Two-Step Tree
c) Multi-Step Binomial Tree Model

In the case of Multi-Step Binomial Tree Model, the first and second movements are same which we discussed above and it also continues in the same manner but only difference is that size of tree may increase from step two to n-time. The result is that the length of time also increased from \((T + \Delta t), (T + 2\Delta t), (T + 3\Delta t)\) to \([T + (n - 1)\Delta t]\). On the basis of this discussion multi-step binomial tree pattern is elaborated to define the proper mathematical statistics.

At time zero (0), the stock price is \(P_s\), and at the time \((T + \Delta t)\), there are two possible stock prices \(P_s u\) and \(P_s d\) and with continuous change in time, the stock price will also change (see in figure 3.4)

\[ P_s u^i d^{j-i} \text{, where } i = 0, 1, \ldots, j \]  

(3.11)

Where: \(u =\) up tree, and \(d =\) down tree; (Also illustrated in One-Step tree)

\[
\text{If } u = \frac{1}{d}, \text{ then } P_s u^2 d = P_s u.
\]

\[ \text{Up Tree} = P_s, P_s u, P_s u^2, P_s u^3, P_s u^4, \ldots, \ldots, P_s u^{n-1} \]

\[ \text{Down Tree} = P_s, P_s d, P_s d^2, P_s d^3, P_s d^4, \ldots, \ldots, P_s d^{n-1} \]

Figure 3.4

Stock and Option Price in a General Multi-Step Tree

Suppose that the life of put or call option on a non-dividend paying stock divided in to ‘\(n\)’ sub interval of length\(\Delta t\). If carry ‘\(i\)’ th node at time \(j\Delta t\) as the \((j, i)\) node, where \(0 \leq j \leq n\) and \(0 \leq i \leq j\). It states that \(F_{j,i}\) give the value of call or put option at the node \((j, i)\). The stock price given at the \((j, i)\) node is \(P_0 u^i d^{j-i}\). Then \(F_{j,i} = F_{n,i}\), because ‘\(n\)’ replaces ‘\(j\)’ according to change
in time \((T + \Delta t)\), and value of put or call option at expiration date is \(\max(D_p - P_T, 0)\), where

\[P_T = P_s u^i d^n - i \text{ and } D_p = \text{delivery price in the contract at time } t\].

Therefore:

\[F_{n,i} = \max(D_p - P_s u^i d^n - i, 0), \text{ where } i = 0,1,2,3 \ldots n\]  

(3.12)

Above equation (3.12) also has a probability to move from \((j, i)\) point. It may happen or not happen, most of the time it depend on change in time.

### 3.5.2.4 Monte-Carlo Simulation

In terms of mathematical finance, a Monte Carlo option model uses Monte Carlo methods to calculate the value of an option with various sources of improbability or with problematical features. Even though the term 'Monte Carlo method' was coined by Stanislaw Ulam in the 1940s, a few traces of this method were developed in the 18th century by French naturalist ‘Buffon’. While discussing a main question he asked about the results of dropping a needle randomly on a striped floor or table (See Buffon's needle). The first function to option pricing was given by Phelim Boyle in 1977 (for European options) which was based on Monte Carlo Method. M. Broadie and P. Glasserman (1996) showed how to price Asian options by Monte Carlo method. In 2001 F. A. Long staff and E. S. Schwartz also tried to develop a practical Monte Carlo method for pricing American-style options.

Monte Carlo Simulation is also recognized as Simulation method. This technique is based on the use of random numbers and probability statistics to investigate problems, consequently the name stochastic technique. This method tends to be more accurate and also more proficient than other option valuation methods but it is only applicable when there are three or more stochastic variables. This all happens because the time taken to bear out Monte Carlo Simulation which increases roughly up to the number of variables. Some valuable trend the time taken for most other measures increase exponentially with the number of variable. Solitary key benefit of Monte-Carlo simulation is that it can provide a standard error for the estimates that it makes. An additional fact is that it is an approach that can put up complex payoffs and complex stochastic process. In most of the cases it can be used when the payoff depends on some function of the whole path followed by a variable, which was not a terminal value according to Hull (2002).

To smooth the progress of an understanding of the technique we shall look at how Monte Carlo Simulation has been used to price standard European Options. Obviously we know that the
Black-Scholes Model is the correct method of pricing the option so Monte Carlo Simulation is not really needed. It is however useful and the researcher tried to give some meaningful results or techniques related.

**Monte Carlo Simulation Calculation Process**

Monte-Carlo Simulation measured a derivative depends on a single market variable $P$ that provides a payoff at time $T$. Assuming the initial rate are constant, we can value the derivatives as follows.

I. Sample a random path for $P$ in a risk-neutral world.

II. Calculate the payoff for the derivative.

III. Repeat step I and II to get many sample value of the payoffs from the derivative in a risk neutral world.

IV. Calculate the mean of the sample payoffs to get an estimate of the expected payoff in risk-neutral world.

V. Discount the expected payoff in risk-free rate to get an estimate of the value of Derivative.

Presume with the purpose of developing the process followed by the underlying market variable in a risk neutral world. The equation becomes

$$dP = \mu P dt + \sigma P dz$$  \hspace{1cm} (3.13)

Where

$dz = $ Winner process;

$\mu = $ Expected return in a risk-neutral world; and

$\sigma = $ Volatility;

To simulate the path followed by $P$, life of the derivative was divided in to $N$ short interval of length $\delta t$ and the above equation (3.13) become

$$P(t + \delta t) - P(t) = \mu P(t) \delta t + \sigma P(t) \varphi \sqrt{\delta t}$$ \hspace{1cm} (3.14)

In the above equation (3.14), $P(t)$ indicate the value of $P$ at time $t$ and $\varphi$ is a random sample from a normal distribution with mean zero and standard deviation one. This facilitates the value of $P$ at time the $\delta t$ to be calculated for initial value of $P$. The value of time $2\delta t$ to be calculated from the value of time $\delta t$. First simulation experiment involves constructing a complete path for $P$ using $N$ random sample from a normal distribution. In practice it is usually more accurate to simulate $\ln P$ rather than $P$. From It$\hat{o}$s Lemma the procedure followed by $\ln P$ is.
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\[ d \ln P = \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \]

Or

\[ \ln P(t + \delta t) - \ln P(t) = \left( \mu - \frac{\sigma^2}{2} \right) \delta t + \sigma \varphi \sqrt{\delta t} \]

Or

\[ \ln \left( \frac{P(t + \delta t)}{P(t)} \right) = \left( \mu - \frac{\sigma^2}{2} \right) \delta t + \sigma \varphi \sqrt{\delta t} \]

Or

\[ P(t + \delta t) = P(t) e^{\left( \mu - \frac{\sigma^2}{2} \right) \delta t + \sigma \varphi \sqrt{\delta t}} \quad (3.15) \]

This equation is true to construct a path for \( P \) in a similar way to equation (3.14). While equation (3.14) is true only in the limit as \( \delta t \to 0 \), equation (3.15) is exactly true for all \( \delta t \).

The problem of Monte Carlo Simulation is that it can be used while the payoff depends on the course followed by the basic variable \( P \) in addition to that it depends only on the final value of \( P \). Payoff can crop up at several times throughout the life of the derivative relatively than at the end any stochastic process for \( P \) can be accommodated. Additionally shown the procedure can also be extended to accommodate situation where the payoff from the derivative depends on several underlying market variables. The major drawbacks of Monte Carlo simulation are that it is computationally extremely time-consuming and cannot easily handle situations at the point where early exercise opportunities happen.

According to the research paper of Tilley (1993) who was the first to apply simulation to American option pricing, using a bundling technique and a backward introduction algorithm. His working direction is a ‘Single Pass’ algorithm, in which all simulation are passed out first prior to the algorithm is functional. In contrast, the approach of Grant, Vora and Weeks (1977) is chronological in its use of simulation, scheduled inductively backwards to estimate the exercise boundary at each early exercise point, finally estimating the price in a forward simulation based on the obtained boundaries. This approach was also used by Fu and Hu (1995) in their research.

3.5.2.5 Finite Differences Method

Finite difference method assessment a derivative by solving the differential equation and this differential equation should be satisfied by derivative. The differential equation is transformed
into a set of dissimilarity equation and these dissimilarity equations are solved integratively (see Brennan and Schwartz, 1977-1978)

If we try to demonstrate this approach, we start from Black-Scholes partial differential equation (PDE):

$$\frac{\partial V_{op}}{\partial t} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V_{op}}{\partial p^2} + R_f p \frac{\partial V_{op}}{\partial p} - R_f V_{op} \leq 0 (3.16)$$

Presume that the life of the option is $T$. We divide this into $N$ equal spaced intervals of length $\frac{T}{N}$. A total of $(N+1)$ time are considered. Then series of change in small time become

$$0, \delta t, 2\delta t, 3\delta t, 4\delta t \ldots T$$

Similarly if the $P_{max}$ is a stock price sufficiently high when it is reaches at that point put options has virtually no value. We define $\delta p = \frac{P_{max}}{M}$ and consider a total of $(M+1)$ uniformly spaced stock price. Then the series of change in price become

$$0, \delta P, 2\delta P, 3\delta P, 4\delta P \ldots P_{max}$$

The level of $P_{max}$ is selected so that individual of these is the present stock price.

The time point and stock price points define a grid consisting of a total of $[(M + 1) \times (N + 1)]$ as shown in figure 1.5 and 1.6; The $(i, j)$ point on the grid is the point that corresponds to time $\delta t$ and stock price $j\delta p$. On the basis of that we will use the variable $V_{op(i,j)}$ to denote the value of the option at the $(i, j)$ point.

Finite Difference Methods are of two Types:

a) Explicit finite difference method (Forward Difference); and

b) Implicit finite difference method (Backward Difference);

a) Explicit Finite Difference Method

If we apply forward difference in equation (3.16) then equation can be changed to these patterns

$$\frac{\partial V_{op}}{\partial t} \quad \text{Change to} \quad \frac{V_{op(i+1,j)} - V_{op(i,j)}}{\delta t}$$

For the space grid we can apply central difference for all the order of derivative

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V_{op}}{\partial p^2} \quad \text{become} \quad \frac{1}{2}\sigma^2 p^2 \frac{[V_{op(i+1)(j+1)} + V_{op(i+1)(j-1)} - 2V_{op(i+1),j}]}{\delta p^2}$$

$$R_f p \frac{\partial V_{op}}{\partial p} \quad \text{become} \quad R_f p \left[\frac{V_{op(i+1)(j+1)} - V_{op(i+1),j}}{2\delta p}\right] \quad \text{and} \quad R_f V_{op} \quad \text{become} \quad R_f V_{op(i+1),j}$$
Then final explicit difference equation becomes
\[
\frac{V_{op(i+1),j} - V_{op(i),j}}{\delta t} + \frac{1}{2} \sigma^2 \pi^2 \left[ \frac{V_{op(i+1)(i+1)} + V_{op(i+1)(j-1)} - 2V_{op(i+1)}}{\delta p^2} \right] + R_t \rho \left[ \frac{V_{op(i+1)(i+1)} - V_{op(i+1)(j-1)}}{2\delta p} \right] = R_t V_{op(i+1)}
\]
Or
\[
V_{op(i+1),j} = X^* \left( V_{op(i+1)(j-1)} \right) + Y^* \left( V_{op(i+1),j} \right) + Z^* \left( V_{op(i+1)(j+1)} \right)
\]
For \( j = 1, 2, 3, \ldots, (M - 1) \) and \( i = 0, 1, 2, 3, \ldots, (N - 1) \)

Where
\[
X^* = \left( 1 + R_t \delta t \right) \left( \frac{-\frac{1}{2} R_t \rho \delta t + \frac{1}{2} \sigma^2 \pi^2 \delta t} {1 + R_t \delta t} \right);
\]
\[
Y^* = \frac{1}{1 + R_t \delta t} \left( 1 - \sigma^2 \pi^2 \delta t \right); \text{ and}
\]
\[
Z^* = \left( 1 + R_t \delta t \right) \left( \frac{1}{2} R_t \rho \delta t + \frac{1}{2} \sigma^2 \pi^2 \delta t \right)
\]

In the above case \( X^*, Y^* \) and \( Z^* \) represent the probability of each node. This case is essentially comparable to the trinomial tree where probability can be assigned to the likelihood of an up move, a down move as well as no move. It can also be shown as in figure 3.5.

**Figure 3.5**
Explicit Finite Difference Method
b) Implicit Finite Difference Method

This method is also similar to the explicit finite difference method but only difference is that instead of moving forward direction, you have to move in backward direction (see in figure 3.6). So, on the basis of that reason most of the functions work in reverse direction. In comparison, the Implicit Finite Difference Method is characteristically more steady and convergent than the Explicit Finite Difference Method. On the other hand it is frequently more computationally demanding according to Hull and White (1990).

If we apply backward difference in equation (3.16) then equation can be changed to these patterns

\[
\frac{\partial V_{op}}{\partial t} \text{ Change to } \frac{V_{op(i+1,j)} - V_{op(i,j)}}{\delta t}
\]

For the space grid we can apply central difference for all the order of derivative

\[
\frac{1}{2} \sigma^2 p^2 \frac{\partial^2 V_{op}}{\partial p^2} \text{ become } \frac{1}{2} \sigma^2 p^2 \frac{V_{op(i,j+1)} + V_{op(i,j-1)} - 2V_{op(i,j)}}{\delta p^2},
\]

\[
R_t P \frac{\partial V_{op}}{\partial p} \text{ become } R_t P \frac{V_{op(i,0,j+1)} - V_{op(i,0,j-1)}}{2\delta p} \text{ and } R_t V_{op} \text{ become } R_t V_{op(i,j)}
\]

Then final explicit difference equation become

\[
\frac{V_{op(i+1,j)} - V_{op(i,j)}}{\delta t} + \frac{1}{2} \sigma^2 p^2 \frac{V_{op(i,j+1)} + V_{op(i,j-1)} - 2V_{op(i,j)}}{\delta p^2} + R_t P \frac{V_{op(i,0,j+1)} - V_{op(i,0,j-1)}}{2\delta p} = R_t V_{op(i,j)}
\]

In the above equation ‘Control Variable Technique’ can be used in juxtaposition with implicit difference method. On the basis of that technique same grid is used to value an option which was under consideration in Partial Differential Equation (PDE). In which the value of \( T \) is \( \max(K - P_T, 0) \), where \( P_T \) the stock price is at time \( T \). Therefore: \( V_{op(N,j)} = \max(K - j\delta P, 0) \), where \( j = 0,1,2,3 \ldots, M \). In this case the value of put option when stock price is zero is \( K \). It means \( V_{op(i,0)} = K \), where \( i = 0,1,2,\ldots,N \). On the basis of above discussion we also assume that the put option is worth zero when \( P = P_{\max} \), as a result \( V_{op(i,M)} = 0 \), where \( i = 0,1,2 \ldots, N \).
3.5.2.6 GARCH Model

In this section the researcher has discussed GARCH option pricing model proposed by Bollerslev in 1986. This model considered a discrete time economy and let $A_{P_t}$ survive the asset price at time $t$. In this case well thought-out one-period rate of returns is supposed to be temporarily log-normally distributed under probability computed ($P_m$). That is to say

$$
\ln \frac{A_{P_t}}{A_{P_{t-1}}} = R_{f_t} + C_{R_P} \sqrt{V_{C_t}} - \frac{1}{2} V_{C_t} + M_{0_t}
$$

(3.17)

Where

$M_{0_t} =$ Mean Zero at time $t$;

$V_{C_t} =$Conditional variance under measure $P_m$;

$R_{f_t} =$ Constant one-period risk-free rate of return (which was continuously compounded);

$C_{R_P} =$ A constant unit risk premium;

On the basis of above mentioned condition of temporary log-normality and one plus the conditionally expected rate of returns equal to $e^{(R_{f_t} + C_{R_P} \sqrt{V_{C_t}})}$. It consequently recommended that $C_{R_P}$ can be interpreted as the unit risk premium. On the basis of GARCH model we further
assume that $M_0$, pursue a GARCH $(p, q)$ procedure of Bollerslev (1986) on the basis of probability measure $P_m$. Formally

$$M_0 \Rightarrow \varnothing_{t-1} \sim N(0, V_c), \text{ under measure } P_m$$

Therefore

$$V_{c_t} = \alpha_0 + \sum_{i=1}^{q} \alpha_i M_{0_{t-i-1}}^2 + \sum_{i=1}^{p} \beta_i V_{c_{t-i}} \quad (3.18)$$

Where $\varnothing_t$ is the information set ($\sigma-$Field) of all information up to and including time $t$;

$$p \geq 0, q \geq 0 \text{ and } \alpha_0 > 0, \alpha_i \geq 0$$

Where $i = 1, 2, 3, 4 ..., q; \beta_i \geq 0, i = 1, 2, 3, ..., q$

According to the explanation, the conditional variance is a linear function of the past squared disturbances and the past conditional variance. Without a doubt you may say that $V_{c_t}$ is $\varnothing_t$ predictable. On the basis of that analysis the option pricing results to be developed in this section depend upon conditional normality. While using an alternative specification for $V_{c_t}$, for example EGARCH of Nelson (1991) or that of Glosten et al. (1993) will not be able to change the fundamental option pricing results on condition that conditional normality remain in place.

To make sure that the Covariance Stationarity of the GARCH $(p,q)$ method, $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$ is supposed to be less than one, because the sum equal to one in the case of GARCH(1,1) process, this procedure is referred to as the integrated GARCH process. Nelson (1990) showed that the IGARCH process is stationary and argotic even though the variance is unbounded. Actually, the GARCH process may still be strictly stationary if the sum of $\alpha_i$’s and $\beta_i$’s is greater than one.

The adequate and compulsory condition for its strict Stationarity is related to the top Lyapunov exponent of a particular progression of unsystematic matrices (see Bougerol and Picard, 1992). The GARCH process specified in equation (3.17) and (3.18) reduce to the standard homoskedastic log-normal process in the Black-Scholes Model if $p = 0$ and $q = 0$. This situation indicates that the Black-Scholes Model is out of the ordinary case.

### 3.5.2.7 Artificial Neural Network (ANN) Model

The stock brokers and investors started taking interest in Neural Network after this concept was introduced by McCulloch and Pitts (1943). This model was used as generalizations of mathematical models of natural systems. In this model there are various essential processing fundamentals of neural networks termed as ‘Artificial Neurons or Nodes’. According to the
In the literature, the Neural Network consists of three types of neuron layers: input, hidden, and output layers. In case of a feed-forward network, the direction of information is from input to output units, in a unidirectional manner. Artificial Neural Network (ANN) is a dynamic system and it can be changed according to the requirement of investors, brokers and any individual as per the application.

The communication of signals among neurons is facilitated using commencement functions which are helpful for the input to determine the output from a neuron. This ANN model function tries to set up a relationship between the input variables and the output desired. The well-liked transfer functions are the sigmoid, the hyperbolic tangent, the Gaussian and their variants. The shift function also assists to establish non-linear relationship in the modelling. In figure (3.7), the input signals are customized by multiplying a weight to each signal and the customized signals are supplementary mutually to find out the joint strength of their output using following activation function. Figure (1.7) generate the function of single flow

\[ R = f(A_1B_1 + A_2B_2 + A_3B_3 + \ldots + A_nB_n) \]

Where:

\[ A_1, A_2, \ldots, A_n = \text{Input}; \]
\[ B_1, B_2, \ldots, B_n = \text{Direction node between Input and Output and it called adjustment weight}; \]
and
\[ R = \text{Output} \]

Or

\[ R = f(\sum_{i=1}^{n} A_iB_i), \text{where } i = 1, 2, 3, \ldots, n; \]  \hspace{1cm} (3.19)

**Figure 3.7**

Single Flow ANN Model

![Single Flow ANN Model](image-url)
A multiplayer network consists of several layers and in presented in figure 3.8. The input variable are shown to the input layer of processing elements, which sends signal that propagates from beginning to end by the network layer by layer till it produce final output.

![A Multilayer ANN Model](image)

The Black-Scholes Model uses five parameters as inputs to calculate approximately the theoretical option price. ANN model also uses the same five parameters to calculate the value of option by adding a multiplying factor attached to each of the parameter (See, Mitra, 2012). Input parameter are: \(P, P_E, R_f, t\) and \(\sigma\);

Where:

- \(P\) = Spot price of the security;
- \(P_E\) = Exercise price of call option;
- \(R_f\) = Risk free rate of interest;
- \(t\) = Time left until option expiry (Date in year fraction); and
- \(\sigma\) = A measure of implied volatility (calculate on the basis of past 60 days daily return of underlying security)

Here in the absence of standard procedure to measure volatility(\(\sigma\)), it was estimated by calculating standard deviation from past 60 day’s returns. Even though the GARCH based methodologies are capable of calculating time varying volatilities in better ways. This ANN model was also tried to expect same kind of result by measurement based on standard deviation. According to the study of Hull (1999) they measure volatility on daily basis which is annualized assuming 252 trading days in year. Figure (1.9) measure five standard input of Black-Scholes Model which tries to facilitate the network to auto adjustments and each input is multiplied by an
adjusting weight ($B_1$ to $B_5$). According to the model only one hidden layer is there which contain two nodes ($C_1$ and $C_2$) to maintain similarity to the original Black-Scholes Model.

**Figure 3.9**
**Structure of Proposed ANN Model on the Basis of Black-Scholes Model Parameter**

Above ANN model is based on the foundation of Black-Scholes Model but modification factors are used at signal transmission point. The five inputs of the model are multiplied by subsequent five adjustment factors ($B_1$ to $B_5$) and the $C_1$ and $C_2$ value in the unseen node are measured as follows. From the above model it may also be noticed that there is no change taking place in the original equation other than the introduction of adjustment weight.

$$C_1 = \frac{\ln\left(\frac{B_5 P}{B_2 P_E}\right) + \left(B_3 R_E + \frac{(B_5 \sigma)^2}{2}\right) B_4 t}{B_5 \sigma \sqrt{B_4 t}}$$  \hspace{1cm} (3.20)$$

$$C_2 = C_1 - (B_5 \sigma \sqrt{B_4 t})$$ \hspace{1cm} (3.21)$$

After all modification and alteration, the value of call option as the output of the network is estimated using the following function:

$$R = B_6 P N(C_1) + B_7 P_E e^{-R t} N(C_2)$$  \hspace{1cm} (3.22)$$

In the above equation (3.22) $B_6$ and $B_7$ are additional adjustment factors, $N(C_1)$ and $N(C_2)$ are standard normal cumulative distribution of $C_1$ and $C_2$ value. We also found that the above input-output relationship of ANN model closely resembles the original Black-Scholes Model apart from that each parameter is multiplied by an adjusting weight. If we initialized the values of
adjusting weights (B₁ to B₇) to one, then the above model gives output which was similar to the original model of Black-Scholes Model, despite the fact that the ANN network weights (B₁ to B₇) will be altered so as to minimize differences between the network output and quoted option price.

3.6 Swap literature
According to the definition of Fletcher (1997), swaps is the most generic séance possible, a swap can be simply defined as an agreement between two parties to exchange, at some futures point, one product, either physical or financial, for another. Kolb (2003), defined that swap is an agreement between two parties to exchange sequences of cash flows over a period in the futures, and the parties that agreed to the swap are known as counter parties. Satay-jit (2004), discussed the counter party risk on the swap process. More recently, Huang and Zhang (2008) investigate the determinant of variation in the yield spreads between Japanese’s yen interest rate swaps and Japan government bond for a similar period of study (from 1977 to 2005), although they use a smooth transition vector auto-regression model to analyse the impact of various economic shocks on swap spreads. According to Taylor (2006), cash flow from the counter parties is generally bound to the value of debt instruments or to the value of foreign currencies.

3.7 Research Gap
While collecting the literature of different financial derivative products, the researcher found that Black-Scholes Option Pricing Model has a possibility of modifications in calculation of call and put value of options. Because Black-Scholes Model is based on various assumptions, short details are given above in literature review section of options model. The structure of Black-Scholes Option Pricing Model is complicated in nature due to the use of applied mathematics. The various researchers derived various outcomes of that formula which have been covered at length in the literature section. On the basis of these results the researcher tried to go for the re-derivation of Black-Scholes Option Pricing Model formula, which turned to be the primary research objective of this thesis. Further in the literature review section researcher found that there is no perfect method to calculate risk free interest rate as it varies according to time and place. This risk free interest rate also became the one of the major assumption of Black-Scholes Option Pricing Model and thus
turned to be the **second primary research objective** of this thesis, in which the researcher tried to incorporate modification in Black-Scholes Option Pricing Model formula by adding some new variable on the basis of given assumption related to risk-free interest rate. According to such kind of research, based on modification in some existing model or formal, empirical testing will be required on the basis of **modified formula**. This resulted in the **third primary objective** of this thesis.