3. ON RAYLEIGH WAVE IN GENERALIZED MAGNETO THERMOELASTIC MEDIA WITH HYDROSTATIC INITIAL STRESS

3.1 INTRODUCTION

The classical dynamical coupled theory of thermoelasticity was extended to generalized thermoelasticity theories by Lord and Shulman (1967) and Green and Lindsay (1972). These theories consider heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Ignaczak and Ostoja-Starzewski (2009) presented the analysis of above two theories in their book on “Thermoelasticity with Finite Wave Speeds”. The representative theories in the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak (1999). Surface waves in elastic solids were first studied by Lord Rayleigh (1885) for an isotropic elastic solid. Thermoelastic Rayleigh waves in semi-infinite isotropic solids are studied by Lockett (1958), Deresiewicz (1961), Nayfeh and Nemat-Nasser (1971), Carroll (1974), Agarwal (1978), Dawn and Chakraborty (1988), and many others with various additional parameters. Initial stresses in solids have significant influence on the mechanical response of the material from an initially-stressed configuration and have applications in geophysics, engineering structures and in the behaviour of soft biological tissues. Initial stress arises from processes, such as manufacturing or growth, and is present in the absence of applied loads. Montanaro (1999) formulated the isotropic thermoelasticity with hydrostatic initial stress. Singh et al. (2006), Othman et al. (2007), Singh (2008), and
many others have applied Montanro (1999) theory to study the plane harmonic waves in context of generalized thermoelasticity.

In this chapter, the governing equations given by Montanaro (1999) are modified in context of Lord and Shulman (1967) and Green and Lindsay (1972) theories with uniform magnetic field. These equations are solved for the surface wave solutions, which satisfies the required boundary conditions at the free surface and we obtain the frequency equation for the Rayleigh wave in the half-space. The frequency equation is approximated for small thermal coupling and small reduced frequency. The velocity of propagation and amplitude-attenuation factor of Rayleigh wave are computed numerically for a particular material. Effects of magnetic field and hydrostatic initial stress on the velocity of the propagation and amplitude-attenuation factor are shown graphically.

3.2 FORMULATION OF THE PROBLEM

We consider a homogenous, isotropic thermoelastic half space with hydrostatic initial stress initially at uniform temperature \( T_0 \). We take origin of the co-ordinate system \((x, y, z)\) at any point on the plane horizontal surface and \(z\)-axis pointing vertically downward into the half-space, which is represented by \( z \geq 0 \). The surface \( z = 0 \) is assumed to be subjected to stress free, thermally insulated or isothermal boundary conditions. We choose \(x\)-axis in the direction of wave propagation so that all particles on a line parallel to \(y\)-axis are equally displaced. Therefore, all the field quantities will be independent of \(y\)-coordinate.
For Rayleigh type wave in the half space \( z \geq 0 \), we take representation of displacement components as

\[
    u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_2 = 0, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}
\]  

(3.2.1)

where \( \phi \) and \( \psi \) are functions of \( x, z \) and \( t \). With the help of (3.2.1) equations (2.1.5) and (2.1.6) reduces to

\[
    \frac{\partial^2 \phi}{\partial t^2} = c_1^2 \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} \right) - \frac{\gamma}{\rho_0} (T + a \dot{T})
\]  

(3.2.2)

\[
    \frac{\partial^2 \psi}{\partial t^2} = c_2^2 \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right)
\]  

(3.2.3)

\[
    \rho_0 c_1^4 (T + a \dot{T}) + \gamma T_0 (1 + \Delta a \frac{\partial}{\partial t} \left[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} \right]) = K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right)
\]  

(3.2.4)

where,

\[
    c_1^2 = \frac{\lambda + 2\mu + \mu_H}{\rho_0}, \quad c_2^2 = \frac{\mu - \rho}{2 \rho_0}, \quad \gamma = \frac{\alpha}{k_f}
\]

The following non–dimensional quantities are considered

\[
    x' = \frac{x}{(c_1/\omega)}, \quad z' = \frac{z}{(c_1/\omega)}, \quad t' = t\omega^*,
\]

\[
    u_1' = \frac{u_1}{(c_1/\omega)}, \quad u_3' = \frac{u_3}{(c_1/\omega)}, \quad T' = \frac{\gamma T}{\rho_0 c_1^2},
\]
\[ \phi = \frac{\phi}{(c_i^2/\omega)^2}, \quad \psi = \frac{\psi}{(c_i^2/\omega)^2}, \quad a^* = a\omega^*, \]

\[ a^* = a^*w^*, \quad w^* = \frac{\rho_c \epsilon_i^2}{K}. \]

With the help of these non-dimensionl quantities, we get the following expressions

\[ \frac{\partial \phi}{\partial x} = \eta \frac{\partial \phi'}{\partial x}, \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi'}{\partial x^2}, \quad \frac{\partial \phi}{\partial z} = \eta \frac{\partial \phi'}{\partial z}, \]

\[ \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial^2 \phi'}{\partial z^2}, \quad \frac{\partial \phi'}{\partial t} = \eta^2 w^* \frac{\partial \phi'}{\partial t}, \quad \frac{\partial^2 \phi'}{\partial t^2} = \eta^2 w^* \frac{\partial^2 \phi'}{\partial t^2}. \]

\[ \frac{\partial \psi}{\partial x} = \eta \frac{\partial \psi'}{\partial x}, \quad \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi'}{\partial x^2}, \quad \frac{\partial \psi}{\partial z} = \eta \frac{\partial \psi'}{\partial z}, \]

\[ \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2 \psi'}{\partial z^2}, \quad \frac{\partial \psi'}{\partial t} = \eta^2 w^* \frac{\partial \psi'}{\partial t}, \quad \frac{\partial^2 \psi'}{\partial t^2} = \eta^2 w^* \frac{\partial^2 \psi'}{\partial t^2}. \]

\[ \frac{\partial T}{\partial t} = \frac{\rho_c \epsilon_i^2 w^*}{\gamma} \frac{\partial T'}{\partial t}, \quad \frac{\partial^2 T}{\partial t^2} = \frac{\rho_c \epsilon_i^2 w^*}{\gamma} \frac{\partial^2 T'}{\partial t^2}, \quad \frac{\partial T}{\partial x} = \frac{\rho_c \epsilon_i^2 w^*}{\gamma} \frac{\partial T'}{\partial x}, \]

\[ \frac{\partial^2 T}{\partial t^2} = \frac{\rho_c \epsilon_i^2 w^*}{\gamma} \frac{\partial^2 T'}{\partial t^2}, \quad \frac{\partial T}{\partial z} = \frac{\rho_c \epsilon_i^2 w^*}{\gamma} \frac{\partial T'}{\partial z}, \quad \frac{\partial^2 T}{\partial z^2} = \frac{\rho_c \epsilon_i^2 w^*}{\gamma} \frac{\partial^2 T'}{\partial z^2}. \]

Using the above expressions in the equations (3.2.1)-(3.2.4) and suppressing the primes, we obtain the equations in non-dimensional form as

\[ u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_2 = 0, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \quad (3.2.5) \]
\[
\frac{\partial^2 \phi}{\partial t^2} = \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} \right) - (T + a \dot{T}) \tag{3.2.6}
\]

\[
\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{v^2} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right) \tag{3.2.7}
\]

\[
(T + a \dot{T}) + \epsilon(1 + \Delta a) \frac{\partial}{\partial t} \left[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} \right] = \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{3.2.8}
\]

where, the thermoelastic coupling is given by

\[
\epsilon = \frac{\gamma^2 T_0}{\rho c^2 c_1^2}, \quad v^2 = \frac{c_1^2}{c_2^2} \tag{3.2.9}
\]

The mechanical and thermal conditions at the boundary \(z=0\) are

\[
\sigma_{13} + \bar{\sigma}_{13} = 0, \quad \sigma_{33} + \bar{\sigma}_{33} = -p, \quad \frac{\partial T}{\partial z} + hT = 0 \tag{3.2.10}
\]

where,

\[
\sigma_{ij} = -p(\delta_{ij} + w_{ij}) + \bar{\lambda}e_{ppj} \delta_{ij} + 2\mu e_{ij} - \gamma(T + a \dot{T})\delta_{ij}
\]

\[
\bar{\sigma}_{ij} = \mu_\epsilon [H_j h_j + H_j h_j - (H\bar{H})\delta_{ij}]
\]

\[
\sigma_{13} = 2\bar{\mu} \frac{\partial^2 \phi}{\partial x \partial z} + \left( \bar{\mu} - \frac{p}{2} \right) \frac{\partial^2 \phi}{\partial x^2} - \left( \bar{\mu} + \frac{p}{2} \right) \frac{\partial^2 \psi}{\partial x^2}
\]

\[
\sigma_{33} = \bar{\lambda} \left( \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \psi}{\partial x \partial z} \right) + \left( \bar{\lambda} + 2\bar{\mu} \right) \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z} - (T + a \dot{T})c_1^2
\]

and \(\bar{\sigma}_{13} = 0\)
\[- \sigma_{33} = \mu \nu H_\nu^2 \left[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} \right] \]

With the help of above equations, the boundary conditions (3.2.10) are written in non-dimensional form as

\[
(2 + p_1) \frac{\partial^2 \phi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \left( 1 + p_1 \right) \frac{\partial^2 \psi}{\partial z^2} = 0 \quad (3.2.11)
\]

\[
(1 - \frac{2}{v^2} - p_2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} - \left( T + aT \right) + \left( \frac{2}{v^2} + p_2 \right) \frac{\partial^2 \psi}{\partial x \partial z} = 0 \quad (3.2.12)
\]

\[
\frac{\partial T}{\partial z} + hT = 0 \quad (3.2.13)
\]

where,

\[ p_1 = \frac{p}{\rho_0 c_2^2}, \quad p_2 = \frac{p}{\rho_0 c_1^2} \]

and \( h \to 0 \) corresponds to the thermally insulated surface and \( h \to \infty \) corresponds to the isothermal surface.

### 3.3 SOLUTIONS AND THE FREQUENCY EQUATION

For thermoelastic surface waves in the half-space propagating in x-direction, the functions \( (T, \phi, \psi) \) may be taken in the form

\[
\{T, \phi, \psi\} = \{\hat{T}(z), \hat{\phi}(z), \hat{\psi}(z)\} \exp\{i(\eta x - \chi t)\} \quad (3.3.1)
\]

Using equation (3.3.1) in equation (3.2.7), we obtain
\[-\chi^2 = \frac{1}{v^2}(-\eta^2 + D^2),\]

\[D^2 = \eta^2 - \chi^2 v^2,\]

\[D = \pm \eta \sqrt{1 - c^2 v^2},\]

\[D = \pm \eta \beta_3\]

where, \[\beta_3^2 = (1 - c^2 v^2),\quad c^2 = \frac{\chi^2}{\eta^2}\]

Therefore, \[\hat{\psi}(z) = C \exp(-\eta \beta_3 z) + D \exp(\eta \beta_3 z)\] and from equation (3.3.1) we get

\[\hat{\psi}(z) = [C \exp(-\eta \beta_3 z) + D \exp(\eta \beta_3 z)] \exp(\eta x - \chi t)] \quad (3.3.2)\]

With the help of equation (3.3.1) in equations (3.2.6) and (3.2.8) imply

\[(\chi^2 - \eta^2 + D^2) \hat{\phi}(z) - (1 - \eta a) \hat{T}(z) = 0 \quad (3.3.3)\]

\[\tau \varepsilon \chi (1 - \Delta a^* \chi)(\eta^2 - D^2) \hat{\phi}(z) + (-i \chi - a^* \chi^2 + \eta^2 - D^2) \hat{T}(z) \quad (3.3.4)\]

Eliminating \[\hat{\phi}(z)\] and \[\hat{\psi}(z)\] from (3.3.3) and (3.3.4), we get

\[
\begin{vmatrix}
(-\eta^2 + \chi^2 + D^2) & -(1 - \eta a \chi) \\
\tau \varepsilon \chi (1 - \Delta a^* \chi)(\eta^2 - D^2) & (-i \chi - a^* \chi^2 + \eta^2 - D^2)
\end{vmatrix} = 0,
\]

or

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\[
\begin{vmatrix}
(-\eta^2 + \chi^2 + D^2) & i\chi_1 \chi \\
\chi^2 \varepsilon \chi_1 (\eta^2 - D^2) & -\chi^2 \chi_2 + \eta^2 - D^2
\end{vmatrix} = 0,
\]

or
\[
\frac{D^4}{\eta^4} + \frac{D^2}{\eta^2} [-2 + \frac{\chi^2}{\eta^2} (1 + \chi_2 - i\chi \varepsilon \chi_3 \varepsilon)]
+ 1 - \frac{\chi^2}{\eta^2} (1 + \chi_2 + i\chi \varepsilon \chi_3 \varepsilon) + \frac{\chi^4}{\eta^2} \chi_2 = 0
\]

or
\[
\beta^4 + \beta^2 [-2 + c^2 (1 + \chi_2 - i\chi \varepsilon \chi_3 \varepsilon)]
+ 1 - c^2 (1 + \chi_2 + i\chi \varepsilon \chi_3 \varepsilon) + c^4 \chi^2 = 0
\]

(3.3.5)

where,
\[
\beta^2 = \frac{D^2}{\eta^2}, \quad \chi_1 = \text{a} + \frac{i}{\chi}, \quad \chi_2 = \text{a}^* + \frac{i}{\chi}, \quad \chi_3 = \text{a}^* + \Delta \frac{i}{\chi}.
\]

Equation (3.3.5) is the quadratic equation in \( \beta^2 \) and \( \beta_1^2, \beta_2^2 \) are the roots of the following equation with \( \text{Re}(\beta) > 0 \)

\[
\beta_1^2 + \beta_2^2 = -2 + c^2 (1 + \chi_2 - i\chi \varepsilon \chi_3 \varepsilon)
\]

and
\[
\beta_1^2 \beta_2^2 = 1 - c^2 (1 + \chi_2 + i\chi \varepsilon \chi_3 \varepsilon) + c^4 \chi^2
\]

Then \( \hat{\phi}(z) \) and \( \hat{T}(z) \) are written as
\[ \hat{\phi}(z) = [A \exp(-\eta\beta_1 z) + A^* \exp(\eta\beta_2 z) + B \exp(-\eta\beta_1 z) + B^* \exp(\eta\beta_2 z)] \]

(3.3.6)

\[ T(z) = \frac{(\chi^2 - \eta^2 + D^2)}{(1 - t \alpha x)} [A \exp(-\eta\beta_1 z) + A^* \exp(\eta\beta_2 z) + B \exp(-\eta\beta_1 z) + B^* \exp(\eta\beta_2 z)] \]

(3.3.7)

For surface waves, \( \hat{T}, \hat{\psi}, \psi \to 0 \) as \( z \to \infty \) in equations (3.3.2), (3.3.6) and (3.3.7), the solutions is reduce to

\[ \phi(z) = [A \exp(-\eta\beta_1 z) + B \exp(-\eta\beta_2 z)] \exp[i(\eta x - \chi t)] \]

(3.3.8)

\[ \psi(z) = C \exp[(-\eta\beta_1 z) + i(\eta x - \chi t)] \]

(3.3.9)

\[ T(z) = \frac{(\chi^2 - \eta^2 + D^2)}{(1 - t \alpha x)} [A \{\chi^2 + \eta^2(\beta_1^2 - 1)\} \exp(-\eta\beta_1 z) + B \{\chi^2 + \eta^2(\beta_2^2 - 1)\} \exp(-\eta\beta_2 z)] \exp[i(\eta x - \chi t)] \]

(3.3.10)

The solutions (3.3.8) to (3.3.10) satisfies the boundary conditions (3.2.11)-(3.2.13), and we obtain the following equation

\[ (2 + p_1)\beta_1 A + (2 + p_1)\beta_2 B - \iota((1 + p_1)\beta_3^2 + 1)C = 0 \]

(3.3.11)

\[ \left(\frac{2}{v^2} + p_2 - c^2\right)A + \left(\frac{2}{v^2} + p_2 - c^2\right)B - \iota\left(\frac{2}{v^2} + p_2\right)\beta_3 C = 0 \]

(3.3.12)
\(-\eta\beta_1 + h)[c^2 + (\beta_1^2 - 1)]A + (-\eta\beta_2 + h)[c^2 + (\beta_2^2 - 1)]B = 0 \quad (3.3.13)

Eliminating A, B, C from (3.3.11) to (3.3.13), we get

\[
\begin{vmatrix}
(2 + p_1)\beta_1 & (2 + p_1)\beta_2 & (1 + p_1)\beta_3^2 + 1 \\
\frac{2}{v^2} + p_2 - c^2 & \frac{2}{v^2} + p_2 - c^2 & \left(\frac{2}{v^2} + p_2\right)\beta_3 \\
(-\eta\beta_1 + h)(c^2 + (\beta_1^2 - 1)) & (-\eta\beta_2 + h)(c^2 + (\beta_2^2 - 1)) & 0
\end{vmatrix} = 0
\]

or

\[
[2 + (p_2 - c^2)v^2][2 - c^2v^2 + p_1(1 - c^2v^2)][\beta_1^2 + \beta_2^2 + \beta_3(\beta_1 + \beta_2) - 1 + c^2]
\]

\[-[4 + 2p_1 + (2p_2 + p_1p_2)v^2]\beta_1\beta_2\beta_3(\beta_1 + \beta_2)
\]

\[
= -\frac{\eta}{h}[(\beta_1^2 + \beta_2^2)(2 + (p_2 - c^2)v^2)[2 - c^2v^2 + p_1(1 - c^2v^2)]
\]

\[-(4 + 2p_1 + (2p_2 + p_1p_2)v^2)\beta_3(\beta_1^2\beta_2^2 + 1 - c^2)]
\]

\[(3.3.14)\]

**3.4 LIMITING CASES**

(a) Small Thermal Coupling:

For most of the materials, \(\varepsilon\) is small at normal temperature. Therefore, we approximated the frequency equation by assuming \(\varepsilon << 1\). For \(\varepsilon << 1\), we obtain from equation (3.3.5) the approximated expressions for \(\beta_1\) and \(\beta_2\) as
\[ \beta_1 = (1 - c^2)^{1/2} \left[ 1 + \frac{\varepsilon c^2 \chi_1 \chi_3}{2(\chi_2 - 1)(1 - c^2)} \right] \] (3.4.1)

\[ \beta_2 = (1 - \chi_2 c^2)^{1/2} \left[ 1 - \frac{\varepsilon c^2 \chi_2 \chi_1 \chi_3}{2(\chi_2 - 1)(1 - \chi_2 c^2)} \right] \] (3.4.2)

These approximated expressions for \( \beta_1 \) and \( \beta_2 \) are inserted in equation (3.3.14) to obtain the approximated frequency equation.

(b) Small Reduced Frequency \( \chi << 1 \):

For small reduced frequency \( \chi << 1 \), we obtain from equation (3.3.5), the following approximated expressions

\[ \beta_1^2 + \beta_2^2 = 2 - i \frac{c^2}{\chi} (1 + \varepsilon) - c^2 [1 + a^* + \varepsilon(a + \Delta a^*)] \] (3.4.3)

\[ \beta_1 \beta_2 = \frac{1 - i}{\sqrt{2\chi}} (1 - c^2 + \varepsilon)^{1/2} \]

\[ + \frac{1 + i}{2\sqrt{2}} \frac{[1 + a^* + \varepsilon(a + \Delta a^*)]}{(1 - c^2 + \varepsilon)^{1/2}} \chi^{1/2} \] (3.4.4)

3.5 SPECIAL CASES

(a) In absence of initial stress and magnetic field the frequency equation (3.3.14) reduces to the following equation
\[
[2 - c^2 v^2] [\beta_1^2 + \beta_2^2 + \beta_1 \beta_2 - 1 + c^2] - 4 \beta_1 \beta_2 \beta_3 (\beta_1 + \beta_2) \\
= -\frac{h}{\eta} [(\beta_1^2 + \beta_2^2) (2 - c^2 v^2)] - 4 \beta_3 (\beta_1^2 \beta_2^2 + 1 - c^2)]
\] (3.5.1)

which agrees with the equation (20) of Dawn and Chakraborty (1988).

(b) In the absence of initial stress, magnetic field and thermal parameters, the frequency equation (3.3.14) reduces to

\[
(2 - c^2 v^2) = 4 \sqrt{(1 - c^2)} \sqrt{(1 - c^2 v^2)}
\] (3.5.2)

which is the frequency equation of Rayleigh wave in an isotropic elastic solid half space.

### 3.6 Numerical Analysis of the Frequency Equation

If we put \( c^2 = c^* + \varepsilon(\xi_1 + t\xi_2) \), where \( c^* \) is the classical Rayleigh wave velocity and \( \xi_1 \) and \( \xi_2 \) are two reals depending on the reduced frequency \( \chi \), and \( a, a^* \) then

\[
\eta = \frac{\chi}{c^*} \left(1 - \frac{\varepsilon \xi_1}{2c^*} - t \frac{\varepsilon \xi_2}{2c^*}\right)
\] (3.6.1)

The velocity of propagation is equal to \( (c^* + \frac{\varepsilon \xi_1}{2c^*}) \) and the amplitude – attenuation factor is equal to \( \exp(\frac{\varepsilon \xi_2}{2c^*}) \) with \( \xi_2 < 0 \). The velocity of
propagation and amplitude-attenuation factor are computed for the following material parameters of Aluminium.

\[ E = 6.9 \times 10^{10} \text{Nm}^{-2}, \quad \sigma = 0.33, \]
\[ \rho_0 = 2700 \text{Kg.m}^{-3}, \quad c_v = 987.9 \text{J.Kg}^{-1}.\text{K}^{-1}, \]
\[ K = 205.85 \text{Jm}^{-1}.\text{s}^{-1}.\text{K}^{-1}, \quad \alpha = 0.01, \]
\[ k_r = 0.05, \quad \omega = 2\text{s}^{-1}, \quad T_0 = 293 \text{K}, \]
\[ x = 0.01m, \quad a = 0.05s, \quad a^* = 0.2s, \]
\[ \varepsilon = 0.05, \quad \mu_e = 1, \quad c^* = 0.9554, \quad \chi = 0.1 \]

The generalized Lame’s constant \( \bar{\lambda} \) and \( \bar{\mu} \) are related as

\[ \bar{\lambda} = \frac{E\sigma}{\zeta(1+\sigma)(1-2\sigma)}, \quad \bar{\mu} = \frac{E}{2\zeta(1+\sigma)} \quad (3.5.2) \]

where \( \zeta \) is initial stress parameter, \( E \) is the Young’s modulus and \( \sigma \) is Poisson ratio. \( \zeta = 1 \) corresponds to the isotropic elastic medium.

The velocity of propagation and amplitude-attenuation factor are plotted against magnetic field parameter \( H \) in Figs. 1 and 2, respectively, when \( p = -2, \) 0 and 2. For \( p = -2 \), the velocity is \( 0.4774 \times 10^3 \text{m/s}^{-2} \) near \( H=0 \) and it decreases slowly with the increase in the magnetic field. It attains its value \( 0.4751 \times 10^3 \text{m/s}^{-2} \) at \( H = 20 \times 10^5 \) oe. The variation of the velocity for \( p = -2 \) is shown by solid line in Figs. 1. For \( p=2 \), the velocity is
0.73964×10³ m/s² near \( H=0 \) and it decrease sharply with the increase in the magnetic field. It attains its value 0.48064×10³ m/s² at \( H=20\times10^5 \) oe. The variation of the velocity for \( p=2 \) is shown by solid line with circle as center symbols in Figs.1. In absence of initial stress, the variation for \( p=-2 \) and 2 reduce to that shown by solid line with triangles as centre symbols in Figs.1. The amplitude-attenuation factors \((\times10^3)\) for \( p=-2, \ 0 \) and 2 are shown graphically against magnetic field in Figs. 2. From Fig.1 and Fig. 2, it is observed that the velocity of propagation and amplitude-attenuation factors are significantly affected by initial stress parameter for lower range of magnetic field parameter.

The velocity of propagation and amplitude-attenuation factor are plotted against initial stress parameter \( p \) in Fig. 3 and Fig. 4, respectively, when \( H=0, \ 10\times10^5 \) oe and \( 100\times10^5 \) oe. For \( H=10\times10^5 \) oe, the velocity is 0.4749×10³ m/s² at \( p=-2 \) and it decreases slowly to its minimum value 0.47481 at \( p=-1.7 \). Thereafter, it attains its maximum value 0.48851×10³ m/s² at \( p=2 \). The variation of the velocity for \( H=10\times10^5 \) oe, is shown by solid line with triangles as centre symbols in Fig. 3. For \( H=100\times10^5 \) oe, the velocity is 0.47775×10³ m/s² at \( p=-2 \) and it increase slowly for the given range of \( p \) and attains its maximum value 0.47783×10³ m/s² at \( p=2 \). The variation of the velocity for \( H=100\times10^5 \) oe, is shown by solid line with circle as center symbols in Fig. 3. In absence of magnetic field, the variation for \( H=10\times10^5 \) oe and \( 100\times10^5 \) oe reduce to that shown by solid line without centre symbols in Fig.3. The amplitude-attenuation factors \((\times10^3)\) for \( H=0, \ 10\times10^5 \) oe and \( 100\times10^5 \) oe are shown graphically against initial stress in Fig. 4. From Fig. 3 and Fig. 4, it is
observed that the velocity of propagation and amplitude-attenuation factors are significantly affected by magnetic field at each value of initial stress parameter.
Figure 3.1. Variations of the velocities of propagation of Rayleigh wave versus the magnetic field parameter, when $p = -2, 0$ and $2$. 
Figure 3.2. Variations of the amplitude-attenuation factors of Rayleigh wave versus the magnetic field parameter, when $p = -2, 0$ and $2$. 
Figure 3.3. Variations of the velocities of propagation of Rayleigh wave versus the hydrostatic initial stress parameter, when $H = 0$, $10 \times 10^5$ oe and $100 \times 10^5$ oe.
**Figure 3.4.** Variations of the amplitude-attenuation factors of Rayleigh wave versus the hydrostatic initial stress parameter, when $H = 0$, $10 \times 10^5$ oe and $100 \times 10^5$ oe.