2

Analysis of energy conversion efficiency and related issues using Lagrangian MHD simulations

2.1 Introduction

In this chapter, we present results from two-dimensional MHD simulations to explore the possibility of direct energy conversion of plasma kinetic energy into electrical pulses. Details of the proposed concept and a brief summary of related works are given in Chapter 1. The major differences with earlier works described in Chapter 1 are in terms of the parameter regime, the external source of the seed magnetic field and the computational model, as explained in Chapter 1. For the present study, we assume that the load is predominantly inductive – this is a major difference from earlier works [25–27], where a resistive or capacitive load was considered. A brief discussion on a few important theoretical and technical issues
that have to be addressed are given in the following sections. Also, we report the
overall efficiency of the system with different load conditions.

2.2 Computational model

During the expansion phase, \( L_n \gg r_{Li}, c/\omega_{pi} \) and \( r_D \); where \( L_n \) is the characteristic
scale length of the plasma, \( r_{Li} \sim v_i/\Omega_i \) is the ion Larmor radius, \( v_i \sim (T_i/m_i)^{1/2} \)
is the ion thermal speed, \( \Omega_i \) is the ion cyclotron frequency, \( \omega_{pi} \) is the ion plasma
frequency and \( r_D \) is the Debye radius. Similarly, the time scale (plasma radial
expansion time \( \sim t_s \) ) is longer than an ion cyclotron period. Therefore, a single-
fluid MHD model (assuming quasi-neutrality) can be used to describe the plasma.
In addition to this, we assume that the plasma behaves like an ideal gas. The
governing equations are as follows [39]:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{2.1}
\]

\[
\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \vec{J} \times \vec{B} \tag{2.2}
\]

\[
\frac{\partial}{\partial t} \left( \rho I + \frac{\rho u^2}{2} \right) + \nabla \cdot \left[ \rho \vec{u} \left( I + \frac{u^2}{2} \right) + p \vec{u} \right] = 0 \tag{2.3}
\]

where, \( \rho \) is the density, \( \vec{u} \) is the velocity vector, \( p \) is the pressure, \( \vec{J} \) is the current
density, \( \vec{B} \) is the magnetic field and \( I \) is the internal energy. We have neglected
the resistive heating term required in Eq. 3.3, because of the high conductivity of
the plasma and short time scales involved. Similarly, the energy flux from thermal
heat conduction is neglected.

The two-dimensional MHD equations are solved using a locally-developed 2D-
2.2. Computational model

Lagrangian code based on the formulations given in [38]. The code uses an explicit finite-difference scheme for hydrodynamic equations.

For the present geometry setup, only the $\theta$-component of current exists in both coil and plasma. A filamentary model, similar to the formulations given by Novac et al. in Refs. [49–51], is used to update the coil current and the induced currents in the plasma. This model includes the effect of plasma dynamics. For this, each coil turn is broken up into a number of coaxial circular loops. Similarly, each computational cell used to describe the plasma region in the hydrodynamic calculation is assumed to be a circular loop with a rectangular cross-section. Each such loop may carry a different current, which is updated self-consistently using coupled circuit equations given below:

\[
\begin{align*}
L_T \frac{dI_c}{dt} + \sum_{j=1}^{N_a} M_{cj} \frac{dI_j}{dt} &= -I_c R_T - \sum_{j=1}^{N_a} I_j \frac{dM_{cj}}{dt} \\
M_{cj} \frac{dI_c}{dt} + L_j \frac{dI_j}{dt} + \sum_{i,j=1; i \neq j}^{N_a} M_{ij} \frac{dI_i}{dt} &= S_j \\
S_j &= -I_j R_j - \sum_{i,j=1; i \neq j}^{N_a} I_i \frac{dM_{ij}}{dt} - I_c \frac{dM_{cj}}{dt} - I_j \frac{dL_j}{dt}
\end{align*}
\]  

(2.4)  

(2.5)  

(2.6)

where, $L_T$, $R_T$ and $I_c$ are the total inductance (coil+stray+load), total resistance and coil current respectively. $L_j$, $R_j$ and $I_j$ are the self-inductance, resistance and induced current respectively for the $j^{th}$ plasma filament. $M_{ij}$ is the mutual inductance between the plasma filaments $i$ and $j$. Similarly, $M_{cj}$ is the mutual inductance between coil and $j^{th}$ plasma filament. The coupled Eqs. 2.4 and 2.5 are solved simultaneously. The cell-centered current density is obtained by dividing the current obtained using Eq. 2.4 and 2.5 by the area of the computational cell in the R-Z plane, which represents a fluid element. The node-averaged current
density is then calculated to estimate the Lorentz force. The self-inductance of a circular loop and the mutual inductance between the loops are calculated using analytical equations in terms of elliptic integrals. The magnetic field at any given point in the computational domain is calculated using the contributions from all plasma filaments and the coil loops. The field from a circular filament loop is calculated using an analytical formula in terms of elliptic integrals. The self and mutual inductances of the current loops can be calculated more accurately by using standard Finite-Element method [141]. However, for the present system involving large number of current carrying filaments, the application of Finite-Element method [141] to calculate the self and mutual inductances of the filaments at each time-step will be computationally expensive. Therefore, we have used analytical formula in terms of elliptic integrals. The estimated maximum error w.r.t. to the results of Finite-Element method [141] was about 2–3 %.

2.3 Initial conditions

A brief discussion on typical initial conditions used in this work has been given in Chapter 1. However, some important parameters are given below for the sake of completeness. The initial plasma kinetic energy and mass are taken as 140 MJ and 6 milligrams (mg) [25–27, 31, 32] respectively. Initially, the plasma undergoes free expansion in the applied magnetic field, since the kinetic pressure, $p_k$ is far higher than the magnetic pressure, $p_B$ (high $\beta = p_k/p_B$ plasma). Considering this fact, we start our simulation with an initial plasma radius of about 1.0 cm. We have taken the coil axial length as approximately 2-4 times the radius. The coil radius ($r_c$), length ($l_c$) and no. of turns ($n_c$) are 1.5 m, 4.5 m and 30 respectively. For higher efficiency, the plasma should stagnate at a radius, $r_p \sim r_c$. The seed
2.3. Initial conditions

field should be sufficient to stop the plasma expansion within this distance, which means that the initial magnetic field has a strong effect on the recovery efficiency. An initial magnetic field of $\sim 5$ T is assumed, corresponding to an initial current of $\sim 750$ kA.

Ideally, the magnetic field profile inside the plasma sphere, at the time simulation starts, must be evaluated by considering magnetic field diffusion into the sphere as it expands rapidly across the magnetic field, from an initial radius $\sim 200 \mu$m to 1 cm. However, magnetic field diffusion into the plasma sphere during those early time-scales can be neglected, due to high temperature and electrical conductivity. Hence we assume a uniform magnetic field through the sphere at $t=0$. The subsequent evolution of this field, as the sphere expands, is calculated using the frozen-field approximation, which conserves total magnetic flux ($\propto B_0$) connected with each plasma mass element. We have examined the difference in plasma dynamics as well as overall system efficiency for two cases. The first case assumes that the initial magnetic field inside the plasma is (and therefore remains) zero at all times. The second case assumes a constant initial magnetic field in the plasma, which is evolved according to the frozen-field approximation. We have found that the choice of initial $B_0$ has negligible effect on the overall system dynamics and efficiency for the system parameters considered in this work. Hence, in the rest of this chapter, we have assumed a uniform initial magnetic field in the plasma.

We have also analyzed the dependence of overall system efficiency on the initial radius of the plasma sphere, $r_p$. Negligible variation is observed for $0.01 \text{m} \leq r_p \leq 0.1 \text{m}$. Therefore, in the present study, we have assumed an initial plasma radius of $r_p = 1 \text{cm} \ll r_c$. The initial coil and load inductances ($L_c$ and $L_L$) were $\sim 1.4$ millihenry (mH) and 1.0 mH respectively.
2.4 Issues to be addressed

In this section, we discuss a few theoretical and technical issues that need to be addressed.

2.4.1 MHD instabilities

Performance degradation due to MHD interchange instabilities of the expanding plasma are a major concern with this concept. The plasma expanding into the magnetic field can undergo Rayleigh-Taylor (RT) instability. The instability amplitude should be small enough so that the expanding plasma layer is stable at least during first expansion phase. From MHD theory, the RT instability growth rate $\gamma = \sqrt{g/L_n}$; for $kL_n \gg 1$ and $\gamma = \sqrt{g}$; for $kL_n \ll 1$, where, $k$ is the wave number, $g$ is the deceleration and $L_n$ is the density scale-length.

From the simulation, the average deceleration and density scale-length are found to be $1 \times 10^{14}$ ms$^{-2}$ and 0.2 m respectively. The large Larmor radius (LLR) effect on RT instability growth rate [47, 48] can be neglected during the first expansion phase for the present case. This is because the ratio $r_{Li}/L_n \ll 1$, where $r_{Li} \sim 10^{-3}$ m is the time-averaged ion Larmor radius, defined as the ratio of thermal expansion velocity to the cyclotron frequency based on the magnetic field. The growth rate thus evaluated for the wavelength perturbations in the range $10^{-3}$ to 0.5 m is found be of the order of $10^7$s$^{-1}$, which is comparable with the inverse of plasma expansion time. Therefore, we expect the RT instability may not be so critical for small amplitude initial perturbations during the first expansion phase of the plasma.

Similarly, in Ref. [40], Winske has reported a detailed theoretical and numerical study which could explain the experimentally observed results (see references given
2.4. Issues to be addressed

in Ref. [40]) on the development of flute modes (similar to RT instability) on expanding plasma clouds. From various experimental results observed and numerical simulations performed, Winske [40] has concluded that the single most important parameter which determines the evolution of flute mode instability is the ratio of ion Larmor radius, $r_{Li}$, to the plasma confinement radius, $R_w$. When $r_{Li}/R_w > 1$, large flute modes are observed which shows nonlinear dynamics. For $r_{Li}/R_w \leq 1$, the flute modes are smaller and the non-linear surface modes appear only at the time of maximum expansion. When $r_{Li}/R_w \ll 1$, only a weak instability is detected. According to Winske [40], such results are consistent with linear theory. A detailed discussion, including non-linear effects, can be found in Ref. [40]. For the present case, the ratio $r_{Li}/R_w$ is $\sim 10^{-3}$. Therefore, only a weak instability is expected during the first expansion phase of the plasma.

The MHD interchange instabilities, however, need to be analysed separately with different initial conditions of applied perturbations. In Chapter 4, we have presented this analysis in detail.

2.4.2 Choice of the load

Depending on the application, the load can be purely resistive, inductive, capacitive or a combination of these. Most of the earlier works have examined the case of a purely resistive load. Higher efficiency is achieved by using an optimized capacitor-diode load where a capacitor is switched to the coil circuit at the moment of peak current in the coil [26]. However, this demands a coil discharge on a timescale which is short relative to that of plasma expansion time [27] ($\frac{\sqrt{L_c C}}{t_p} \ll t_p$, where $t_p$ is the plasma expansion time $\sim 10^{-7}$ s, $L_c$ is the coil inductance and $C$ is the capacitance).

For the present study, we assume that the load is predominantly inductive – this is an important difference as compared to earlier works [25–27], where a resistive or
2.4. Issues to be addressed

Capacitive load was considered. The inductive load could be electrically decoupled from the pickup coil after the completion of first expansion phase of the plasma and could be subsequently switched into a different load (not shown in Fig. 1.4) for $t \geq t_p$. The decoupling of the inductive load from the pickup coil will be necessary due to the fact that, after the first expansion phase of the plasma, the amplitude of perturbations on the irregular surface of the plasma caused by RT instability is likely to be so large that the plasma can not maintain its stability [33]. The remaining plasma energy in the form of inductive energy of its diamagnetic current will convert into plasma heating due to the penetration of the field into the plasma at a late stage.

In this work, we focus on system performance with an inductive load. However, to facilitate comparison, we have also performed a sample simulation with a resistive load.

2.4.3 Coil inter-turn breakdown

Since the inductive energy recovery system proposed here has a total operational time less than $\sim 0.2 \, \mu s$, extremely high voltages (few hundreds of MV) across the coil are expected. Ultra-high voltages $\sim 500 \, \text{MV}$ were predicted in earlier work [25, 26]. One possible method to prevent the inter-turn voltage break-down is to use magnetic insulation. The concept of magnetic insulation has been investigated by many authors, such as Hirsch [43] and Winterberg [41]. Mima et al. [25] has discussed the effect of magnetic insulation to reduce the possibility of electric breakdown between neighboring pickup coil segments. Therefore, in Ref. [25], they expected the breakdown voltage to be very high ($\sim 10 \, \text{MV}$) for their design. The application of this concept in high voltage transformers have been investigated by Winterberg [42], Novac et al. [44] and Istenic et al. [45]. Magnetic insulation
has proved to be the practical technique for use in the development of high-voltage components like transmission lines and plasma opening switches etc (see Ref. [44] and References therein). The magnetic field required for insulation can either be provided externally or by the system itself (magnetic self-insulation). Magnetic self-insulation is more appropriate for the present system. More details on magnetic self-insulation method used in high voltage helical transformer coils can be found in Ref. [44,45].

From our simulation results for the present case, the maximum average inter-turn voltage is found to be $\sim 25$ MV, which is higher than the break-down voltage $\sim 0.5$ MV obtained using the equations provided in Ref. [45,46] without considering magnetic insulation. Considering magnetic insulation, the relation between the breakdown voltage ($V_b$) and magnetic field is given by [43,45]

$$B \geq \sqrt{\frac{2mE}{ed_g}} \quad (2.7)$$

where, $E = V/d_g$ is the electric field, $V$ is the voltage and $m$ and $e$ are the mass and charge of an electron. Using this formulation, Novac et al. [44,45] have studied the application of magnetic self insulation to eliminate breakdown between the coil turns of a helical transformer. For helical coils, the inter-turn electric field is along the axial ($E_z$) direction. Therefore, the radial component of magnetic field ($B_r$) should be used in Eq. 2.7, assuming axisymmetry. From the simulation results, we have found that the magnetic field component, $B_r \geq 0.25$ T between the coil turns. Therefore, the breakdown voltage $V_b \geq 125$ MV for $d_g = 0.15$ m. This is higher than the average inter-turn voltage ($\sim 25$ MV) obtained in our simulation. A detailed discussion, including the effect of field variation along the pitch of the coil, is given in Sec. 2.5.

Finally, with an optimized design having longer plasma expansion time, e.g. by
increasing the coil mean radius, and increasing inter-turn separation (pitch) of the coil, the voltage level can be reduced to an acceptable level. A brief discussion of this is provided in later sections.

In short, we have listed a few methods to mitigate the problem of inter-turn break-down. However, a detailed optimization study in this direction may be required to maintain the internal voltage levels within the acceptable range of existing technologies.

2.5 Results and discussion

The initial conditions used are within the range of parameters given in Sec. 2.3. The plasma sphere is assumed to be centered at \((r,z) = (0,0)\) and because of the symmetry only one quarter of the system is simulated. Fig. 2.1 shows the different stages of plasma expansion.

In the early phase of plasma dynamics, the plasma sphere has high initial directed velocity \(\sim 10^7 \text{ ms}^{-1}\) and moderately high plasma \(\beta\) value. Therefore, it expands freely across the magnetic field (radial direction) as well as along the magnetic field (axial direction). Expansion perpendicular to the field is decelerated by the progressive increase in magnetic field due to magnetic flux compression. Expansion along the axial direction, at radii close to the axis, is not significantly affected. This is because motion along the \(z\)-direction is opposed by \(B_r\), and \(B_r \to 0\) for \(r \to 0\). Hence the plasma sphere tends to elongate in the direction of the magnetic field. This can be readily seen from the density contour plots of Fig. 2.2 at different times. This is consistent with the earlier reported results [25–27]. Pure Lagrangian computational scheme fails when Large deformations occurs. Consequently, the simulations presented here are performed with a coarse mesh.
(only near the axis) and the calculations are stopped when the radial expansion of the plasma comes to a halt at \( z = 0 \) location. This does not significantly change the calculated system efficiency. We see that plasma expansion nearly stops at a radius \( \sim 1.23 \) m at \( z = 0 \) location. The plasma stopping radius can be roughly obtained by equating the magnetic energy excluded by the plasma to the initial plasma energy; \( 4\pi r_{\text{max}}^3/3)(B^2/2\mu_0) \sim E_p \), where \( E_p \) is the initial plasma energy.

This leads to \( r_{\max} \sim 8.5 \times 10^{-3}(E_p/B^2)^{1/3} \). The maximum radius thus evaluated is \( \sim 1.5 \) m, which is higher than the \( r_{\max} \) obtained from simulation. The analytical estimate for \( r_{\max} \) is likely to over estimate the radius since it has neglected the field amplification due to magnetic flux compression. However, considering the average magnetic field during the expansion phase (\( \sim 6.3 \) T), the \( r_{\max} \) evaluated is \( \sim 1.28 \) m, which is close to the numerical result.

The plasma expansion radius along the field lines can be roughly estimated (neglecting the deceleration produced by magnetic field component, \( B_r \)) using the typical expansion velocity \( \sim 10^7 \) m/s and total radial expansion time \( \sim 2 \times 10^{-7} \) s, which is found to be \( \sim 2 \) m and is consistent with the simulation value \( \sim 1.99 \) m.

The time evolution of plasma radius and the magnetic field on the surface of the plasma at \( z=0 \) are shown in Fig. 2.3. As mentioned earlier, because of the high initial directed velocity and \( \beta \) value, the early stage of plasma expansion is less affected by the magnetic pressure. However, as the plasma expands, its pressure and density decreases continuously and the magnetic field inside the compression volume increases. Therefore the deceleration by the magnetic pressure dominates in the later stages of expansion. It is clear from Fig. 2.3 that the magnetic pressure tends to dominate at a time \( t \sim 0.1 \) \( \mu \) s.

Fig. 2.5 shows the radial velocity profile at different times for an axial location \( z=0 \). The initial velocity profile is almost linear and then decreases due to magnetic
2.5. Results and discussion

pressure. The velocity at the outer radius of the plasma decreases faster (high magnetic field on the surface) and goes to zero at the ‘stagnation point’. The magnetic field on the plasma surface at the stagnation point is 7.6 T. The radial profile of the normalized density, for the z=0 plane, and at different times, is shown in Fig. 2.4. The plasma forms a near shell-like geometry at the stagnation point, where the outer surfaces slow down due to magnetic pressure and the inner region catches up with the outer region, as shown in Fig. 2.2 and 2.4. The plasma density scale length, $L_n$ varies over the range ~0.1 to 0.2 m.

One important parameter which determines the efficiency of the flux compression system is the ratio of magnetic diffusion time ($t_d = \mu_0 \sigma L^2$) to the compression time ($t_c \sim 10^{-7}$); where $L$ is the system dimension and $\sigma$ is the conductivity. The necessary condition is that this ratio, $R_e = t_d/t_c \gg 1$. For the present case $R_e \sim 10^7$ (for $L \sim 0.1$ m and resistivity at stagnation point $\sim 10^{-8}$ $\Omega$m), indicating negligible magnetic field diffusion over the time-scales of interest.

Fig. 2.6 shows the spatial variation of the magnetic vector potential, $A_\theta$. This represents field line contours inside the coil, including the plasma region, at different times. The magnetic field is given by: $B_z = \frac{1}{r} \frac{\partial \psi}{\partial r}$, where $\psi = rA_\theta$ is the stream function. The magnetic field outside the plasma is amplified by magnetic flux compression, while the field inside the plasma gets reduced by the diamagnetic ($\theta$) current produced by the plasma. As expected, the maximum magnetic field between the coil and plasma, inside the flux compression volume, is observed at the axial midplane $z=0$.

Fig. 2.7 shows the time variation of normalized electrical energy stored in the load, coil current and inductance. The normalizations are performed with respect to their initial values (initial inductive energy in the load is $E_{L0}=281$ MJ). The plasma energy conversion efficiency is defined by $\eta_p = \frac{E_{L_f}-E_{L0}}{E_p}$, where $E_{L_f}$ and
2.5. Results and discussion

Figure 2.1: The Lagrangian mesh which shows the different stages of plasma expansion ($t=0.03 \mu s$, $0.08 \mu s$, $0.15 \mu s$ and $0.18 \mu s$). The square dots represent the coil loop locations. Only one quarter of the system is shown because of the symmetry.

Figure 2.2: Evolution plasma density ($kg/m^3$) contours (the contour lines are spaced linearly) at different times during expansion phase. Only one quarter of the system is shown because of the symmetry.
2.5. Results and discussion

Figure 2.3: The variation of plasma radius at $z = 0$ and normalized magnetic field with respect to initial magnetic field ($B_0 \sim 5$ T).

Figure 2.4: The radial variation of normalized density at different times for $z=0$ plane. The normalization factor is the maximum value of the density along the radial direction for a given time.

$E_p$ are the final load and initial plasma energy. The plasma energy recovered in the inductive load is 97 MJ, corresponding to an efficiency of 69%. The inductive energy increase in the coil and the residual plasma energy (kinetic + internal energy) were 15 MJ and $\sim 28$ MJ respectively. We have defined the overall system efficiency, $\eta_s$, for $t \leq t_p$ as below:

$$\eta_s = \frac{E_L}{E_i + E_p} \quad (2.8)$$
2.5. RESULTS AND DISCUSSION

Figure 2.5: The radial velocity profile at different times for \( z=0 \) plane.

Figure 2.6: Contours of magnetic field lines represented by magnetic vector potential \( (A_\theta \text{ in Tesla-meter}) \), at different times \( (t=0, 0.056\mu s, 0.08\mu s \text{ and } 0.18\mu s) \)

where, \( E_i = \frac{1}{2}(L_c + L_L)I_0^2 \) is the total initial electrical energy in the system. Note that \( E_i \) is defined after the exclusion of the capacitor by the simultaneous switching action of switches \( s_1 \) and \( s_2 \), and \( t_p \) is the time required to complete the first expansion phase. The above expression assumes that the inductive energy remaining in the coil (not the load) cannot be recovered after the completion of first expansion phase. Also, we assume the load is decoupled from the system.
2.5. Results and discussion

for $t \geq t_p$, as discussed earlier. The overall system efficiency, $\eta_s$, calculated using Eq. 2.8 is found to be equal to 56\%. The peak coil current obtained is 870 kA.

The effect of load inductance on the conversion efficiencies ($\eta_p$ and $\eta_s$) is shown in Table 2.1. It is clear from the table that higher efficiency is achieved as load inductance increases. For the cases with low load inductance compared to the coil inductance, the energy efficiency in the load is found to be low. This is because most of the energy will be inductively stored in the coil. It is worth noting that as the load inductance increases, despite higher $\eta_s$ value, the initial electrical energy requirement increases since we are keeping the initial magnetic field constant at $\sim 5$ T. This is because we assume that the load is active in the circuit during the priming of the system.

Next, for the sake of comparison, we have analysed the system efficiency with different resistive loads. Table 2.2 summarizes these results. The efficiency increases as the load resistance increases and tends to saturate, which is consistent with the observations reported in Ref. [27], where the maximum efficiency was reported with a resistive load of $\sim 1400 \, \Omega$. In contrast to inductive loads, the initial energy required for different cases listed in Table 2.2 are the same $\sim 400 \, \text{MJ}$.

Next, we have examined the variation of plasma stopping radius, $r_{\text{max}}$, at the
Table 2.1: System performance for different inductive load conditions. \( L \) is the load inductance, \( E_i \) is the initial electrical energy, \( E_L \) is the inductive energy stored in the load, and \( \eta \) is the conversion efficiency.

<table>
<thead>
<tr>
<th>( L ) (mH)</th>
<th>( E_i ) (MJ)</th>
<th>( E_L ) (MJ)</th>
<th>( \eta_p ) (%)</th>
<th>( \eta_s ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>407</td>
<td>8</td>
<td>5.7</td>
<td>5.5</td>
</tr>
<tr>
<td>0.1</td>
<td>421</td>
<td>16</td>
<td>11.5</td>
<td>10</td>
</tr>
<tr>
<td>0.3</td>
<td>478</td>
<td>41</td>
<td>29</td>
<td>26</td>
</tr>
<tr>
<td>0.5</td>
<td>534</td>
<td>63</td>
<td>45</td>
<td>38</td>
</tr>
<tr>
<td>0.7</td>
<td>590</td>
<td>79</td>
<td>56</td>
<td>47</td>
</tr>
<tr>
<td>1.0</td>
<td>675</td>
<td>97</td>
<td>69</td>
<td>56</td>
</tr>
</tbody>
</table>

Table 2.2: System performance for different resistive loads. \( R \) is the load resistance, \( E_L \) is the resistive energy across the load, and \( \eta \) is the conversion efficiency. The total initial energy is 400 MJ.

<table>
<thead>
<tr>
<th>( R ) (( \Omega ))</th>
<th>( E_L ) (MJ)</th>
<th>( \eta_p ) (%)</th>
<th>( \eta_s ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>8</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>250</td>
<td>31</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>500</td>
<td>61</td>
<td>43</td>
<td>15</td>
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<td>90</td>
<td>64</td>
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<td>24.7</td>
</tr>
<tr>
<td>1000</td>
<td>101</td>
<td>72</td>
<td>25</td>
</tr>
</tbody>
</table>

axial midplane \((z=0)\) for different initial magnetic fields. The coil radius, length and number of turns are 3 m, 9 m and 30 respectively. The initial magnetic field is varied from 1.4 to 6.3 T. A comparison of simulation results with a simple analytical expression described earlier this section is shown in Fig. 2.8. Reasonable agreement is found, except for the cases with low initial magnetic field. The difference in results for the cases with low magnetic field is are due to the fact that the analytical expression neglects the effect of field amplification, and hence over-estimate the stopping radius. This effect will be higher for the cases with low initial magnetic fields, where the radial plasma expansion will be higher and likely to have considerable difference between initial and final magnetic fields. The variation of \( B_f/B_0 \) is shown in Fig. 2.8, where \( B_0 \) and \( B_f \) are the initial and final magnetic fields. Another reason for the difference in simulation results and
analytical result is the assumption of uniform expansion of the plasma sphere while deriving the expression for $r_{max}$. In reality, the plasma sphere expands non-uniformly, expanding more along the axial direction. This leads to underestimation of $r_{max}$ at $z=0$.

Figure 2.8: Variation of plasma stopping radius for different values of initial magnetic fields

The time evolution of coil inter-turn voltage for typical system dimensions described in the beginning of this section (Case-1) is shown in Fig. 2.9. The change in flux inside the coil is high during the period where the magnetic deceleration is small ($t \leq 0.1 \mu s$). The voltage increases to a peak value of about $\sim 27$ MV (peak electric field of 180 MV/m, for pitch of the coil equal to 0.15 m) during this period and then decreases continuously due to the magnetic pressure driven deceleration of the plasma expansion. This is because the change in flux decreases due to deceleration of the plasma. An optimized design with appropriate coil dimensions and initial magnetic field can reduce the average inter-turn voltage. For example, with an increase of 33 % in coil radius (case-2), with an initial magnetic field of 3.8 T and coil length equal to three times the coil radius, the inter-turn voltage decreases to 19 MV (E=95 MV/m); corresponds to a 42 % decrease in the voltage (see Fig. 2.9, case-2). The electric field between the coil turns are reduced by 2.8 times. However, with increased coil dimensions the initial energy required will
increase by 20\%. Similarly, keeping the system dimensions are the same as case-1 and increasing the magnetic field from 5 to 6.4 T (case-3) the voltage and electric field are reduced by 12.5\% despite the short operational time (see Fig. 2.9, case-3). Even though the internal voltages are within the range of values discussed in Sec. 2.4.3, a detailed optimization study in this direction is needed to maintain the voltage levels within the acceptable range without reducing overall system efficiency.

Finally, we have analysed the magnetic self-insulation process for the entire pulse duration. Eq. 2.7 can be written as $B_r d_g \geq \sqrt{2mV/e}$, where $V$ is the inter-turn voltage. Integrating this equation along the pitch of the coil turn (to account for the variation of $B_r$ along the axial direction between coil turns), the following equation can be derived as a necessary condition to satisfy magnetic self-insulation criteria [44,45].

$$V = \left( \sqrt{2mV/e} \right) / \left( \int_0^{p/2} B_r(r_c, z)dz \right) \leq 1 \quad (2.9)$$

where, $r_c$ is the coil radius and $p$ is the pitch. It is known that that the magnetic component, $B_r$, produced by a helical coil is smallest near the center of the coil. Therefore, the present analysis concentrates on this region. However, for purposes
of comparison, we have provided the results obtained for two turns located at the axial end of the coil. The variation of $\alpha_V$, the ratio given in Eq. 2.9, for two different locations (center and axial end point) are shown in Fig. 2.10. It is clear from the figure that the criteria for magnetic self-insulation is fairly satisfied. It is worth noting that the factor $\alpha_V$ is high (~a factor of 2) for the turns located at the center of the coil.

Figure 2.10: Time evolution of $\alpha_V$ defined in Eq. 2.9 for two different locations. It is clear that, the ratio $\alpha_V \leq 1$. See text for the discussion.

2.6 Limitations of the study

In the present work we have mainly focused on the introduction of the concept, the dynamics of plasma across the magnetic field and the energy conversion efficiency. Therefore, similar to earlier works, we have started our simulation from the time when plasma is created and the initial magnetic field is setup. Also, we have not considered the implications of D-3He systems for inertial fusion energy (IFE) drivers and targets. We have assumed that the fast opening switches capable of absorbing hundreds of MJ energies can be realized with existing or near-term engineering technologies. We assume such systems and technologies will be devel-
2.7. Conclusions of this study

A conceptual study of magnetic flux compression inside a cylindrical coil by an expanding fusion plasma sphere have been performed numerically using 2D magneto-hydrodynamic (MHD) simulations. Preliminary theoretical analysis shows that, for an unperturbed initial plasma, MHD interchange instabilities would not grow during the first expansion phase of the plasma for typical system parameters examined here. A few important theoretical and technical issues that need to be addressed have been discussed.

It is observed that during the final stage of MFC, the plasma shape becomes distorted (non-spherical) due to non-uniform deceleration caused by the magnetic field outside the plasma sphere. In particular, there is elongation of the plasma in the axial direction. These effects, collectively, lead to a non-spherical expansion of the plasma with large deformation.

The concept can be used as a method to convert a part of fusion plasma kinetic
energy into pulsed electrical energy. An overall system efficiency of ~56\% obtained for a typical system with appropriate load conditions. Approximately 78\% of plasma kinetic energy is converted into electrical energy with appropriate inductive load conditions. The system performances with different inductive and resistive load conditions are studied.

The simulation results indicate that the proposed system is promising in terms of overall efficiency. However, ultrahigh coil inter-turn voltages (~25 MV) are predicted. Therefore, the application of magnetic self-insulation to avoid coil inter-turn break-down is considered. Even though the voltage levels are within the theoretically acceptable range, a detailed optimization study is required to avoid coil inter-turn break-down without reducing the system efficiency.