CHAPTER : 5

A ROTATING BIANCHI TYPE II COSMOLOGICAL MODEL WITH VISCIOUS FLUID AND HEAT FLUX
5.1: INTRODUCTION

Recently there has been a renewed debate on the problem of universal rotation (Birch (1982)). It has been stated that there is some observational evidence that the universe is rotating. The concept of a rotating universe with vanishing expansion and shear, appeared with the advent of Godel (1949) universe. But it is well known that our universe is in the expanding phase. So it seems reasonable to construct and investigate rotating, expanding cosmological models.

Usually the material content of cosmological model is assumed to be perfect fluid. But perfect fluid distribution is an idealization. It does not explain the dissipative phenomena such as viscosity, heat flux etc. which are present in the early stages of the evolution of the universe. The currently observed large entropy per baryon and the remarkable isotropy of the background radiation in the present epoch can be explained satisfactorily by considering the effects of viscosity and heat flow. (Misner(1968), Caderni and Fabbri(1978)). Therefore the models containing viscous
fluid and heat flow are more realistic than the usual perfect fluid models.

The anisotropic cosmological models of a Bianchi group admitting the viscosity and heat flux are relatively few in the literature. Banerjee and Sanyal (1988) have investigated an irrotational Bianchi type V viscous fluid world model with heat flow. Koppar and Patel (1988a) have discussed some non-static Godel-type universes with viscous fluid and heat flow. They (Koppar S S and Patel L K (1988b) ) have also investigated a rotating Bianchi type II cosmological model containing viscous fluid and heat flow.

The main purpose of this chapter is to discuss a new rotating Bianchi type II world model with viscous fluid and heat flow which is different from that discussed by Koppar and Patel (1988b).

5.2 : THE METRIC AND THE FIELD EQUATIONS

In general case finding exact solutions of the Einstein's field equations describing rotating universes is associated with many mathematical difficulties. Therefore one must make some simplifying assumptions to derive such exact soluitons.

Therefore the line element corresponding to rotating Bianchi type II universe is taken in the form ( Singh, Koppar and Patel, 1989 )
\[ ds^2 = \left[ dt + \alpha A_0 B^a (dr + 4m^2 d\phi) \right]^2 - A_o B^{2a} \left[ dr + 4m^2 d\phi \right]^2 - B^2 \left[ \text{Cosec}^2 \theta \, d\theta^2 + d\phi^2 \right] \]  

...(5.2.1)

where \( B \) is a function of time \( t \), \( a \), \( \alpha \) and \( A_0 \) are constants and \( m \) is a function of \( \theta \) satisfying the differential equation

\[ 4m \, \text{Sin} \theta \frac{dm}{d\theta} = \lambda \]  

...(5.2.2)

Here \( \lambda \) is a non-zero constant.

Let us choose the following orthogonal tetrad \( \theta^a \) (a=1,2,3,4) for the metric (5.2.1):

\[ \theta^1 = A_0 B^a \left[ dr + 4m^2 d\phi \right] \quad \theta^2 = b \, \text{Cosec} \theta \, d\theta \]

\[ \theta^3 = B \, d\phi \quad \theta^4 = dt + \alpha \theta^1 \]  

...(5.2.3)

The metric (5.2.1) can be expressed in the Cartan's frame (5.2.3) as

\[ ds^2 = [\theta^4]^2 - [\theta^1]^2 - [\theta^2]^2 - [\theta^3]^2 = g_\langle ab \rangle \, \theta^a \, \theta^b \]  

...(5.2.4)

where \( g_{\langle ab \rangle} = e^{i}_{(a)} e^{k}_{(b)} g_{ik} \),  

\[ e^{i}_{(a)} \theta^a = dx^i \]

are the tetrad components of the metric tensor \( g_{ik} \).

Using (5.2.3),(5.2.4) and the Cartan's equations of structure one can find the tetrad components \( R^a_{bcd} \) of the curvature tensor for the metric (5.2.1). From \( R^a_{bcd} \) the tetrad components \( R_{\langle ab \rangle} = R^c_{abc} \) of Ricci tensor can be easily obtained. This is a routine
calculation and hence we shall not enter into the details here.

The non-vanishing $R_{(ab)}$ are listed for ready reference:

\[
R_{(14)} = -2\alpha \left( \frac{B}{B} - a \left( \frac{\dot{B}}{B} \right)^2 - A_o^2 \lambda^2 B^{2(a-2)} \right)
\]

\[
R_{(22)} = R_{(33)} = \left[ \alpha^2(a+1) - (a+2) \right] \left( \frac{\dot{B}}{B} \right)^2 + (\alpha^2-1) \left( \frac{\ddot{B}}{B} \right)
\]

\[
+ 2(1-\alpha^2)A_o^2 \lambda^2 B^{2(a-2)}
\]

\[
R_{(11)} = \left[ \alpha^2(a+2) - a \right] \left( \frac{\dot{B}}{B} \right)^2 - 2A_o^2 \lambda^2 B^{2(a-2)} + a \left[ \alpha^2(a-1) - (a+1) \right] \left( \frac{\ddot{B}}{B} \right)^2
\]

\[
R_{(44)} = \left[ 2 + a - a^2 \right] \left( \frac{\dot{B}}{B} \right)^2 - 2A_o^2 \lambda^2 B^{2(a-2)} + a \left[ a-1 + a^2(a+1) \right] \left( \frac{\ddot{B}}{B} \right)^2
\]

...(5.2.5)

Here and in what follows an overhead dot denotes differentiation with respect to $t$.

We assume that the material filling the universe is a viscous fluid with heat flow. The energy momentum tensor for such a distribution is given by

\[
T_{ik} = \left[ \bar{p} + \rho \right] \nu_i \nu_k - pg_{ik} - \eta \mu_{ik} + [q_i \nu_k + q_k \nu_i]
\]

...(5.2.6)

with

\[
\nu_i \nu_i = 1, \quad q_i \nu_i = 0
\]

...(5.2.7)

\[
\mu_{ik} = \left[ \nu_{i;k} + \nu_{k;i} \right] - \left[ \nu_i f_k + \nu_k f_i \right], \quad f_i = \nu_k \nu_{i;k}
\]

...(5.2.6)
and
\[ \tilde{p} = p - \left[ \xi - \frac{2}{3} \eta \right] \nu^i. \] \hspace{1cm} \ldots(5.2.9) \]

Here semicolon denotes covariant derivative. In the above equations \( p \) and \( \rho \) are respectively the thermodynamic pressure and the material density, \( \tilde{p} \) is the effective pressure and \( \eta \) and \( \xi \) are shear viscosity and bulk viscosity coefficients, respectively.

We use co-moving co-ordinates and take the heat flow in the \( \theta^i \)-direction. Therefore the tetrad components \( \nu_{(\omega)} \) and \( q_{(\omega)} \) of the flow vector \( \nu_i \) and the heat flow vector \( q_i \) are
\[ \nu_{(\omega)} = (0,0,0,1), \quad q_{(\omega)} = (q,0,0,0) \] \hspace{1cm} \ldots(5.2.10) \]
where \( q \) is a function of time to be determined from the field equations.

The components \( \nu_i, \nu^i \) and \( q_i \) can be determined from
\[ \nu^i = e^i_{(\omega)} \nu_{(\omega)}, \quad e^i_{(\omega)} \theta^a = dx^i \]
\[ \nu_i = e^i_{(\omega)} \nu_{(\omega)}, \quad q_i = e^i_{(\omega)} q_{(\omega)}, \quad e^i_{(\omega)} dx^i = \theta^a \]

They are given by
\[ \nu^i = (0,0,0,1), \quad \nu_i = \left[ \alpha A_0 B^a, 0, 4m^2 \alpha B^a, 1 \right] \]
\[ q_i = \left[ A_0 B^a, 0, 4m^2 A_0 B^a q, 0 \right] \] \hspace{1cm} \ldots(5.2.11) \]
The co-ordinates are labelled as $x^1 = \theta$, $x^2 = \phi$, $x^3 = t$. It is easy to see that $\nu_i$ and $q_i$ given by (5.2.11) satisfy the condition (5.2.7). The acceleration vector $f_i$ can be easily computed. It is given by

$$f_i = \alpha a B^{a\mu} B(1,0,0,0)$$

...(5.2.12)

The quantities $\mu_{ik}$ can be determined from (5.2.8). For the sake of brevity, we shall not give the expressions for $\mu_{ik}$ here. But the surviving tetrad components $\mu_{ab} = e^{a(i)} e^{k(i)} \mu_{ik}$ are listed below for ready reference:

$$\mu_{ab} = -2a \left( \frac{B}{B^2} \right), \quad \mu_{22} = \mu_{33} = -2 \left( \frac{B}{B^2} \right)$$

...(5.2.13)

We shall use the field equations

$$R_{ik} = \frac{1}{2} g_{ik} R + \Lambda g_{ik} = -8\pi T_{ik}$$

where $T_{ik}$ is given by (5.2.6). These equations may be written in the tetrad basis as

$$R_{(ab)} = -8\pi \left[ (p + \rho) \nu_{(a)} \nu_{(b)} - \frac{1}{2} \left[ \rho - p - 2 \eta^{*} \right] g_{(ab)} \right]$$

$$- 8\pi \left[ q_{(a)} \nu_{(b)} + q_{(b)} \nu_{(a)} \right] + 8\pi \eta \mu_{(ab)} + \Lambda g_{(ab)}$$

...(5.2.14)

where $\eta^{*}$ is the expansion scalar and is given by

$$\eta^{*} = \nu^i \nu_{i}$$
Using the usual definitions of the shear scalar $\sigma$ and the rotation $\Omega$ of the flow vector $\nu_i$, we have obtained $\sigma$ and $\Omega$ for $\nu_i$ given by (5.2.11).

$\theta^*$, $\sigma$ and $\Omega$ are given by

$$
\theta^* = (a+2) \left[ \frac{B}{B} \right]
$$

$$
\sigma^2 = \left[ \frac{1}{3} \right] (1-a)^2 \left[ \frac{B}{B} \right]^2
$$

$$
\Omega^2 = \alpha^2 A_o^2 \lambda^2 B^{2(a-2)} \quad \ldots(5.2.15)
$$

Clearly

$$
\left( \frac{\sigma}{\theta^*} \right) = Z(a), \text{ where } Z(a) = \frac{(1-a)}{\sqrt{3}(a+2)}
$$

Thus $\sigma/\theta^*$ is constant for our model. Here it should be mentioned that the condition $(\sigma/\theta^*) = \text{constant}$ is extensively used by Collins, Glass and Wilkinson (1980) in connection with spatially homogeneous cosmological models.

In view of the results (5.2.4), (5.2.5), (5.2.10), (5.2.13) and (5.2.14) we obtain the following non-trivial relations.

$$
8\pi \eta = \frac{R_{(11)} - R_{(22)}}{\mu_{(11)} - \mu_{(22)}} \quad \ldots(5.2.16)
$$

$$
8\pi q = -R_{(14)} \quad \ldots(5.2.17)
$$
where $R_{(ab)}$ are given by (5.2.5).

In the above analysis, the coefficient $\xi$ does not occur explicitly, so we take $\xi = 0$ for simplicity. We have a system of four equations (5.2.16), (5.2.17), (5.2.18) and (5.2.19) for five unknowns $\bar{p}, \rho, q, \eta$ and $B$. Thus the number of unknowns is greater than the number of equations at hand. Therefore we have to make one reasonable assumption for getting an explicit solution.

We close this section by a very brief mention of the Raychaudhuri (1955) equation

$$\theta^i_\nu + \frac{1}{3} \theta^* \theta^2 - f^i_\nu + 2 \left[ \sigma^2 - \Omega^2 \right] = R_{ik} \nu^i \nu^k \tag{5.2.20}$$

Now clearly assuming $\Lambda = 0$,

$$-R_{ik} \nu^i \nu^k = 4\pi \left[ \rho + 3\bar{p} + 2\eta \theta^* \right]$$

Therefore the Hawking-Penrose (1970) energy conditions are satisfied if

$$R_{ik} \nu^i \nu^k \leq 0, \text{ i.e., } \rho + 3\bar{p} + 2\eta \theta^* \geq 0 \tag{5.2.21}$$
5.1: SOLUTIONS OF THE FIELD EQUATIONS

It is well-known that, in an early stage of the universe when neutrino decoupling occurs, the matter behaves like a viscous fluid. It is also a fact that the coefficient $\eta$ of the shear viscosity decreases as the universe expands. So it seems reasonable to assume that the coefficient of shear viscosity varies directly as the scalar of expansion. Therefore we assume

$$\eta = K \theta^*$$  \hspace{1cm} \ldots(5.3.1)

where $K$ is an arbitrary constant.

Substituting $\eta$ from (5.2.16) and $\theta^*$ from (5.2.15) we obtain the following differential equation for the metric potential $B$

$$\left[1 - a + \alpha^2(1 + a)\right]\left[\frac{B'}{B}\right] + 2\lambda^2A_0^2(a^2 - 2)B^2(a^2 - a^2)$$

$$+ \left[1 - a^2 + \alpha^2(a^2 - a) - 16\pi K [2a^2 - a]\right] \left[\frac{B'}{B}\right]^2 = 0$$  \hspace{1cm} \ldots(5.3.2)

The general solution of (5.3.2) seems to be complicated. The general case is discussed in the appendix. Here we discuss a particular solution of (5.3.2) which gives us a rotating generalisation of the Bianchi type II viscous fluid model discussed by Patel and Singh (1989).

Let us now introduce the new time co-ordinate $T$ defined by

$$dT = B^Ldt$$  \hspace{1cm} \text{where $L$ is a real constant. Performing this transformation and choosing the constants $L$ and $K$ as}
\[ L = a - 2 \]

\[ 16\pi n(a-1)(a+2) = (2a-1)(a-1) - \alpha^2 [2a - 2a^2 + 3] \] ...(5.3.3)

the equation (5.3.2) reduces to

\[ \frac{d^2 B}{dT^2} - \frac{2A_0^2 \lambda^2 (2-\alpha^2)B}{1-a+\alpha^2(1+a)} = 0 \] ...(5.3.4)

We assume that \(0 < a < 1, \quad 0 < \alpha^2 < 2\). So the quantity \(\frac{2A_0^2 \lambda^2 (2-\alpha^2)B}{1-a+\alpha^2(1+a)}\) is positive, say \(b^2\).

Therefore the solution of (5.3.4) is given by

\[ B = \exp(bT) \] ...(5.3.5)

For the above value of \(B\), the physical parameters \(\eta, \quad \rho, \quad \bar{\rho} \quad q\) are given by

\[ 16\pi \eta(1-a) = bF(a) \exp[b(a-2)T] \] ...(5.3.6)

\[ 8\pi \rho = -\Lambda + [3b^2 G(a)/4] \exp[2b(a-2)T] \] ...(5.3.7)

\[ 8\pi \bar{\rho} = \Lambda - b^2 H(a) \exp[2b(a-2)T] \] ...(5.3.8)

\[ 8\pi q = -2\alpha [b^2 + A_0^2 \lambda^2] \exp[2b(a-2)T] \] ...(5.3.9)

where the function \(F(a), \quad G(a)\) and \(H(a)\) are defined by
\[ F(a) = \alpha^2 \left[ 2a - 2a^2 + 3 \right] - (2a - 1)(a - 1) \quad \ldots (5.3.10) \]

\[ G(a) = \left[ \alpha^2 [5a + 3] - 3[3a + 1] \right] (\alpha^2 - 2)^{-1} \quad \ldots (5.3.11) \]

and

\[ H(a) = \frac{\left[ \alpha^2 (2a^2 + 5a - 5) - 8a^2 - 7a + 7 \right]}{2(\alpha^2 - 2)} \quad \ldots (5.3.12) \]

Using the above results we have obtained

\[ 8\pi \left[ \rho + 3\bar{p} + 2\eta \theta^* \right] = 2A + b^2 J(a) \exp[2b(a - 2)T] \quad \ldots (5.3.13) \]

where \( J(a) \) stands for

\[ J(a) = \left[ \frac{3}{4} [\alpha^2 - 2] \right] \left[ \alpha^2 [-5a + 13 - 4a^2] + 16a^2 + 5a - 17 \right] - \left[ \frac{2 + a}{a - 1} \right] F(a) \quad \ldots (5.3.14) \]

Let us assume \( \Lambda = 0 \). Then clearly \( \bar{p} \) becomes a constant. It is given by

\[ \bar{p} = \frac{-4H(a)}{3G(a)} = -[L(a)]^{-1} \text{ say.} \]

Therefore the physical requirements \( \bar{p} \geq 0, \rho > 0 \) and \( \rho + 3\bar{p} + 2\eta \theta^* > 0 \) give the following restrictions:

\[ L(a) \leq 0, H(a) \leq 0, G(a) \leq 0, J(a) \leq 0 \quad \ldots (5.3.15) \]

We now give some numerical estimates of the fluid parameters. If the universe is rotating, then the rotation must be very slow. Therefore \( \alpha \) is very small. For \( \alpha^2 = 0.01 \) and for various values of
a satisfying $0.4 < a < 0.6$ the values of $F(a)$, $G(a)$, $H(a)$, $Z(a)$, $J(a)$ and $L(a)$ are tabulated in table.

(See TABLE at the end of the Chapter)

From the table it is clear that $F(a)$ remains positive for the range $0.4 < a < 0.6$ and it is a decreasing function of $a$. Here it should be noted that for the non-rotating counterpart of our solution the range for $'a'$ was $0.5 < a < 0.6$. Thus the introduction of rotation increases the range for the constant $a$. From the table it is easy to see that the physical requirements (5.3.13) are satisfied for $a$ satisfying $0.4 \leq a \leq 0.58$. Between the values $0.58$ and $0.60$, the function $H(a)$ changes its sign. Thus $\bar{p}$ becomes zero between these two values of $a$. From the table it is also clear that $\bar{p}$ is always less than $\rho$. For the values of $a$ not satisfying $0.4 \leq a \leq 0.6$, we have to take the non-zero cosmological constant $\Lambda$.

Let us discuss some general features of our solution. It is easy to see that our model has non-zero expansion and shear. If $\alpha \neq 0$, then the rotation of the stream lines of the viscous fluid is also non-zero. The fact $\alpha \neq 0$ implies that $f_i \neq 0$ and $q \neq 0$. Thus the stream lines of the viscous fluid filling our universe are not geodetic. Thus the non-geodetic nature of the stream lines and the heat flow are intimately linked with the rotation of the flow vector.

The present upper limit for the ratio $\sigma/\theta^*$ is $10^{-3}$, obtained
from indirect arguments concerning the isotropy, of the premordial black body radiation (Collins, Glass and Wilkinson, 1980). From table it is clear that for $0.4 \leq a \leq 0.58$, we have $0.144 \leq Z(a) \leq 0.093$. Thus $Z(a)$ is considerably greater than $10^{-3}$. This in turn implies that our solution represents an early stage of the universe. This seems quite alright because the dissipative effects due to viscosity and heat flow are significant only in the early stages of the evolution of our universe.

Two particular cases of our solutions are note-worthy.

CASE: [1]

Let us assume that

$$\alpha^2 = \frac{(2a - 1)(a - 1)}{[2a + 3 - 2a^2]}$$

...(5.3.16)

We can get admissible values of $a$ satisfying (5.3.16). For example $\alpha^2$ remains positive for $0 \leq a \leq \frac{1}{2}$. In this case $\eta = 0$. The parameters $\bar{p}, \rho$ and $q$ can be easily calculated. For the sake of brevity we shall not give their expressions here. In this case we get a rotating perfect fluid model with non-zero heat flux. This gives us a rotating generalisation of the perfect fluid model discussed by Damiao Soares (1978) admitting heat flux. When $a = \frac{1}{2}, \alpha^2 = 0$. In this case we recover the solution of Damiao Soares.

CASE: [2]
Let us assume that \( a = 1/2 \) from the beginning. In this case we obtain

\[
8\pi \eta = \frac{7a^2 b}{2} \exp(-3bT/2)
\]

\[
8\pi \rho = -\Lambda + \frac{3b^2 [15 - 11a^2]}{8[2 - a^2]} \exp(-3bT)
\]

\[
8\pi p = \Lambda - \frac{b^2 [3 - 4a^2]}{4[a^2-2]} - \exp(-3bT)
\]

\[
8\pi q = -2a [b^2 + A_o^2 \lambda^2] \exp(-3bT)
\]

\[
b^2 = \frac{4A_o^2 \lambda^2 [2 - a^2]}{[1 + 3a^2]} \quad \text{...(5.3.17)}
\]

This gives us viscous fluid generalisation of the solution of Damiao Soares (1978) with heat flow. Further when \( a = 0 \), we recover the non-rotating Bianchi II perfect fluid model of Damiao Soares (1978).

5.4 : CONCLUDING REMARKS

Some further simple particular solutions of the differential equations (5.3.2) for the function \( B \) can easily obtained.

Assuming \( a^2 = 2 \), the equation (5.3.2) can be integrated. The solution can be expressed as
\[ B = \left( D t + C \right) \left( (a-2) + 16\pi K(a+2) \right)^{1/(a+3)} \]

where \( C \) and \( D \) are constants of integration.

Choosing the constants \( K \) as

\[ 16\pi K \left[ 2 - a^2 - 1 \right] = 1 - a^2 + \alpha^2 \left[ a^2 - a - 1 \right] \]

in (5.3.2), the coefficient of \( B_z \) in (5.3.2) becomes zero. Under this assumption, the equation (5.3.2) admits the solution

\[ B_z = \pm \left[ \frac{2(2-a^2)}{(1-a)[1-a^2+1+a]} \right]^{1/2} t + E \]

where \( E \) is an arbitrary constant.

The associated fluid parameters \( \bar{p}, \rho, q \) and \( n \) for the above two cases can be easily calculated. For brevity we shall not give these details here.

When rotation is zero, the heat flux vanishes. But the converse is not true. Thus \( R_{(14)} \) can be made zero even in the case \( \alpha=0 \). Thus rotating viscous fluid models without heat flux are possible. But we shall not explore this possibility here.

In the next chapter, we shall consider rotating Bianchi type II, VIII, IX space-times in connection with the field equations\[ R_{ik} = \Lambda g_{ik}.\]
APPENDIX:

The differential equation (5.3.2) is

\[
\frac{\dot{B}}{B} + M \left( \frac{B}{\dot{B}} \right)^2 - NB^{2(\alpha-2)} = 0 \quad \ldots (A.1)
\]

Where

\[
M \left[ 1 - a + a^2 (1+a) \right] = \left[ 1 - a^2 + a^2 (a^2 - a - 1) \right] - 16\pi K \left[ 2 - a^2 - a \right]
\]

\[
N \left[ 1 - a + a^2 (1+a) \right] = 2A_0^2 \lambda [2 - \alpha^2] \quad \ldots (A.2)
\]

The first integral of (A.1) is given by

\[
B^2 = \frac{NB^{2(\alpha-1)}}{a+M-1} + CB^{-2M} \quad \ldots (A.3)
\]

Where \( C \) is a constant of integration. Also the constants \( a, N, M \) and \( C \) should be choosen in such a way that \( B \) remains positive. We now use \( B \) as the new time co-ordinate and replace \( dt^2 \) by

\[
dB^2 \left[ \frac{NB^{2(\alpha-1)}}{a+M-1} + CB^{-2M} \right]^{-1} \]. The associated fluid parameters \( \bar{p}, \rho, \eta \) and \( q \) are determined as follows:
\[ 8\pi p = \Lambda - 2K(a-2) \left[ \frac{NB^2(a-2)}{a+H-1} + CB^{-(M+1)} \right] - \left[ \frac{(2a^2-1)NB^2(a-2)}{a+H-1} \right] + CB^{-2(M+1)} + A_0^2 \lambda^2 B^2(a-2) \] ... (A.4)

\[ 8\pi p = -\Lambda + \left[ \frac{(2a+1)NB^2(a-2)}{a+H-1} + CB^{-2(M+1)} - A_0^2 \lambda^2 B^2(a-2) \right] \] ... (A.5)

\[ 8\pi \eta = K(a+2)B^{-1} \left[ \frac{NB^2(a-1)}{a+H-1} + CB^{-2M} \right]^{1/2} \] ... (A.6)

\[ 8\pi q = -2\alpha \left[ A_0^2 \lambda^2 B^{2(a-2)} + aCB^{-2(M+1)} + \frac{4NB^2(a-2)}{(a+H-1)} \right] \] ... (A.7)
REFERENCES:

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