1. INTRODUCTION

The concept of stability plays very important role in analysis of any systems. In the state space representation of the system, stability is determined by computing the eigenvalues of the system matrix $A$ since the system is represented by linear state variable model given by the equation

$$\dot{X} = AX + BU$$

$$Y = CX$$

Where $A$ is the system matrix, $B$ is the input matrix and $C$ is the output matrix.

For the determination of the stability, necessary and sufficient conditions are the eigenvalues must belong to the open left half of the s-plane. There exist several methods available in control literature to compute eigenvalues. In determining the eigenvalues numerically with the help of digital computer, the existing methods take a lot of computations. Also, the eigenvalues of the system matrix cannot be determined accurately due to finite word length problem, rounding off errors, truncation errors, machine epsilons that are present in the computer.

When the power system stability is carried out on the system problems with the eigenvalues that are nearer to the imaginary axis, eigenvalues at the origin, also eigenvalues at the right hand side of the s-plane play a significant role.

Several methods have been developed for to compute the eigenvalues of the system matrix. Some of these methods apply to symmetric matrices, to tri-diagonal matrices and a few can be used for general matrices.

These methods can be classified into two categories

(i) These methods in this category work with original matrix $A$ and its characteristic polynomial, one such method is the Faddeev – Leverrier procedure. Once the coefficients of the polynomial are known, the
method uses the root finding technique, such as Lin–Bairstow method to
determine the eigenvalues of the matrix.

(ii) These methods in this category reduce the original matrix to tri-diagonal
form when the matrix is symmetric or to Hessenberg form when \( A \) is
non-symmetric by orthogonal transformation or elementary similarity
transformations. Then apply successive factorization procedure such as
LR or QR algorithms to compute the eigenvalues.

In the following, several existing methods have been described one by one.

1.1 Literature review of the existing methods

Power method [1, 15] is an iterative method and it computes the largest
eigenvalue of the system matrix. It can be easily applied to the sparse matrix,
symmetric matrix since the only operation required in this method is multiplication.
The power method is used in shifting of origin. The largest eigenvalue in the new
system will be the smallest eigenvalue in the original system. The disadvantage of
the Power method is the slow rate of convergence and also to compute the both
largest and smallest eigenvalues transformation of the coordinates system is required
which takes computations. Jacobi method computes [15] the eigenvalues of the
symmetric matrix. This method uses the plane rotation and also orthogonal
transformation to the given matrix to transform it to diagonal matrix. The
eigenvalues of the symmetric matrix are real. The disadvantages of this method is
that orthogonal transformations involve matrix multiplication which takes lots of
computations. During the process of orthogonal transformation, the elements of the
matrix above and below the principle diagonal should be made zero which may not
necessarily become zeros during subsequent transformation. The elements that
becomes zero in Jacobi method is not disturbed in Given method [1] during the
plane rotations. The finite number of plane rotation taken in this method is
\( \frac{n(n-1)(n-2)}{2} \) to reduce a given matrix to tri diagonal form. The total
number of operation counts required in this method is \( O \left( \frac{4}{3} n^3 \right) \). The
disadvantages of this method is that it involves orthogonal transformation which
transforms the given symmetric matrix to tri diagonal which takes operation counts.
For this tri diagonal matrix strum sequences are generated to compute the bound in
which the eigenvalues lie which involves computations. In Rutihauzer method [1],
the given matrix is transformed to upper triangular matrix by expressing the given
matrix as LU where L is the lower triangular matrix and U is the upper triangular
matrix. The eigenvalues are computed for the upper triangular matrix. The
disadvantages of the above method is that the given matrix is transformed to upper
triangular matrix and all elements below the principle diagonal should be made zero
which may not necessarily become zero during subsequent transformations. Also,
LU decomposition method has some practical difficulties in its application. Hence
to overcome these difficulties the lower triangular matrix L is replaced by unitary
matrix Q. Also as mentioned in [1], the QU algorithm is also not simple for
practical application. In Gauss elimination method [1], the given matrix is
transformed to upper triangular matrix using row transformations which require lots
of operation counts. For large value of n operation counts required are $O\left(\frac{n^3}{3}\right)$.
Gauss elimination method is also used to solve the simultaneous equations. The
disadvantages of the above method is that all the elements must be made zero which
does not become zero in the subsequent transformation. Hence the eigenvalues that
are computed will not be accurate. The Gauss Jordan reduction method [1] is an
extension of the Gauss elimination method. It reduces a set of n equations from its
canonical form of $AX = C$ to the diagonal set of the form $IX = C'$ where I is the
unit matrix in this equation and it is identical to $X = C'$, the solution vector.
Gauss Jordan reduction method applies the same series of elementary row operation
that are used by the Gauss elimination method. This method applies these operations
both below and above the diagonal in order to reduce all the off–diagonal element
of the matrix to zero. Operation counts for Gauss Jordan Reduction method
are $O\left(\frac{n^3}{3}\right)$. Gauss seidal [1] method is an iterative method. The Gauss seidal
substitution method requires an initial guess of the value unknowns $x_1$ to $x_n$. These
values are used to begin evaluation of new estimates of the $x_i$’s. Each newly
calculated $x_i$ replaces its previous value in subsequent calculations. The iterations
continue until all the newly calculated $x_i$’s converge to within a convergence
criterion of their previous values. This method is also called successive
displacement. The rate of convergence of the gauss seidal is twice that of Jacobi
method. The rate of convergence of the iterative method depends on the eigenvalues
of the iterative matrix. The Inverse method [2] is the simplest transformation for the
computation of the eigenvalues that can be performed. An interesting transformation is obtained if we use the matrix \((A - pI)\) instead of \(A\). The eigenvalues of the matrix \((A - pI)\) are \((\lambda_i - p)\). Dominant eigenvalues of \((A - pI)\) is either \((\lambda_1 - p)\) or \((\lambda_n - p)\) convergence to the largest eigenvalue is maximum for this value of \(p\) and this eigenvalue corresponds to the smallest eigenvalue of the original system (i.e., matrix \(A\)). The disadvantage of this method is that the choice of \(p\) is difficult unless we know a prior estimate of the eigenvalues. The Lanczo’s algorithm [2] is an iterative algorithm invented by Cornelius Lanczo’s that is an adaptation of power methods to find eigenvalues and eigenvectors of the square matrix or the singular value decomposition of a rectangular matrix. It is particularly useful for finding decompositions of very large sparse matrices. Petor Montgomery published in 1995, an algorithm based on Lanczo’s algorithm for finding elements of the null space of a large sparse matrix, since the set of people interested in large sparse matrices over finite fields and the set of people interested in large eigenvalue problems. This is also called the block Lanczo’s algorithm. The disadvantage of this method is that this is applicable for only large sparse matrices. In Faddeev–Leverier [3] method the co-efficients of the characteristic polynomial are evaluated. Once the co-efficients of the polynomial are known by the root finding technique, such as Lin Bairstow method is used to compute the eigenvalues of the given matrix. The disadvantages of the Faddeev–Leverier method involve matrix multiplication in calculating the co-efficients of the characteristic polynomial. For large order matrix, multiplications requires lots of time. While computing the co-efficients errors may be due to round off or chopping of numbers. Hence the computed co-efficients may need to represent the actual value. Further, the number of iterations taken by Lin Bairstow method to compute all the roots of the characteristic polynomial depends upon the initial approximations to the co-efficients of the quadratic factor. The disadvantage of this method is that it takes large number of iterations to compute all the roots. The QL method [1] is an iterative process which is more stable than the Jacobi method and it is applicable to tri diagonal matrix. In this method, the symmetric matrix is reduced to tri diagonal form using a technique known as Householder method. The QL method will converge on the systems eigenvalues by reducing the tri diagonal matrix to diagonal form. The diagonal elements of this diagonal matrix are the eigenvalues. The disadvantage of this method is that during
the transformation of the values the originality will not be maintained due to round
off and chopping of numbers. QR-method [1] is used to find the eigenvalues of the
system when the square matrix is unsymmetrical. The given matrix must be reduced
to tri diagonal form before applying this method and a upper triangular matrix with
one diagonal below the main diagonal. This form is known as Hessenberg form and
this achieved by applying the Householder’s method to non symmetric square
matrix. The disadvantage of the above method is the convergence of the system of
eigenvalues is, if the sum of the squares of the elements below the main diagonal of
the matrix $H_{new}^{i}$ is smaller than that of its predecessor. $H_{new}^{i-1}$ . Rayleigh quotient
iteration [2] is an eigenvalue algorithm which extends the method of inverse
iteration by using the Rayleigh quotient to obtain increasingly accurate eigenvalues
estimates. Rayleigh quotient is an iterative method i.e. it must be repeated until it
converges to an answer (i.e. the true eigenvalue algorithm). In this method very rapid
convergence occurs and not more iterations are required. The Rayleigh quotient
algorithm converges cubically. In numerical linear algebra, the Arnoldi iteration [2]
is an eigenvalue algorithm on the non Hermitian matrices. Lanczo’s iteration is an
eigenvalue algorithm for Hermitian matrices. The Arnoldi iteration was invented in
1951. Arnoldi algorithm is an iterative method. Arnoldi belongs to a class of linear
algebraic algorithm that gives a partial result after a small number of iteration. This
algorithm is in a contrast to direct method. Arnoldi iteration is a large sparse matrix
algorithm. It does not access the elements of the matrix directly, but rather makes
the matrix map vectors and makes its conclusion from their images. Divide and
conquer [2] is an eigenvalue algorithm for Hermitian or real symmetric matrices that
have recently become competitive in terms of stability and efficiency, with more
well known algorithm such as QR algorithm. The basic concept of this algorithm is
the famous divide and conquers approach from computer science. An eigenvalue
problem is divided into two problems, half the size, each of these is solved
recursively and the eigenvalues of the original problem are computed from the
results of the smaller problems. The problem of finding the number of eigenvalues
of a matrix in different regions of the complex plane arises in many practical
situations. The region widely studied in both mathematics and control literature is
the half planes and the unit circles. The half plane problems are commonly known as
inertia problem in mathematics.
1.2 Numerical stability and computational problems [15]

A computer has a finite word length and so only a fixed number of digits are stored and used during computation. Most of the difficulties in computation on a digital computer derive from the inherently finite word length used to represent real or complex numbers. It manifests itself in two important ways in arithmetical calculations finiteness of precision and finiteness of range. While this may be the source of great frustration to the average user of computing facilities, it does provide as essentially limitless supply of challenging problems for the numerical analyst. Some very convenient mathematical properties that we sometimes take for granted hand computation are no longer valid on a computer. For example, the associative law for addition of real numbers is no longer generally true.

A particular number of which we shall make frequent use is machine epsilon. This is defined to be the small positive number $\epsilon$ which when added to 1 on our computing machine gives a number greater than represent able number $\delta$ less than $\epsilon$ gets “ round off “ when added 1 to give exactly 1 , again as the rounded sum. The number $\epsilon$ varies of course depending on the computer being used and the precision with which the computations are being done i.e., single precision, double precision etc.

Thus in obtaining the solution of a problem using numerical techniques, it cannot be said perfectly whether the solution obtained is accurate or not.

Using numerical data and processing which involve implementation of algorithms, and the final output of the result, two types of errors are generally introduced. They are mainly truncation errors and round off errors. Besides these, errors in the implementation of the algorithm must be taken into account. We also have absolute errors and percentage errors.

**Round off Errors:** Round off errors occur when a fixed number of digits are used to represent exact numbers or in other words when a number have an inconvenient number of digits, a better technique for accuracy is called “Round off”. In this technique, the last retained digit is adjusted depending upon the value of the value of the succeeding digit. The last retained digit is retained, as it is if the
succeeding digit is less than 5 and increased by one if the succeeding digit is greater than or equal to 5.

Example: 2.365432, 0.1236604 are rounded off to 2.365, 0.124 respectively.

**Truncation Errors:** When a number has too many digits, it becomes inconvenient or even impossible either to store the number in the computer memory as it is. In this case, after a convenient number of digits, the remaining digits in the number are “chopped off”. For example $\frac{1}{3} = 0.333333\ldots$ may be taken as 0.333 depending on the requirement of the number of places after decimal. The error due to such chopping off is called the truncation error. The truncation error is caused at various stages i.e., during input of data to the computer, computations in every step of the algorithm of the numerical techniques used and finally in the output of the computing result.

Truncation errors are also caused while converting a decimal number into its binary. For example: 0.1 is a terminating decimal number. While its binary representation 0.000110011 is non terminating sine the computer has finite word length and also fixed number of digits are stored and used during computation. This results in an error due to truncation.

**Algorithmic Errors:** Suppose we have some mathematical defined problem represented by $f$ which acts on a data $a \in R$ where $R$ is the same set of solutions. Given $x \in R$ we desire to compute $f(x)$. Suppose $x^*$ is some approximation to $x$. If $f(x^*)$ is “near” to $f(x)$, the problem is said to be well conditioned.

If $f(x^*)$ vary from $f(x)$, even when $x^*$ is near to $x$, then the problem is said to be ill conditioned.

Let $f^*$ denote an algorithm implemented to evaluate $f$. Given $x$, $f^*(x)$ represents the results of applying the algorithm to $x$. The algorithm $f^*$ is said to be numerically stable if for all $x \in R$, there exists $x^* \in R$ such that $f^*(x)$ is near to $f(x^*)$, if the problem is well conditioned, then $f^*(x)$ will be near to $f(x)$ i.e., $f^*$ does not introduce much error.
One cannot obtain a stable algorithm to solve an ill conditional problem, but an unstable algorithm can produce poor solution even to a well conditional problems. Thus two separate factors to be considered in determining the accuracy of a computed solution $f^*$ (x). First if the algorithm is stable, $f^*$ (x) is near to $f(x^*)$. Thus $f^*$ (x) is near to $f(x)$ we have an accurate solution. Thus errors are caused by a wrong choice of algorithm for a numerical/computational technique. While the algorithm may be mathematically perfect, the computations involved in the computations can cause errors due to various factors including truncation and round off errors. Further, some algorithm though mathematically sound when set to work on a computer or calculator at come intermediate stage set struck up since the number involved at that stage may be either too small or too large, beyond the range of computing device.

**Absolute Errors:** An absolute error is the numerical difference between the true value of a quantity and its approximate value. If the absolute error is more for a problem then the required solution is far away from the estimated solution. This happens when the problem are ill conditional.

If $p$ is the exact value required and $p^*$ is the approximated value, then

$$\text{Absolute error } = |p - p^*|$$

$$\text{Relative error } = \frac{|p - p^*|}{|p|} \text{ provided } p \neq 0.$$  

In all the methods explained above, are computationally expensive.

Therefore a graphical technique namely Gerschgorin method to compute eigenvalues and decide the stability of the system to avoid computational burdens. A few work carried using Gerschgorin circle technique are,

Algorithms have been developed in [13] to identify the eigenvalues with positive real part of the system matrix using the Gerschgorin theorem. In [14], although a new approach is suggested to identify the presence of a eigenvalue of a matrix with positive real part, using Gerschgorin Theorem. After plotting the graph from the system matrix A, the limits within which all the real eigenvalues of the system matrix A lie can be obtained. Using these limiting values and trace of the
matrix an algorithm is developed to test, whether or not system has an eigenvalue with the positive real part. The proposed method in [15] works efficiently to decide about the instability of the system of any order, when it and eigenvalue on positive real axis of the s-plane. Some interesting results on the Gerschgorin circles are obtained. In [24], Gerschgorin theorem and Kharitonov’s theorem are used to solve various problems in Control theory. When the Gerschgorin circle is isolated a similarity transformation is used to improve the Gerschgorin bound. Based on the eigenvalues of the system matrix, definiteness of the real symmetric matrix [24] has been discussed. In this method the eigenvalues has to be computed. Analytical approaches have been developed to identify the real eigenvalues on the Gerschgorin bound.

In addition to the above graphical technical implemented, few novel algorithms are developed in this thesis to compute the eigenvalues and also decide the stability of the system matrix as far as possible to overcome the difficulties.

1.3 Gerschgorin circle theorem [1, 28]

Given a \((n \times n)\) matrix \(A = [a_{ij}] \in \mathbb{C}^{n \times n}\) \(i, j = 1, 2, ..., n\) with \(n \geq 2\) [1] then the eigenvalue of \(A\) is equivalent of \(A\) to finding \(n\) zeros of the associated with the characteristic polynomial

\[
P_n(S) = \det(\lambda I - A) \tag{1.3.1}
\]

Where \(I\) is the identity matrix. But for large \(n\), finding these zeroes are very a daunting problem. Hence in order to estimate eigenvalues of \(A\), without finding the characteristic polynomials, Russian Mathematician Semyon Aranovich Gerschgorin in 1931 establishes eigenvalue inclusion result.[2] As reported in [3], this result was sensation at that time, which created a great enthusiasm. This theorem is stated as follows

**Theorem: 1.3.1:**

The largest eigenvalue in modulus of square matrix \(A\) cannot exceed the largest sum of the module the elements along any row or column.
Since the eigenvalues of $A^T$ are same those of $A$, the theorem is also true for columns.

**Theorem 1.3.2:**

Let $p_k$ be the sum of moduli of the elements along the $k^{th}$ row excluding the diagonal element $a_{k,k}$. Then every eigenvalue of $A$ lies inside or on the boundary of at least one of the circles in the $s$-plane $(|\lambda - a_{k,k}| = p_k)$

Proof: Let $\lambda_i$ be an eigenvalue of the $(nxn)$ matrix $A$ and $x_i$ be the corresponding eigenvector with the components $v_1, v_2, ..., v_n$. Then the relation $Ax_i = \lambda_i x_i$ is

$$a_{i,1}v_1 + a_{i,2}v_2 + ... + a_{i,n}v_n \rightarrow (1.3.3)$$

$$a_{2,1}v_1 + a_{2,2}v_2 + ... + a_{2,n}v_n \rightarrow (1.3.4)$$

$$a_{2,1}v_1 + a_{2,2}v_2 + ... + a_{2,n}v_n \rightarrow (1.3.5)$$

Let $v_k$ be the largest modulus of $v_1, v_2, ..., v_n$ and selecting the $k^{th}$ equation and dividing by $v_k$, we get

$$\lambda_k = a_{k,1} \left( \frac{v_1}{v_k} \right) + a_{k,2} \left( \frac{v_2}{v_k} \right) + ... a_{k,k} + ... + a_{k,n} \left( \frac{v_n}{v_k} \right) \rightarrow (1.3.6)$$

Hence,

$$|\lambda_i - a_{kk}| = |a_{k,1} \left( \frac{v_1}{v_k} \right) + a_{k,2} \left( \frac{v_2}{v_k} \right) + ... + a_{k,n} \left( \frac{v_n}{v_k} \right)| \leq |a_{k,1}| + |a_{k,2}| + ... + |a_{k,n}| \rightarrow (1.3.7)$$

$$|\lambda_i - a_{kk}| \leq p_k$$

$$|\lambda_i - a_{kk}| \leq \sum_{i=1,i\neq k}^n |a_{k,i}| = P_k, = 1, 2, ..., n \rightarrow (1.3.8)$$
where $a_{k,i}$ is the center of the circle with radius $P_k$. Denoting each of these $n$ circles by $R_k$, we get

$$R_k = \{ |\lambda_i - a_{kk}| \} = P_k, k = 1, 2, ..., n$$

Therefore each of the eigenvalues of matrix must lie in the union of ‘S’ these $n$ circles in the Plane i.e.,

$$S = \bigcup_{k=1}^{n} R_k \quad \rightarrow (1.3.9)$$

It is also noted that $A$ and $A^T$ have the same eigenvalues. By applying the theorem [] to $A^T$ it yields another set $\hat{S}$. Thus

$$\sigma (A) = S \cap \hat{S} \quad \rightarrow (1.3.10)$$

In case of real symmetric matrix, $[A]$ and $[A^T]$ are same. Thus,

$$\sigma (A) = S \text{ or } \hat{S} \quad \rightarrow (1.3.11)$$

The eigenvalues of the matrix $A$ will be in the intersection of these two unions $S$ and $\hat{S}$ in the plane. By considering the intersection of these Gerschgorin circles, bounds under which real eigenvalues will lie, are obtained. The bounds obtained are nothing but the extreme ends of the intersection of Gerschgorin circles. Let these bounds are denoted by ‘a’ and ‘b’ where ‘a’ is the extreme left bound and ‘b’ is the extreme right bound.

If $a, b \in \mathbb{C}^-$, then $\forall \lambda_i \in \mathbb{C}^- , i = 1, 2, ..., n$, (all the eigenvalues lie on the left of the s-plane.) i.e., the system is stable.

If $a, b \in \mathbb{C}^+$, then $\forall \lambda_i \in \mathbb{C}^+ , i = 1, 2, ..., n$, (all the eigenvalues lie on the right of the s-plane.) i.e., the system is unstable.

The bounds, which lie on the real axis, can be determined for system matrix $A$ using Gerschgorin theorem as follow.

Consider $[A]_{n \times n} \in \mathbb{R}^{n \times n}$, calculating (for row wise circles)
Where $L_{rk}$ is left bound for each row-wise circle,
$L_r$ is the extreme left for row wise circle.
$R_{rk}$ is right bound for each row-wise circle and
$R_r$ is the extreme right bound for row wise circle.

Now calculating for column-wise circles

$L_{ck}= a_{kk} - \sum_{j=1, j \neq k}^{n} |a_{kj}|, k = 1, 2, ..., n, \quad \rightarrow (1.3.16)$

$L_c = \min(L_{ck}), k = 1, 2, ..., n \quad \rightarrow (1.3.17)$

$R_{ck} = a_{kk} + \sum_{i=1, i \neq k}^{n} |a_{ik}|, k = 1, 2, ..., n \quad \rightarrow (1.3.18)$

$R_c = \max(R_{ck}), k = 1, 2, ..., n \quad \rightarrow (1.3.19)$

Where

$L_{ck}$ is left bound for each column-wise circle,
$L_c$ is the extreme left for column-wise circle.
$R_{ck}$ is right bound for each column-wise circle and
$R_c$ is the extreme right bound for column-wise circle. \( \rightarrow (1.3.20) \)

Thus determination of Gerschgorin bounds depend upon the values of $L_r$, $L_c, R_r, R_c$.

Left Gerschgorin bound:
If $L_c, L_r < 0$, then the Left Gerschgorin bound $a = \max (L_c, L_r)$

If $L_c, L_r > 0$, then the left Gerschgorin bound $a = \min (L_c, L_r)$

If $L_c \leq 0, L_r > 0$, then the left bound $a = L_c$

If $L_c > 0, L_r \leq 0$, then the left bound $a = L_r$ \(\Rightarrow (1.3.21)\)

Right Gerschgorin bound:

If $R_c, R_r < 0$, then the Right Gerschgorin bound $b = \max (R_c, R_r)$

If $R_c, R_r > 0$, then the Right Gerschgorin bound $b = \min (R_c, R_r)$

If $R_c \leq 0, R_r > 0$, then the Right bound $b = R_r$

If $R_c > 0, R_r \leq 0$, then the Right bound $b = R_c$ \(\Rightarrow (1.3.22)\)

**Example 1.3.1:**

Consider the system matrix $A$ of order (3x3)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix} \Rightarrow (1.3.23)$$

Gerschgorin circle of the above matrix is drawn and is shown in figure 1.1

**Fig 1.1: Gerschgorin bound [-4, 4]**
Using the Gerschgorin theorem () the eigenvalues of the matrix $A$ lie in the region of $s$-plane

$$|\lambda| \leq 5,$$

$$|\lambda| \leq 6,$$

Using Gerschgorin theorem (1.3.1)

(i) The first set union of circles $S$ (Row wise addition)

$$|\lambda - 1| \leq 3,$$

$$|\lambda - 1| \leq 2$$

$$|\lambda + 1| \leq 4$$

So that all the eigenvalues of $A$ lie in the union of the circles or disks.

(ii) The second set of union of circles $\hat{S}$ (Column wise addition)

$$|\lambda - 1| \leq 2$$

$$|\lambda - 1| \leq 5,$$

$$|\lambda + 1| \leq 2$$

So that all the eigenvalues of $A^T$ lie in the union of these circles or disks.

(iii) All the eigenvalues of system matrix $A$ lie in the intersection of these two unions $S$ and $\hat{S}$ in the $s$-plane.

(iv) Using C++ programs are drawn and then by considering the intersection of Gerschgorin circles, we can obtain the limits $a$ and $b$ under which the eigenvalues lie. The limits that are obtained are the extreme ends of the intersection of the Gerschgorin circles. The shaded area in the figure shows the region in which the eigenvalues of the matrix $A$ lies.
1.3 Review of the work done via Gershgorin circles

The Gershgorin circle theorem, which is a graphical technique, gives the bounds in which the eigenvalues lies. Many Researchers have applied this Gershgorin circles theorem in various applications which has been presented.

Given an irreducible complex matrix $A$ order $(nxn)$ which possesses isolated Gershgorin disk, than it is known that by Gershgorin circle theorem [1] that there exists an eigenvalue in this isolated Gershgorin disk. For the improved bounds for this isolated eigenvalue, it is natural to consider positive diagonal similarity transformation allied to , to reduce the radius of the isolated Gershgorin disk and algorithms are available in [ 5 ] and [ 4 ] which yields the smallest such isolated Gershgorin disk under positive diagonal similarity transformations which contains this isolated eigenvalue of $A$. The main objective of [4] is to show that the basic iteration of the first algorithm of [5] majorises a similar iteration which can be applied directly to original matrix of order $(nxn)$.

The most common approach for the problems are: 1) compute the eigenvalues of $A$ explicitly; 2) compute the characteristic polynomial of $A$ explicitly, and then apply a favorite root location technique to the characteristic polynomial; 3) Solve an appropriate matrix equation. Of these the first one is believed to be the best from the numerical point of view. The second approach is not numerically viable because transformation of an arbitrary matrix to companion form is not numerically stable. Normally these cannot be achieved by orthogonal transformation. Solutions of matrix equations using numerically most effective methods are more expensive than computing the eigenvalues. To reduce the cost of computing the eigenvalues, a direct method (method that does not require computation of eigenvalues or the characteristic polynomial or solutions of any matrix equations) was proposed by Carlson and Datta [6] in 1979 to compute the inertia of a non Hermitian matrix. There exists an efficient and numerically stable method due to Householder for the transforming the arbitrary matrix to a Hessenberg matrix by unitary similarity transformation ; and the inertia of an Hermitian matrix $H$ can be efficiently computed by finding the triangular decomposition of $H= LDL^*$. No such direct method seems to exist for eigenvalue location in other regions. In [10] they have extended the Carlson –Datta inertia method to cover several other important regions.
A direct method is proposed [10] for determining eigenvalue distribution of a matrix with respect to several important regions of the complex plane. These regions include half planes, shifted half planes, hyperbolas, sectors, quadrants, imaginary axis, region contains within two straight lines that passes through the origin, etc. The method neither requires computation of characteristic polynomial of the given matrix nor solution of and matrix equations. The method proposed in [6] is more efficient than the eigenvalue and matrix equation methods. In state space representation of power systems dynamic and steady state stability is determined by computing the eigenvalues of the system matrices. There exist several methods in literature as discussed in [1] to compute the eigenvalues of the matrix $A$. However it is sufficient to identify the eigenvalues of the system matrix belonging to the right half of the $s$-plane, to decide the system stability.

Ole, deals with design of decentralized control of interconnected dynamic systems. It is assumed that each sub systems has its own control input and that the interconnections are through the states of the other subsystems. The purpose of the paper [7] is to investigate the possibility of using the so-called block Gerschgorin theorem to evaluate the stability of the total system given the local controllers. This theorem enables to determine inclusion regions for the eigenvalues of the total system and these regions are usually sharper than those obtained by the usual Gerschgorin circle theorem.

The spread of the symmetric matrix is the difference between the largest and smallest eigenvalues. The Gerschgorin circle theorem can be used to bound the extreme eigenvalues of the matrix and hence its spread. It has been shown by David [9], that how inaccurate this bound can be. It is also shown that the ratio between the bound and the spread is bounded by $\sqrt{p + 1}$ where $p$ is the maximum number of off diagonal non zeroes in any row of the matrix. For if all the off diagonal elements are nonzero than the spread is bounded by $\sqrt{n}$. This bound is not sharp for the order greater than two but examples with ratios of $\sqrt{n} - 1$ for all $n$ are given. For banded matrices with $m$ nonzero bands the maximum ratio is bounded by $\sqrt{m}$ independent of size of $n$. This bound is sharp provided only that $n$ is at least $2m$. For the sparse matrices, $p$ it is found that $p$ is small and the Gerschgorin bound may be accurate.
B.N.Datta et al have extended the Carlson –Datta inertia method to cover several other important regions. A direct method is proposed in [10] for determining eigenvalue distribution of a matrix with respect to several important regions of the complex plane. These regions include half planes, shifted half planes, hyperbolas, sectors, quadrants, imaginary axis, region contains within two straight lines that passes through the origin, etc. The method neither requires computation of characteristic polynomial of the given matrix nor solution of and matrix equations.

Vaishya have been developed algorithms which is a new technique to identify the eigenvalues of the power system matrix $A$ with positive real part of the system matrix using the Gerschgorin theorem while computing the characteristic polynomial [11]. The main objective of Kerr, is to show that the basic iteration of the first algorithm of [4] majorises a similar iteration which can be applied directly to the original $(n \times n)$ matrix and the algorithm in [21] actually converges to the isolated eigenvalue of the matrix $A$.V.S. Pusadhkar presents a new approach to identify the presence of real eigenvalues using the Gerschgorin theorem.[14].

Lyapunov stabilization theory and Gerschgorin theorem, a simple generic criterion is derived for global synchronization of two coupled chaotic systems with a unidirectional linear error feedback coupling. The simple criteria is applicable to a large class of chaotic systems, when only a few algebraic inequalities are involved. To demonstrate the efficiency of design, the suggested approach in [16] has applied to some typical chaotic systems with different types of nonlinearities such as the original Chua’s circuit with a sine function, and the Rossler chaotic system. It is proved that these synchronizations are suitably designing the coupling parameters.

Chaos synchronization and parameter identification of single time scale brushless DC motors are studied by Z-M Ge et al [17]. In order to analyze a variety of periodic and chaotic phenomena, they have employed several numerical techniques such as phrase portrait, bifurcation diagram and Lyapunov exponents. By the adaptive control, the improved back stepping method, Gerschgorin theorem, and by addition of a monitor, chaos synchronization of two identical BLDGM systems has been presented. Then by the adoptive method, parameters identification is approached.
The identical two degree freedom loudspeaker systems are discussed for synchronization of chaos by Z-M Ge et al. [18]. Two methods are used to synchronize two identical chaotic systems with different initial conditions, the adaptive control and the Gerschgorin theorem. Finally they have research the parameter identification for two identical two –degrees –of – freedom loud speaker systems by adaptive control and random optimization method.

Shuh –Chuan et al have proposed a new method to design a feedback controller for synchronizing the hyperchaotic circuits. The main idea of the method is to design a controller by applying a suitable Lyapunov function and Gerschgorin theorem. Therefore a feedback controller can guarantee that the signal by chaotic masking from the transmitter can be recovered at the receiver. A scheme of hyperchaotic communication systems is proposed in [19], where the transmitter and the receiver are constituted of the coupled Chua’s chaotic circuits in this paper. On the other hand, to enhance the degree of security in chaotic communication systems, the technique of cipher are also considered in this paper. Therefore, the cryptosystems can be obtained by using the encrypter and decrypter as the chaotic transmitter –receiver pairs. To the purpose of the synchronization, the concepts of identical and generalized synchronizations are used to achieve the aim that the plaintext signals from the encrypter can be retrieved at the decrypter. Finally, the simulation results are given to demonstrate the performance of the proposed cryptosystems.

A set of criteria of asymptotic stability for linear and time invariant systems with multirate point delays. The criteria are concerned with stability local in the delays and stability independent of the delays and are classified in several groups according to the technique dealt by M.de.la.Sen [20]. The technique, used includes both Lyapunov’s and Gerschgorin circle theorem. Lyapunov’s inequalities are guaranteed if a set of matrices of undelayed and delayed dynamics are stability matrices. In this paper, some extension to robust stability has also been considered.

The algorithm developed by R.S. Varga actually converges to the isolated eigenvalue of the matrix A. The symmetric matrix contains real eigenvalues and hence to compute the largest eigenvalue of a symmetric matrix of order (n×n) in
[21] a note has been presented to improve the lower bound for the largest eigenvalue using Lagrange’s series expansion

The feedback delays arise in the control of a computer network-from the information transfer process itself and from the processing of control signals at the network nodes. Flow control of data sources in a computer network often results in time delay control problem. Feedback delay reduces the stability of the system. It has been discussed by James et al [22] how to use the Routh –Hurwitz stability criterion to design and analyze the stability of a flow control algorithm with feedback delay.

The main objective of Danny Gomez was that to show a different presentation of the first part of the Gerschgorin theorem. The proof in [24] is straightforward and understandable compared to the original proof of the Gerschgorin theorem [1]. The regions containing eigenvalues of a matrix are obtained in terms of partial absolute deleted rows and column sums. Further, some sufficient and necessary for H- matrices are derived by Ting et al. Also, an upper bound for the Perron root of nonnegative matrices is presented [25]. The new upper bound with known ones is compared for some examples. The symmetric matrix contains real eigenvalues and hence to compute the largest eigenvalue of the symmetric matrix of order \( n \times n \) Piet Van Mieghem has presented a note to improve the lower bound for the largest eigenvalue [26] using Lagrange’s series expansion.

Tehrani has been concerned with the problem of designing discrete –time control systems with closed loop eigenvalues in a prescribed region of stability. In a state feedback matrix which assigns all the eigenvalues to zero is considered and then by elementary similarity operations and using Gerschgorin Theorem it has been found in a state feedback which assigns the eigenvalues inside a circle with center \( c \) and radius \( r \). The new algorithm has been developed [27] can be employed for large scale discrete –time linear control systems.

The stability of variable parameters double grid second order cell Cellular Neutral Networks linearized in the central linear part of the cell characteristic is investigated by means of Gerschgorin ‘s Theorem using spatial domain as well as
spatial frequency domain descriptions. It has been shown by Iolanda et al., that the stability margin towards the right hand side of the complex plane are identical within both approaches and are larger than expected according to simulations. A conjecture regarding the limits of the characteristic polynomials roots is made and verified through simulations [29].

The spread of the symmetric matrix is the difference between the largest and smallest eigenvalues. The Gershgorin circle theorem can be used to bound the extreme eigenvalues of the matrix and hence its spread. It has been shown by Yuji, how inaccurate this bound can be. It has been also shown that the ratio between the bound and the spread is bounded by $\sqrt{p + 1}$ where $p$ is the maximum number of off diagonal non zero elements in any row of the matrix. For if all the off diagonal elements are nonzero than the spread is bounded by $\sqrt{n}$. This bound is not sharp for the order greater than two but examples with ratios of $\sqrt{n - 1}$ for all $n$ are given. For banded matrices with $m$ non-zero bands the maximum ratio is bounded by $\sqrt{m}$ independent of size of $n$. This bound is sharp provided only that $n$ is at least $2m$. For the sparse matrices, $p$ has been found that found that $p$ is small and the Gershgorin bound may be accurate [30].

Yogesh has algorithm for identification of real eigenvalues on right half of the s-plane for linear systems, hence determining instability of the system. The proposed approach in this paper [31] is based on the Gershgorin theorem and a new approach of Bisection method. The method is efficient, since there is no need to determine all the real eigenvalues and also the characteristic polynomial of the system matrix. It has been found that in some class of control system problems, the method needs minor computations. The proposed approach in this paper is useful particularly in power system applications where the order of the system is large. In this paper the algorithm has been illustrated with the power system examples.

A new approach to estimate the eigenvalues of the images has been presented by Vilas et al. The approach is based on the Gershgorin circle technique. In this paper [32] eigenvalues has been computed using single value decomposition and also the Gershgorin circle technique for some images and the results are compared. The estimation of eigenvalues can be used to extract the important
information of the images. It is a graphical technique and all real eigenvalues need not be computed.

1.5 Objective of the thesis

The main objective of this thesis is to develop an alternative approach for Routh stability criterion using Gerschgorin circles. Secondly, to develop a technique to compute the real eigenvalues of a system matrix as accurately as possible via Gerschgorin circles. In addition to this our aim is also to present some miscellaneous results namely theoretical algorithms to compute eigenvalues and also to examine whether or not we can identify the common eigenvalues between two matrices via Gerschgorin circles.

1.5.1 Contribution to the thesis

1. A heuristic alternative method is developed to decide the stability of a system vis-a-vis the existing Routh test. For Routh test we require a characteristic polynomial with all the co-efficient with same sign. Therefore, the sign of the coefficients of the characteristic polynomial can be decided based on the how Gerschgorin circles are placed in the s-plane.

2. A software program using C++ has been developed to compute the eigenvalues as accurately as possible via Gerschgorin circles. The eigenvalues are computed accurately, since we have applied Bisection method in the Gerschgorin bound.

3. Determination of the stability of the power system based on the Gerschgorin circles.

If the mathematical model of the power system is given, and if the system matrix of the model is known, the Gerschgorin circle is drawn for the system matrix and observing the Gerschgorin circles in the s-plane the stability can be decided, without actually computation of the eigenvalues of the system matrix. In case for any higher order power system example if the Gerschgorin circles are drawn in the s-plane if the...
Gerschgorin bounds are equal on both sides, the system is always unstable. A Novel approach to obtain the spread of the eigenvalues via Gerschgorin circles.

4. Common eigenvalues between two matrices have been computed using Gerschgorin circles.

5. Intervals of the Gerschgorin bound where the eigenvalues exists are compared with bounds computed by Strum sequences.

6. Comparative results have been presented in respect of computations of eigenvalues using the existing method and Gerschgorin circles method. and comparative results have been presented in respect of computation of the root of the polynomial using Bisection method, False position method, Secant method and Gerschgorin circles method by taking the companion form of the polynomial.

7. Some structural matrices have been developed to compute all types of eigenvalues.

8. Determination of the definiteness of the real symmetric matrix using Gerschgorin circles and also determination of the eigenvalues of the matrices possessing strong diagonal dominance using Gerschgorin circles.

9. Computations of eigenvalues of some matrices which are applied in computer science using Gerschgorin circles.

10. Application of Gerschgorin circles to estimate the eigenvalues of the geometrical images and also to damped spring vibrations.

1.6 An overview of the work done to find the stability of the Linear Time Invariant systems.

Leephapreeda has accomplished an overall performance index which is defined as summation of Quadratic cost functions with constraints on Lyapunov’s equations for each system. The quadratic cost functions are used to field both good
performance of the systems and suitable magnitude of the controller output while satisfaction of Lyapunuv’s equations guarantees the stability of the systems. The numerical optimization is then applied to minimize the overall performance index in order to reach a sub-optimal control provided that the solution of the simultaneous stabilization problems exists. The simulation results demonstrate that the technique proposal in the paper [33] can determine one control which stabilizes a set of linear time invariant systems and provides good transient behavior of the systems. Guping has discussed the strict positive systems of linear time invariant systems with multiple time delays [34]. They have presented sufficient conditions for via linear matrix inequalities such that the linear delay system is strictly positive real. Wing have characterized the discrete time single input single output (SISO) linear time invariant systems by a two dimensional kernel function and a filter band structure. Based on the characterization they have investigated the conditions for the stability, the invariability, the causality and the finite response properties of a discrete time SISO linear shift invariant systems. The advantages in [35] for the analysis is that a linear time varying system can be analyzed and designed through finite number of one dimensional kernel functions and LIT filters. Hence, it facilitates the analysis and a design of a LT varying systems, such a L/M is used in digital image processing and digital Video processing.

Ooba has studied in a state space model of a class of linear time shift invariant multivariable multidimensional dynamic systems the problem of internal stability of a system in various aspects. A sequence of equivalent statement is presented to characterize the necessary and sufficient conditions for the internal stability of a multi dimensional dynamics. The statements [36] are generalized to further enhance ones for meeting the stability of a mixed multidimensional and multi circular dynamics, while they degenerate into the stability condition of a circulant matrix when the under lying structure entirely degenerates. As a related topic, a model degree reduction problem has been studied by Ooba by the balancing realization method in a class of linear shift invariant multivariable multidimensional systems. Olgac has considered

A general class of linear time invariant systems with time delay. Here the complexity arises due to the exponential type transcendental terms in their characteristic equations. The transcendility
brings infinitely many characteristic roots. In [37] it starts with the determination of all possible purely imaginary characteristic roots for any positive time delay. To achieve this simplifying substitution is used for the transcendental terms in the characteristic equations. It is proved that the number of such resonant root for a given dynamic is finite. Each one of these root is created by infinitely many time delays which are periodically distributed. Also Olgac has found root crossing direction at these locations are invariant with respect to the delay and dependent only on the cross frequency. Jian et al has applied NyQuist criterion to study the stability of the system [38]. Here the generalized eigenvalue minimization algorithm is used with Bisection search of 10 iterations was performed and the results are compared with several methods. Wen-Liang et al analysis the study of linear time invariant open –loop unstable systems subjected to saturation. Firstly approximating the locally asymptotically stable region of the system for the case where the control is small enough to be saturated, to the case when the control is allowed to be saturated. It is also shown in [39] that, when the Lyapunov descent criterion and the Kuhn tucker theorem is applied, a superior local asymptotically stable region is found. In this work a technique for approximately the locally asymptotically stable region is presented.

M.De.Lasen has derived some criteria for asymptotic stability of Linear Time Invariant systems with constant point delays. Such criteria are concerned with the properties of robust stability related to two relevant auxiliary delays – free systems which are built by deleting the delayed dynamics or considering that the delay is zero. Explicit asymptotic stability results, easy to test, all given for both the unforced and closed –loop systems when the stabilization controller for one of the auxiliary delay free systems is used for the current time delay systems. The proposed in this paper [40] is frequency domain analysis technique. Hyosung et al provides a new analytical robust stability checking method of fractional order linear time invariant interval uncertain systems. Here Lyapunov inequality is utilized for finding the maximum eigenvalue of the Hermitian matrix. Though it finds the stability of the systems accurately, [41] it uses the analytical method to find the robust stability. Srdjan.S. Stankovia’s et al has proposed a method for stabilization of the structurally fixed modes in expansions of linear time invariant dynamic systems in the scope of overlapping Decentralized control design based on the expansion /contraction frame.
work. The method in [42] is a judicious choice of complementary matrices in the expanded space. In this paper from the given matrix it obtains the state transformation matrices which is the block diagonal state feedback matrix and then obtains the eigenvalues of these matrixes to check the stability. Wen Liang analyzed the controllable set (stability region) of a linear time invariant open loop unstable systems. [43]. They have applied Lyapunov descent criterion and Kuchn theorem to the case when the input is allowed to saturate.

Chasi investigated linear systems with polynomial dependence on the time invariant uncertainties constrained in the simplex via homogenous parameter dependent quadratic Lyapunov functions. It is shown in [44] that a sufficient condition for establishing whether the system is either stable or unstable can be obtained by solving a generalized eigenvalue problem. Moreover the condition is also necessary by using a sufficient large degree of the HPD-QLF. Jun – Guo considers the problem of Robust stability and stabilization for the class of fractional order linear time invariant systems with the convex polytypic uncertainty. The stability condition of the fractional order linear time invariant systems without uncertainties is expanded by introducing a new matrix variable. And exhibits a kind of decoupling between the positive definite matrix and the system matrix. Based on the new expanded stability condition, sufficient conditions for the robust stability and stabilization problems are established in terms of linear matrix inequalities by using parameters-dependent positive definite matrix [45]. Suctlane et al. analysis stability for a class of switched linear systems modeled as hybrid automata. It [46] deals with switched linear planar systems, modeled by hybrid automation with one discrete state. The guard on the transition is a line in the state space and the reset map is a linear projection on to the x-axis. They define necessary and sufficient condition for the stability of the switched linear systems with fixed and arbitrary dynamics in the location. Shumafov has considered the static stabilization problem for two dimensional linear time invariant control systems with a delay feedback. They have obtained the necessary and sufficient conditions for the stabilization of the systems under consideration. The theorems proved in this paper [47] shows that such a delay feedback approach is efficient in stabilizing the second order linear systems.
Rosa has introduced the notion of absolutely distinguishable discrete dynamic systems, with particular applicability to linear time invariant systems. The motivation for this novel type of distinguish ability is that, in particular from the stability and performance requirements of the worst cases, a persistence of excitation type of condition and a minimum number of iterations are required to properly distinguish dynamic systems. They [48] have also demonstrated that the former constraints can be written as a lower bound on the intensity of the exogenous disturbances. The applicability of the developed theory is illustrated with a set of examples.

P. Orlowski proposed a novel method for feedback stability evaluation for linear time varying, discrete time control systems. It is assumed that the time varying systems can be described by the general discrete time, time varying state space model and by the equivalent linear input–output operator. The method in [49] extends feedback stability concepts for systems given in a general linear time-varying discrete time form. The Author has selected the short time stability concepts employed for inference about feedback stability of systems defined on an infinite time horizon. Gielan has applied all the Lyapunov methods for delay difference inclusions more recently [50]. Firstly Lyapunov–Krasovskii, secondly the Lyapunov Razmikhin method. Using the above both stability analysis and stabilizing controllers synthesis methods have been proposed.

1.7 Brief view of the Research work carried in our thesis.

1.7.1 An alternative method for Routh stability criteria based on the Gerschgorin circles

The Routh stability criteria states that for a given system matrix to be stable, it is necessary and sufficient that there should be no change in sign in the first column of the Routh array, when it is done through the characteristic polynomial with all its coefficients of same sign, of the matrix $A$. Depending upon the changes in sign in the first column of the array, the technique decides the existence of number of complex conjugate eigenvalues with the positive real part. The Routh criteria considers a characteristic polynomial which contains all the coefficient to have the same sign and the computation of the characteristic polynomial from the given system matrix takes lots of computation especially when the order of the matrix is very large. In our method observing the Gerschgorin circles the sign of the
Coefficient of the characteristic polynomial of the system matrix of any order is found heuristically which requires no computations. Also the structure of sign changes in the first column of the Routh array, has been shown how it is reflected in the complex s-plane when the Gerschgorin circle are drawn for the system matrix. Many illustrations are provided to validate the proposed technique.

1.7.2 Stability Analysis of the system matrix using Gerschgorin circles.

Given a system matrix $A$ of order $(n \times n)$, by drawing the Gerschgorin circles of the matrix if the Gerschgorin bound is very large on the left hand side of the s-plane, then the heuristically we can conclude that system is stable with probability 0.9. If the Gerschgorin bound on both side are equal, then the system is unstable. Similarly if the Gerschgorin bound on the right hand are very much larger than the left hand side of the s-plane, than the system is unstable with probability 0.99. These results are proved analytically and also graphically using Gerschgorin circles. The results are illustrated with suitable examples.

The stability of the system is analyzed by transformation of the centers of the Gerschgorin circles.

The spread of the eigenvalues can be decided using the trace of the matrix and also the Gerschgorin bound. Analytical proof has been developed which gives the distribution of real eigenvalues.

1.7.3 Determination suitable step length has been obtained to compute the eigenvalues of the system matrix accurately.

We start with the given matrix, then we draw the Gerschgorin’s circles of this matrix, and then we find the Gerschgorin bound by taking the intersection of the unions of row wise and column wise circles under which the eigenvalues lie. It is interesting to note that the intersection of all Gerschgorin’s circle lie on the real axis, will give clue whether or not the real eigenvalues exist. Thus we do this by computing the determinant($\lambda_i I - A$) substituting $\lambda_i$ for all the intersection points one by one. During these calculations if the determinant changes the signs alternatively, this leads to the existence of eigenvalues on real axis. But by the above information i.e. the intersection of Gerschgorin’s on the real axis a step size $h$ has
been proposed to compute all the real eigenvalues accurately. In the above procedure the following things have been done. Suppose $-a$ and $b$ are the bounds under which the real eigenvalues exist, then if we start computing the determinant $(\lambda I - A)$ between 0 and $b$ with some step size, like $\frac{ab}{\text{trace}}$ then the change in the determinant sign, implies there exists a real eigenvalue with the positive real part. This means that the above system is unstable. Further, if we cannot find the changes in sign of determinant implying there exists no real eigenvalues with positive real part. Then we start computing the determinant $(\lambda I - A)$ from 0 to $-a$. Then if the determinant changes the sign $p$ times, then this implies the existence of $p$ real eigenvalues with negative real part. If $p = n$, then the system matrix $A$ has $n$ real eigenvalues with in negative real part, implying then the system is stable. In case $p < n$ it has been found that the remaining eigenvalues are repeated if the determinant of the differences between trace and sum of the $p$ real eigenvalues are zero otherwise it is complex conjugate pairs with positive or negative real part.

Likewise different step lengths have been taken and the eigenvalues are computed for different matrices and the results are tabulated. A suitable step length has been obtained which computes the eigenvalues of the system matrix accurately.

In the entire Gerschgorin bound to compute the eigenvalues Bisection method, false position method and Secant method have been applied and the results are tabulated.

1.7.4 **Determination of common eigenvalues using Gerschgorin circles.**

Given two system matrix $A$ and matrix $B$, draw the Gerschgorin circles of the matrix $A$ and matrix $B$, we obtain the area bounded in which the eigenvalues of the matrix $A$ and matrix $B$ lie. Program has been developed to overlap the areas, so that we get the common area. We find that the area is reduced and the common eigenvalues lies in the overlapping area. The common eigenvalues are computed by applying the Bisection method in the common area. This method to compute the common eigenvalues has been applied to compute the common fixed modes in Decentralized control systems.
1.7.5 Computation of eigenvalues using Gerschgorin circles.

Bounds for the existence of eigenvalues are computed using the existing Strum sequences method and also using the Gerschgorin circles and the results are compared.

The eigenvalues of the system matrix have been computed using the existing method available in the literature i.e., Given method, Jacobi method, Rutihasser method, Householder QL-method, Householder QR-method, Power method. The eigenvalues of the system matrix are computed using Gerschgorin circles methods and the results are compared and tabulated.

In the existing method we know that the roots of the polynomial are computed using the Bisection method, false position method and Secant method. In this thesis an attempt has been made to find the roots of the polynomial using the Gerschgorin circles. The companion form of the given polynomial are taken and Gerschgorin circles are drawn for this matrix and at the Gerschgorin bound Bisection method, false position method and Secant method are applied separately and roots are computed and the results are tabulated.

Positive definiteness of the real symmetric matrix is computed using existence method and also using Gerschgorin circles and the results are compared.

1.7.6 Development of structural matrices

Structural matrices of order \((n \times n)\) have been developed and it has been analytically proved using Gerschgoim circle theorem that this matrix possesses eigenvalues at the origin and the remaining \((n - 1)\) eigenvalues are repeated. Also using Gerschgorin circles theorem a formula has been derived to compute the repeated eigenvalues.

In the above structural matrix it has been proved using Gerschgorin circles theorem that if the center of the Gerschgorin circle is shifted by \(\varepsilon\) distance, than one of the eigenvalue is \(\varepsilon\) itself and the remaining eigenvalues are repeated. A formula has been derived using Gerschgorin circle theorem to compute the repeated eigenvalues.
Some structural matrices of order \((3 \times 3)\) has been developed which possesses negative real eigenvalue and a complex conjugate pairs of eigenvalues with the positive real part.

Some structural sparse matrices of order \((n \times n)\) has been developed and it has been heuristically shown that the matrix possesses \((n - 2)\) repeated eigenvalues and the remaining two eigenvalues are complex conjugate pairs with positive real part.

1.7.7 Definiteness of real symmetric matrix via Gerschgorin circles

A matrix is said to be positive definite if all the eigenvalues lie on the right half the s-plane, positive semi definite if one of the eigenvalues is at the origin and all the remaining eigenvalues are positive. Similarly negative definite if all the eigenvalues are negative and negative semi definite if one of the eigenvalue is at the origin and the remaining eigenvalues are negative. If the eigenvalues are spread on the both side of the imaginary axis in s-plane, than the matrix is said to be indefinite. This definiteness of the real symmetric matrix is identified using the Gerschgorin circles.

1.7.8 Eigenvalues of strong diagonal dominance matrices

If the matrix in the principle diagonal elements are larger compared to the elements in the upper and lower triangular matrix than the matrix is said to have strong diagonal dominance. The behavior of the eigenvalues and the Gerschgorin circles are studied for such type of matrices and the results are tabulated.

1.7.9 Application of Gerschgorin circles in computer science and control systems.

The computation of eigenvalues using Gerschgorin circles have been applied for some system matrices which are used in computer science. Eigenvalues have been estimated for some geometrical figures using principle component analysis and also using Gerschgorin circles and time taken by both methods are compared.
A matrix which represents the power system has been considered and the stability is decided using Gerschgorin circles based on the Gerschgorin bound and compared with the other existing results.

Conditions for the damped string vibration have been developed using Gerschgorin circles and it is compared with the existing method. For the vibrating string of fifth order Gerschgorin circles technique have been used to compute the eigenvalues and the results are compared with the existing method.