ABSTRACT

This research work is primarily concerned with the computation of eigenvalues by graphical method to decide the stability of linear time invariant systems. It is known that the linear time invariant systems are modeled as state variable equations. Then the stability depends on the location of the eigenvalues in the s-plane of the system matrix $A$. The system is asymptotically stable if the eigenvalues are in the open left half of the s-plane and away from the imaginary axis. If the eigenvalues of $A$ belongs to the open right half of s-plane, then the system is unstable. So the location of eigenvalues of the system matrix plays very important role in deciding the stability of the system. There exist many techniques in the literature to compute the eigenvalues which take lot of computations. This is due to various similarity transformations of matrix, finite word length, round off errors, truncation errors, machine epsilon etc., Due to these problems eigenvalues computed are not accurate and also takes lot of computations.

In this thesis, to overcome the problems mentioned above, a novel approach which is graphical technique is adopted to locate the eigenvalues of the system matrix and hence decide the stability of the system matrix. This graphical approach is based on the Gerschgorin circles. This novel method is exhaustively explained in the body of the thesis. Different techniques developed via Gerschgorin theorem, are now briefly discussed one by one, and chapter wise.

Chapter 2, describes an alternating heuristic graphical approach for Routh stability criterion. As it is known that the stability of the system using Routh test is decided by observing the changes in sign in the first column of the Routh array. But the Routh test needs characteristic polynomial with positive coefficients and computations of characteristic polynomial from the given system matrix, takes lot of computations if the matrix is of higher order. The Graphical method proposed decides, heuristically, the sign of the coefficients of the characteristic polynomial based on the bounds of the Gerschgorin circles. And secondly, to check the stability in Routh criterion, Routh array is developed and depending on the change in sign in the first column of the Routh array stability is decided. In this thesis, it is shown how this graphical technique shows the changes in sign of the Routh array is reflected in
the Gerschgorin circle. Hence computational burdens are appreciably reduced due to graphical technique.

Chapter 3 discusses about the stability of a system via Gerschgorin circles. When the Gerschgorin circles are drawn for a system matrix, the stability of the system can be decided by observing at the bound of the Gerschgorin circles with respect to the origin. If the right hand side bound is more than the left hand side, then the system is unstable (heuristically). Detailed discussions have been made regarding the stability for various illustrative examples. Some analytical results have also been obtained to decide the stability of a given system matrix using the Gerschgorin circles. A power system example has been presented to decide the stability based on the above discussions.

In chapter 4, a novel software technique has been developed to compute the eigenvalues of the system matrix accurately at the Gerschgorin bound by using a suitable step length which has been found from various step lengths which was considered initially for various matrices. The algorithm is developed to compute the eigenvalues accurately using the Gerschgorin bound without using the step length. Roots of the polynomial are computed mainly by Bisection method, False Position and Secant method. In our thesis, Gerschgorin circles technique have been applied to find the roots of the polynomial at the Gerschgorin bound and the results are compared.

A new approach has been presented to find the common eigenvalues between the two matrices. Programs have been developed to identify and compute the common eigenvalues which can be applied in many Engineering applications like identification of similar images and so on, in control systems etc. Application of common eigenvalues is used in finding fixed modes in Decentralized control systems.

In Chapter 5, contains the comparative results between the existing methods of computing eigenvalues and the proposed Gerschgorin circle technique. In our Gerschgorin circles approach a new idea has been proposed to compute the largest eigenvalues using the Gerschgorin bound and it is compared with the existing Power method which also computes the largest eigenvalue.
In Chapter 6, presents various types of Structural matrices in different cases, along with the analytical proofs using Gerschgorin theorem in some cases to decide the eigenvalues of the system matrix. These structural matrices are very much useful in computer science since some of the matrices are symmetric and also in control system to decide the stability. These matrices can also be used in various other Engineering applications.

In Chapter 7, the Definiteness of Real symmetric matrices which are encountered, particularly, in applications like image processing, Pattern Recognition etc., have been identified based on the Gerschgorin circles. And this approach reduces the computational burdens over the existing methods and also the behavior of the eigenvalues of the strongly diagonal dominated matrices is studied and the results have been presented.

In Chapter 8, some applications in computer science and control systems have been discussed based on the Gerschgorin circles techniques.

Finally, some general conclusions have been made based on the results presented in our thesis, and the suggestions for future work are also included at the end.

Some additional theoretical results that are worked out related to the thesis have all been presented in the appendices.