CHAPTER 2

RESPONSE FUNCTION OF NaI(Tl) DETECTOR:
AN INVERSE MATRIX APPROACH

The response function of gamma radiation detector is an important factor for spectrum analysis because some photons and secondary electrons may escape the detector volume before fully depositing their energy, of course destroying the ideal delta function response. The observed pulse-height spectrum consists of a peak and a continuous distribution of pulses for monochromatic photons incident on a NaI(Tl) scintillation crystal and it becomes an essential need to deconvolute (unfold) the measured pulse-height distribution.

In order to unfold the incident radiation energy spectrum from the measured pulse-height distribution, one needs to know the response function of the detector used, since every detector has its own characteristic response function. The response function can be obtained either experimentally or analytically by using Monte Carlo simulations. In the present study response function of NaI(Tl) scintillation gamma ray detector is obtained experimentally i.e. by taking into account the experimentally determined parameters like peak-to-total ratio, intrinsic efficiency and FWHM (full width at half the maximum height) of the detector. The experimental response features for sodium iodide detector for a monoenergetic gamma-ray source will be quite useful to other investigators because of the importance of this type of detector in industry as well as in research also.

Since the early days of gamma ray spectroscopy, various techniques [57] based on interpolation and extrapolation of standard empirical line shapes (in particularly suitable for energies greater than 2 MeV gamma rays) for the
analysis of complex gamma spectra have been developed. Complex and sophisticated methods are being used for the conversion of observed pulse-height distributions into true gamma ray spectra. A brief write-up for the work already reported by various investigators, for response function of sodium iodide scintillation detector, is given below:

2.1 Review of literature on response function

Berger and Doggett [58] used the Monte Carlo methods to calculate the response of NaI(Tl) detectors, and presented summary results in terms of photo-fraction (ratio of area under the photo-peak to total area under the observed spectrum). The photo fraction values were given for collimated mono-energetic radiations ranging from 0.279 - 4.45 MeV for NaI(Tl) crystals having size ranging from 0.25” (radius) x 5” (length) to 2.5” x 9”. Also a fair agreement with experimental results was indicated, but experimental photo-fraction values were consistently somewhat lower than theoretical values.

Hubbell and Scofield [59-60] expressed the response of a 5” (diameter) x 4” (long) NaI(Tl) crystal, to axially incident 0.01 to 8 MeV gamma rays, by a 28 x 28 matrix. The conversion of observed pulse-height distributions to the true gamma ray spectrum was achieved by multiplying inverse of 28 x 28 matrix with observed pulse-height distributions. Measured pulse-height distributions were combined with Monte Carlo calculated distributions for the input data. This 28 x 28 inverse response matrix covered an energy range from 0.01 to 7.84 MeV. The bin mesh was taken uniform in the square root of energy and each interval including an entire photo-peak. It was concluded that such a response matrix is suitable for the unscrambling of continuous spectra but not for sharp peaks.

Berger and Seltzer [61] calculated the response of 3” x 3” NaI(Tl) detector
by a method that takes into account the multiple scattering and escape of incident gamma rays as well as of the secondary charged particles and bremsstrahlung from the detector. It was suggested that response function can be written as the convolution of energy deposition spectrum and a resolution function multiplied by the detector efficiency. They made a systematic tabulation of the response function of the detectors exposed to broad parallel beams of gamma rays at 21 energies between 100 keV and 20 MeV, and studied the dependence of the response function on the direction of the incident gamma rays. It was concluded that there is a little dependence of response function shape on direction of the incident gamma ray beam.

Sie [62] employed an empirical method for calculating the response function of the 3” x 3” NaI(Tl) detector. The method was based on the decomposition of the response into various parts associated with the various interaction processes in the detector. The smooth dependence of the parameters on the gamma ray energy facilitated interpolations to synthesize response functions for gamma rays. The method was tested with a complex spectrum from a $^{152}$Eu source and its application to a gamma ray study was described as an illustration. For gamma rays energy, greater than 2 MeV, this method offered simplicity while not as accurate a method of interpolation as other techniques.

Corvisiero et al. [63] used a Monte Carlo code to evaluate the response function of large NaI(Tl) crystals to high energy photons (up to 300 MeV). They considered all the involving electromagnetic processes, including radiation energy losses, annihilation in flight and multiple scattering while calculated the response function. The numerical and physical approximations were in good agreement with the available experimental response functions of NaI(Tl) crystals.
to high energy monochromatic photon beams.

Orion and Wielopolski [64] calculated the response functions of the BGO and NaI(Tl) detectors of different sizes and types at 0.662, 4.4, and 10.0 MeV energies using three, EGS4 [65], MCNP [66] and PHOTON [67] Monte Carlo codes. The energy range of 0.662-10 MeV was chosen to cover energies of interest in body-composition studies. The energy deposition in the detector was calculated using 512 bins of 20 keV widths. The main differences in the response function for the detectors of different size and type were found to be in their escape peaks. The superior efficiency of the BGO detectors was weighed against their inferior resolution, and their higher price than that of the NaI(Tl) detectors. Since the price of the BGO detectors strongly depends on the crystal’s size, its optimization was an important component in the design of the entire system.

Itadzu et al. [68] made use of EGS4 code for calculation of response functions of the 16" x 16" x 4" large sized NaI(Tl) detector. Their response matrix was of the order of 24 x 256 on the scale of energy bins versus pulse-height distribution used to unfold gamma ray spectra from the measured data. An agreement between the calculated results and measured data had been found under error limit of ±15%. The unfolding results found to be reasonable in a preliminary analysis on natural background gamma ray spectra.

Ghanem [69] calculated the response features of NaI(Tl) detector, for a mono-energetic gamma photon source, by developing Monte Carlo calculations. The response features calculated were full energy peak, single scattered escape, single escape peak, double scattered escape, double escape peak, besides single scatter, double scatter and triple scatter continuum. These
features were compared for different sizes and geometric efficiencies of the detector. The Monte Carlo simulation provided the weighting factors of the above mentioned eight features of the response for the NaI(Tl) detector that were necessary for response function generation for that type of detectors.

Shi et al. [70] used Berger-Seltzer’s method [61], general Monte Carlo (MC) programs, such as EGS4 [65] and MCNP4B [66] and special MC programs for calculating the response function for the 3”x 3” NaI(Tl) detector. The pulse-height distributions from sources with energies ranging from 0.4118 to 7.11 MeV were compared with the simulated values. The spectra generated by Berger-Seltzer's method and the general MC programs did not agree well with the experimental data whereas PETRANS 1.0 (in addition to scintillation efficiency it considers the single escape peak shift in calculations) generated spectra were found to be in good agreement with experimental data.

Vitorelli et al. [71] calculated the response function of 3” x 3” NaI(Tl) detector using the general purpose code MCNP4B [10] for gamma rays from an $^{241}$Am/Be source capsule and compared this simulated spectral shape with the measured spectral shape obtained using a gamma ray spectrometer with a cylindrical NaI(Tl) gamma detector. A good agreement was found between the simulation and the experimental response function.

Singh et al. [72] and Sabharwal et al. [73] constructed the response matrix for NaI(Tl) detector by inverse matrix approach. They chosen a bin mesh ($E^{1/2}$) of 0.1 (MeV)$^{1/2}$ and 0.05 (MeV)$^{1/2}$, for constructing a 10 x 10 and 16 x 16 response matrices, respectively. The detector response unfolding converting the observed pulse-height distributions to a photon energy spectrum was found to be quite satisfactory for multiple scattering corrections in case of gamma scattering.
measurements. Their results had confirmed that for thick targets, there was significant contribution of multiple scattered radiation, emerging from the scatterer, having energy equal to that of singly scattered Compton process.

Hakimabad et al. [74] calculated the response functions of a 3” x 3” NaI(Tl) detector by using a Monte Carlo based MCNP-4C code. The gamma-rays from radioisotope sources were used in the range from 0.081 to 4.438 MeV for this determination. The calculated results were compared with measured data by using standard gamma-ray sources to check their accuracy. Through the precise modeling of the detector structure, the agreement between results found to be improved. Comparison results showed that in $^{60}$Co spectrum the calculated response agree with the measured one with less than ±3% relative deviation.

Cengiz [75] calculated the response function (using the Monte Carlo method) of a 3” x 3” NaI(Tl) scintillation detector to photons from point gamma-ray sources (in the energy up to 1.5 MeV), placed 10 cm from the scintillator surface, applying simple approximations based on the peak-to-total ratio and the detector resolution. The Compton continuum of the detector response function was assumed as an isotropic (rectangular) region for the photon energies up to 1 MeV. In the energies between 1 and 1.5 MeV, the Compton continuum was obtained assuming a single Compton scattering with free electrons. The photopeak of the detector response function was assumed as a line. Each determined channel of the response function was distributed to Gaussian functions. The obtained response functions were compared with the experimental values and a good agreement was found.

2.2 Theory for response matrix

It is generally assumed that response spectrum is a Gaussian peak, when
detector is bombarded by a mono-energetic beam of the given radiation. But, a Gaussian peak response is not always realized particularly in the case of neutral (gamma) radiation. The main features of the gamma-ray spectra are associated with the processes by which gamma rays interact with matter. The pulse-height spectrum simply reflects the different interactions which occur in the detector volume. The response function of a detector at a given energy is determined by the different interactions which the radiation can undergo in the detector and its design and geometry. During interaction with detector volume, some gamma photons suffer Compton scattering instead of fully depositing their energy thus producing a low energy tail. Due to these events at a lower energy than under the full energy peak, the response function thus consists of a Gaussian peak with a low energy tail determined by the amount of scattering and other energy losses [76] in the detector. The following relation

\[
\frac{d\sigma}{dT} = \frac{\pi r_e^2}{m_e c^2 \gamma^2} \left[ 2^+ \frac{s^2}{\gamma^2 (1-s)^3} + \frac{s}{1-s} \left( \frac{2}{\gamma} \right)^2 \right]
\]  

(2.1)

depicts that the Compton electrons are distributed continuously in energy, of course destroy the ideal delta function response. Where the parameters \( s = \frac{T}{h\gamma} \), \( \gamma = \frac{h\nu}{m_e c^2} \), \( r_e \) is the classical electron radius, \( m_e \) is mass of an electron and \( T \) is energy of Compton electron. Moreover, events interacting via the pair production (for \( E \geq 1.02 \text{ MeV} \)) mechanism will also contribute a structure to the function. Thus there is probability that a photon incident at certain energy may be recorded at lower energies. This probability defines the response function, \( R(E, E') \), where \( E \) is the incident energy and \( E' \) is the recorded energy. Therefore, one can relate the pulse-height distribution, \( P(E') \), to the incident photon energy, \( S(E) \), by
When the spectrum is recorded by a multi-channel analyzer, above equation takes the discrete form

$$P_j = \sum_{i=1}^{N} S_i R_{ij}$$  \hspace{1cm} (2.3)

Where $P_j$'s are the recorded counts in the $j$th channel, $R_{ij}$ is the element of the response matrix coupling the $j$th pulse-height interval with the $i$th energy interval, $S_i$ is the radiation intensity in the $i$th energy interval, and $N$ is the number of energy intervals over which the energy spectrum is to be observed [77]. To determine the gamma ray spectrum $S(E)$, from the measured pulse-height distribution, requires the knowledge of $R(E, E')$. If the pulse-height distribution is measured over $M$ channels, one obtains $M$ sets of equation of the form of equation (2.3) that can be formulated in a matrix form as

$$P = SR$$  \hspace{1cm} (2.4)

Where $P$ is a matrix (or vector) having $M$ components specifying the measured pulse-height distribution, $R$ is an $M \times N$ detector response matrix and $S$ is a vector of dimension $N$ of the energy spectrum in $N$ energy intervals. The response of the detector is obtained by simple matrix multiplication of $P(E')$ and $R^{-1}_{ij}$ (Inverse of matrix $R$ with $N=M$) as:

$$S_i = \sum_{j=1}^{N} P_j R^{-1}_{ij}$$  \hspace{1cm} (2.5)

This procedure of obtaining $S(E)$ from $P(E')$ is known as the unfolding of the measured spectrum. The matrix multiplication of these $P$ and $R^{-1}$ matrices gives another row matrix, which is the true gamma ray spectrum of the detector. The elements of this $S_i$ are the values corresponding to bin meshes chosen.
above in the construction of the matrix. So by dividing each value by its corresponding bin width, the numbers of photons per energy bin are calculated as

\[ S(E) = \frac{S_i}{E_i - E_{i-1}} \]  \hfill (2.6)

Where \( S(E) \) is in units of photons per unit energy interval. The plot of this \( S(E) \) versus the top of each energy bin in terms of the histogram, gives the response-corrected spectrum of detector.

### 2.3 Response matrix construction

For the construction of response function, pulse-height distributions for energies 279 (\(^{203}\)Hg), 320 (\(^{51}\)Cr), 511 (\(^{22}\)Na), 662 (\(^{137}\)Cs) and 834 (\(^{54}\)Mn) keV are obtained experimentally from mono-energetic sources by placing each of the radioactive sources at distances of 10 cm in front of a well collimated NaI(Tl) detector (Fig. 2.1).

![Experimental arrangement for recording pulse-height distribution from mono-energetic sources for construction of response matrix.](image)

Fig. 2.1: Experimental arrangement for recording pulse-height distribution from mono-energetic sources for construction of response matrix.

The pulse-height distributions obtained experimentally for energies 279, 320, 511, 662 and 834 keV are normalized in such a way that the areas under
their photo-peaks are made equal to intrinsic (crystal) efficiency values [78] calculated using the formula

\[ \varepsilon_i(E) = 1 - e^{-\mu_{\text{tot}}(E)t} \]  

(2.7)

Where \( \mu_{\text{tot}}(E) \) is the attenuation coefficient for NaI(Tl) at the mid point of source energy bin and \( t \) is the crystal thickness. These distributions are then smoothed such that peaks resulting only from the interactions after the photon entries into the crystal are included (Fig. 2.2) whereas those from before entry should be subtracted off. The photo-peaks of these smoothed curves are omitted from the

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**Fig. 2.2:** Observed pulse-height distributions from mono-energetic sources, the areas normalized to crystal (intrinsic) efficiency.
spectrum and their theoretical Compton edges 
\[ V_c = \frac{E}{1 + \frac{E}{1+2\alpha}} \]
are noted, Where \( E \) is the Incident photon energy and \( \alpha = \frac{E}{m_0c^2} \). Then all of these five different distributions for different source energies are plotted linearly on \( V/V_c \) scale (Fig. 2.3), where \( V_c \) is theoretical Compton edge for each of energy. Each distribution is divided into energy bins of constant width in terms of \( (E)^{1/2} \) MeV \[3,4,16\]. It is worthwhile to mention here that actually there are 200 cross-cut points (corresponding to forty \( (E)^{1/2} \) energy bins each for five different energies) in Fig. 2.3, but only few are shown to visualise them well separated from one another. Labelling each bin by its top energy, the bin content distributions for the given

![Graph](image)

Fig. 2.3: The transformation to \( V/V_c \) scale, cross-cuts of constant \( V/V_c \) are indicated.
source energies $E$, are plotted as function of $(E')^{1/2}$. These curves are then interpolated to obtain a series curves for each of the bin ranging from $0.025 \,(\text{MeV})^{1/2}$ to $1.0 \,(\text{MeV})^{1/2}$ in the energy range from 0 to 1 MeV as shown in Fig. 2.4. In this figure only few curves are shown (similar as in Fig. 2.3) but actually there are total 40 such curves corresponding to $(E')^{1/2} = 0.025, 0.05, 0.075, 0.1, 0.125, \ldots \ldots \ldots \ldots \ldots 1 \,\text{MeV}$. The curves having different $(E')^{1/2}$ values (Fig. 2.4) are then further divided into energy bins of width equal in $E^{1/2}$ MeV and the bin contents are written in the form of a triangular matrix $(R)$ having elements $R_{ij}$, where the indices $i$ and $j$ refers to incident energy $E$ and pulse-height of each energy bin $E'$. The sum of each row is equated to $\{1 - \varepsilon_p(E)\} \varepsilon_i(E)$, where $\varepsilon_i(E)$ is the intrinsic (crystal) efficiency. The photo-peak efficiency $\varepsilon_i(E)\varepsilon_p(E)$ is then added to the principal diagonal of the matrix, making each row equal to $\varepsilon_i(E)$.

![Fig. 2.4: Interpolated bin content counts as function of incident energy at different values of $(E')^{1/2}$.](image)
For each ith energy, a summation over all j values equals to the crystal efficiency, the resultant matrix \((R)\), having elements \(R_{ij}\), given in Table 2.1 is a desired response matrix, which converts spectra \(S(E)\) into expected measured pulse-height distribution \(P(E')\) (as given by equation 2.3 and 2.4). Mathematica (version 5.1), a computational software program, has been used to invert the response matrix \(R_{ij}\) and for matrix multiplication of \(P(E')\) and \(R_{ij}^{-1}\) (Table 2.2) as per demanded by equation 2.5. A typical observed (after subtracting background) pulse-height distribution \(P(E')\) for 662 keV gamma ray scattering from aluminium block (12 mm in thickness) is shown as curve ‘a’ of Fig. 2.5. The resulting calculated histogram \(S(E)\) (given by equation 2.6) is shown as curve ‘b’ in the figure. It is observed from Fig. 2.5 that low pulse-height counts

Fig. 2.5: Experimentally observed pulse-height distribution, \(P(E')\), curve-a obtained after subtracting background events (unrelated to sample) and resulting calculated histogram(curve-b) of \(S(E)\) converting pulse-height distribution to a true photon spectrum.
<p>| (E'/2) (keV) | 0.025 | 0.050 | 0.075 | 0.100 | 0.125 | 0.150 | 0.175 | 0.200 | 0.225 | 0.250 | 0.275 | 0.300 | 0.325 | 0.350 | 0.375 | 0.400 | 0.425 | 0.450 | 0.475 | 0.500 | 0.525 | 0.550 | 0.575 | 0.600 | 0.625 | 0.650 | 0.675 | 0.700 | 0.725 | 0.750 | 0.775 | 0.800 | 0.825 | 0.850 | 0.875 | 0.900 | 0.925 | 0.950 | 0.975 | 1.000 |
|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| (R_{11}) | 0.025  | 0.050  | 0.075  | 0.100  | 0.125  | 0.150  | 0.175  | 0.200  | 0.225  | 0.250  | 0.275  | 0.300  | 0.325  | 0.350  | 0.375  | 0.400  | 0.425  | 0.450  | 0.475  | 0.500  | 0.525  | 0.550  | 0.575  | 0.600  | 0.625  | 0.650  | 0.675  | 0.700  | 0.725  | 0.750  | 0.775  | 0.800  | 0.825  | 0.850  | 0.875  | 0.900  | 0.925  | 0.950  | 0.975  | 1.000  |</p>
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Table 2.2: Inverted response matrix, $R_q^n$, in the same units as Table 1. Note that $q$ should multiply the numbers.
resulting from partial absorption of higher energy photons have been shifted to the full energy peak after applying response matrix to observed pulse height distribution.

2.4 Summary

Incorrect physical data are obtained from an analysis of recorded gamma-ray spectra, without application of unfolding methods. Analysis of data obtained in the experiments requires accurate knowledge of the shapes of the response functions for a range of gamma ray energies. So, the conversion of observed pulse-height distribution to a true photon energy spectrum is essentially required for gamma spectroscopy.

Although, Monte Carlo calculations provide the detector response, but these are incapable of taking into account the effects of materials surrounding the detector and attenuation of photons in the entrance window. Besides this, the Monte Carlo calculations suffer from various discrepancies most probably due to the fact that the calculations are done for an idealized bare detector by assuming an infinitesimally narrow beam impinging at the centre of the detector whereas in the realistic situation, the beam is of finite size. Thus by taking in to account all these aspects, the present work has been performed for the conversion of a pulse-height distribution to a photon energy spectrum with the help of a response matrix technique. As the finite energy resolution (due to distribution of pulse-height) of the scintillation spectrometer leads to widening of detector peak so a matrix of fine mesh would be required to correct for this small spread of peak.

The present work demonstrates the inverse matrix technique for constructing a 40 X 40 matrix (covering energy range from 0.625 keV to 1.0 MeV) with bin mesh \((E)^{1/2}\) of 0.025 \((\text{MeV})^{1/2}\) that takes in to account the various
experimentally determined parameters. In this context, the inverse response matrix technique takes into account, all the realities occurring in the real world of experiments. Although this method of obtaining experimental response function seems to be time consuming and complicated as it requires number of mono-energetic pulse-height distributions from different gamma ray sources as input. But this method has advantages, over the Monte Carlo calculations of the detector response, as already addressed in the above paragraph.