CHAPTER-7

SIMILARITY SOLUTIONS OF THE THREE DIMENSIONAL
BOUNDARY LAYER EQUATIONS OF A CLASS OF GENERAL
NON-NEWTONIAN FLUIDS

7.0. INTRODUCTION:

The subject of boundary layer flows of non-Newtonian fluids has been a
topic of an investigation from a long time as it has an application in various
industries and in day to day life. Some special work on the topics is given by
Acrivos et al (1960, 1965), Schowalter (1960), Bizzell et al (1962), Hayasi
(1965), Kapur et al (1963), Lee and Ames (1966), Hansen and Na (1967),
Banks et al (1986). Even though considerable progress has been made in our
understanding of the flow phenomena, more work is still needed to understand the
effect of various parameters involving non-Newtonian models and the formulation
of accurate method of analysis for anybody shapes of engineering significance.
The theoretical research on the topic however is hampered because of complex
nature of the equations describing the flows. Moreover, the nature of non-linear
shearing stress and rate of a strain relationship of the various models poses
additional difficulties.

In the analysis of boundary layer problem, the class of solution known as
similarity Solutions places a vital role because it is the only class of exact solution
for the boundary layer equations. For two dimensional flow of Newtonian fluids, it
is well-known that similarity solutions exist for the class of bodies known as the
Falkner-Skan problem, which includes many practical geometries on the other
hand, for non-Newtonian fluids, the non-linear relation between shearing stress
and the rate of strain causes further restrictions on the class of problems which can
be solved by similarity transformations. It is interesting to note that this non-linear
relationship can be mathematically expressed as a functional relationship between stress tensor and rate of deformation tensor and for different non-Newtonian fluids this relationship may be implicit or explicit functional relationship. For the two dimensional case, such problem was investigated by Hansen and Na (1968) and they have drawn the conclusion that for boundary layer flows of non-Newtonian fluids of any model, similarity solutions exist only for the flow passed a 90° wedge.

There are certain aspects of three dimensional boundary layers. In two dimensional flow, since the boundary layer is restricting to more in the direction of the outer flow due to any sufficiently strong opposing pressure gradient. These ultimately results in separation of the flow from the surface and may trigger important re-adjustment of the outer flows. In the three dimensional boundary layer, on the other hand, the flow remains in freedom of choosing the minimum difficult path and strong unfavorable pressure gradient do not necessarily lead to detachment of flow from the surface, but demonstrate themselves by drastic changes in flow directions. Consequently, not only is the boundary layer separation modified in the three dimensional flow but also it carries different implications (regarding over all effects) from those which apply in two dimensional case.

Even though very useful information can be revealed as to the various physical parameters on the boundary layer characteristics from the similarity solution, it is of limited engineering value since for practical purposes bodies other than 90° wedge will most likely be encountered. This point to the need for a general formulation and solution technique which can solve any problem of boundary layer flows of non-Newtonian fluids such as the Reiner – Philippoff fluid treated in this chapter – a topic which seems to have been neglected in the literature. In this Chapter, we will therefore look beyond the similarity solution of the problem by considering shapes other than a 90° wedge. A formulation is given in which the boundary layer equations are transformed to a form which are
suitable for solution by some exact or numerical solution techniques. The formulation is made into such a general form that boundary layer flows of any shape can be treated by entering the expression of the main stream velocity into a general function \( p(x) \) and \( q(x) \) these are also known as body shape functions. Similarity equations will be presented in this chapter for two examples, namely, the similarity solution of the flow over a \( 90^0 \) wedge and the non-similar solution of the flow over a semi-infinite flat plate with mainstream parallel to the plate. The second example is known as Blasius solution, which for the case of two dimensional flows of Newtonian fluids is in well agreement. Deviations from similarity solutions as shown in the present chapter where non-Newtonian fluids are treated therefore show clearly the effects of the various parameters involved in the model.

There are two reasons for studying this particular non-Newtonian fluid model, namely Reiner-Philippoff fluid model. First, this model correctly represents a class of non-Newtonian fluids and yet there seems to be luck of reported literature on the boundary layer flow of such fluids. Second, the present analysis introduces a method of formulation and solution which can be applied to the boundary layer flow of any non-Newtonian fluid over any body shape in which the velocity gradient is expressed explicitly as a function of the shearing stress.

### 7.1 PROBLEM FORMULATION:

The governing differential equations for the three dimensional boundary layer flow of a Reiner – Philippoff non-Newtonian fluid can be written as [Refer. Timol and Kalthia (1986)].

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{7.1}
\]

\[
\rho \left\{ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right\} = \frac{\partial}{\partial y} \tau_{yx} + \rho U_e \frac{dU_e}{dx} \tag{7.2}
\]

\[
\rho \left\{ u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right\} = \frac{\partial}{\partial y} \tau_{yz} + \rho U_e \frac{dW_e}{dx} \tag{7.3}
\]
where \(\tau_{yx}\) and \(\tau_{yz}\) are shearing stresses parallel to Y-direction and acting along X and Z-direction respectively. Hence, \(\tau_{yx}\) and \(\tau_{yz}\) are related explicitly to the velocity gradient by

\[
\frac{\partial u}{\partial y} = \frac{\tau_{yx} \mu_\infty}{\mu_\infty + (\mu_0 - \mu_\infty)^2 \left(1 + \left(\frac{\tau_{yx}}{\tau_\infty}\right)^2\right)} \tag{7.4}
\]

\[
\frac{\partial w}{\partial y} = \frac{\tau_{yz} \mu_\infty}{\mu_\infty + (\mu_0 - \mu_\infty)^2 \left(1 + \left(\frac{\tau_{yz}}{\tau_\infty}\right)^2\right)} \tag{7.5}
\]

The boundary conditions are:

\[
y = 0: \ u(x) = 0; \ v(x) = 0; \ w(x) = 0 \tag{7.6}
\]

\[
y = \infty: \ u(x) = U_e(x); \ w(x) = W_e(x) \tag{7.7}
\]

Let us introduce the following dimensionless quantities

\[
\bar{x} = \frac{x}{L}; \ \bar{y} = \frac{y}{\sqrt{R_e}}; \ \bar{u} = \frac{u}{U_\infty}; \ \bar{v} = \frac{v}{U_\infty} \sqrt{R_e} ;
\]

\[
\bar{w} = \frac{w}{W_e}; \ U_e = \frac{U_\infty U_e}{U_e}; \ \bar{\tau} = \frac{\mu}{\mu_\infty}; \ \bar{R_e} = \frac{U_\infty L}{\sqrt{V}} ;
\]

\[
\bar{W}_e = \frac{W_e}{U_\infty}; \ \bar{\tau}_{yx} = \frac{\tau_{yx} \sqrt{R_e}}{\rho U_\infty^2}; \ \bar{\tau}_{yz} = \frac{\tau_{yz} \sqrt{R_e}}{\rho U_\infty W_e} ;
\]

\[
\bar{\mu}_\infty = \frac{\tau U_\infty L}{R_e}; \ \bar{\mu}_\infty = \frac{\rho W_e L}{R_e} \tag{7.8}
\]

And a stream function, \(\bar{\psi}\) such that

\[
\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}, \ \bar{v} = -\frac{\partial \bar{\psi}}{\partial \bar{x}} \tag{7.9}
\]

Equations (7.1)-(7.7) become

\[
\frac{\partial \bar{\psi}}{\partial \bar{y}} \frac{\partial^2 \bar{\psi}}{\partial \bar{x} \partial \bar{y}} - \frac{\partial \bar{\psi}}{\partial \bar{x}} \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} = \frac{\partial}{\partial \bar{y}} \left(\bar{\tau}_{yx} \bar{x}\right) + \bar{U}_e \frac{d \bar{U}_e}{d \bar{x}} \tag{7.10}
\]

\[
\frac{\partial \bar{\psi}}{\partial \bar{y}} \frac{\partial \bar{w}}{\partial \bar{x}} - \frac{\partial \bar{\psi}}{\partial \bar{x}} \frac{\partial \bar{w}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left(\bar{\tau}_{yz} \bar{z}\right) + \bar{U}_e \frac{d \bar{W}_e}{d \bar{x}} \tag{7.11}
\]
\[
\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} = \frac{\bar{\tau}_{y\bar{x}}}{1 + \left(\frac{\bar{\tau}_{y\bar{x}}}{\bar{\tau}_{y\bar{x}}\bar{y}}\right)^2} \tag{7.12}
\]

\[
\frac{\partial \bar{w}}{\partial \bar{y}} = \frac{\bar{\tau}_{y\bar{z}}}{1 + \left(\frac{\bar{\tau}_{y\bar{x}}}{\bar{\tau}_{y\bar{z}}\bar{y}}\right)^2} \tag{7.13}
\]

Subject to the boundary conditions:

\[
\bar{y} = 0 : \quad \frac{\partial \bar{\psi}}{\partial \bar{y}} (\bar{x}, 0) = 0; \quad \frac{\partial \bar{\psi}}{\partial \bar{x}} (\bar{x}, 0) = 0; \quad \bar{w} (\bar{x}, 0) = 0
\]

\[
(7.14)
\]

\[
\bar{y} = \infty; \quad \frac{\partial \bar{\psi}}{\partial \bar{y}} (\bar{x}, \infty) = \bar{U}_e; \quad \bar{w} (\bar{x}, \infty) = \bar{W}_e
\]

\[
(7.15)
\]

Equation (7.10)-(7.15) represents a system of non-linear partial differential equations, the solution of which is quite difficult. One major simplification can be achieved by using the similarity transformation where the system of non-linear partial differential equations is reduced to a system of ordinary differential equations. Such transformations are of the limited to some special forms of the mainstream velocities. For the case of boundary layer flows of general non-Newtonian fluids, it was proved by Timol and Kalthia (1986) that similarity solutions exist only if the mainstream velocities are given by

\[
\bar{U}_e (\bar{x}) = (\bar{x})^{\frac{1}{3}} \tag{7.16}
\]

\[
\bar{W}_e (\bar{x}) = (\bar{x})^{\frac{1}{3}} \tag{7.17}
\]

This corresponds to the three dimensional boundary layers flow past a 90° wedge. For other shapes, the flow is not self similar and a transformation will be introduced in this chapter to reduce equations (7.10)-(7.15) to a form which can be solved by some suitable numerical technique.
7.2 GROUP THEORETIC ANALYSIS:

Similarity analysis by the group theoretic method is based on concepts derived from the theory of continuous transformation group. This method was first introduced by Birkhoff (1950) and Morgan (1952) and later on lot of contribution was made in these techniques by various research workers [Bluman and Cole (1974), Bluman and Kumai (1989) and Bluman and Anco (2002).]

For the present problem here we select following one parameter linear group of transformation:

\[ G: x = A^{\alpha_1} \bar{x}; \quad y = A^{\alpha_2} \bar{y}; \quad \psi = A^{\alpha_3} \bar{\psi}; \quad \tau_{yx} = A^{\alpha_4} \bar{\tau}_{y\bar{x}}; \]

\[ U_e = A^{\alpha_5} \bar{U}_e; \quad \tau_{yz} = A^{\alpha_6} \bar{\tau}_{y\bar{z}}; \quad w = A^{\alpha_7} \bar{w}; \quad W_e = A^{\alpha_8} \bar{W}_e. \]

Where \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8 \) are constants and \( A \) is a parameter of group transformation. We now seek relations among the \( \alpha \)'s such that the basic equation will be invariant under this group of transformation. This can be achieved by substituting the transformation into equation (7.10)-(7.15). Thus, we obtain

\[ A^{2\alpha_3-2\alpha_2-\alpha_1} \left( \frac{\partial \bar{\psi}}{\partial \bar{y}} \cdot \frac{\partial^2 \bar{\psi}}{\partial \bar{x} \partial \bar{y}} - \frac{\partial \bar{\psi}}{\partial \bar{x}} \cdot \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} \right) = A^{\alpha_4-\alpha_2} \frac{\partial}{\partial \bar{y}} (\bar{\tau}_{y\bar{x}}) + A^{2\alpha_5-\alpha_1} \bar{U}_e \frac{d\bar{U}_e}{dx} \]

(7.18)

And

\[ A^{\alpha_3+\alpha_7-\alpha_2-\alpha_1} \left( \frac{\partial \bar{\psi}}{\partial \bar{y}} \frac{\partial \bar{w}}{\partial \bar{x}} - \frac{\partial \bar{\psi}}{\partial \bar{x}} \frac{\partial \bar{w}}{\partial \bar{y}} \right) = A^{\alpha_6-\alpha_2} \frac{\partial}{\partial \bar{y}} (\bar{\tau}_{y\bar{z}}) + A^{\alpha_5+\alpha_3-\alpha_1} \bar{U}_e \frac{d\bar{W}_e}{dx} \]

(7.19)

\[ F_1 \left( A^{\alpha_4} \bar{\tau}_{y\bar{x}}, A^{\alpha_3-2\alpha_2} \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} \right) = 0 \]

(7.20)

\[ F_2 \left( A^{\alpha_6} \bar{\tau}_{y\bar{z}}, A^{\alpha_7-\alpha_2} \frac{\partial \bar{w}}{\partial \bar{y}} \right) = 0 \]

(7.21)

\[ A^{\alpha_3-\alpha_2} \frac{\partial \bar{\psi}}{\partial \bar{y}} = A^{\alpha_5} \bar{U}_e \]

(7.22)
\[ A^{a_7} \bar{w} = A^{a_8} \bar{W}_e \quad (7.23) \]

From equations (7.10)-(7.15), it is seen that if the basic equation are to be invariant under the \( G \) group of transformation, the powers of \( A \) in each term should be equal. Thus, invariance of equations (7.18) to (7.23) under the group \( G \) gives following relations among \( \alpha \)'s.

\[
\begin{align*}
2\alpha_3 - 2\alpha_2 - \alpha_1 &= \alpha_4 - \alpha_2 = 2\alpha_5 - \alpha_1 \\
\alpha_3 + \alpha_7 - \alpha_2 - \alpha_1 &= \alpha_6 - \alpha_2 = \alpha_5 + \alpha_8 - \alpha_1 \\
\alpha_4 &= 0; \quad \alpha_3 - 2\alpha_2 = 0; \quad \alpha_6 = 0; \\
\alpha_7 - \alpha_2 &= 0; \quad \alpha_3 - \alpha_2 = \alpha_5; \quad \alpha_7 = \alpha_8
\end{align*}
\]

(7.24)

Solving equations (7.24), we get

\[
\begin{align*}
\frac{\alpha_2}{\alpha_1} &= \frac{\alpha_5}{\alpha_1} = \frac{\alpha_7}{\alpha_1} = \frac{\alpha_8}{\alpha_1} = \frac{1}{3} \\
\frac{\alpha_3}{\alpha_1} &= \frac{2}{3}; \quad \alpha_4 = \alpha_6 = 0
\end{align*}
\]

(7.25)

The next step in this method is to find the so-called “absolute invariants” under group of transformation. Absolute invariants are function having the same from before and after the transformation.

\[
\begin{align*}
\frac{\bar{y}}{x} = \frac{\bar{y}}{(x)^\frac{1}{3}}; \quad \frac{\bar{w}}{x} = \frac{\bar{w}}{(x)^\frac{2}{3}}; \quad \frac{\bar{w}}{x} = \frac{\bar{w}}{(x)^\frac{1}{3}}; \\
\tau_{yx} = \bar{\tau}_{y\bar{x}}; \quad \tau_{yz} = \bar{\tau}_{y\bar{z}}
\end{align*}
\]

(7.26)

Therefore, these functions are absolute invariants under this group of transformation. We therefore obtain the transformed independent and dependent variables are:

\[
\eta = \frac{\bar{y}}{(x)^\frac{3}{5}}; \quad f_1(\eta) = \frac{\bar{w}}{(x)^\frac{2}{5}}; \quad f_2(\eta) = \frac{\bar{w}}{(x)^\frac{1}{5}};
\]

163
\[ g_1(\eta) = \bar{\tau}_{\eta x} ; \quad g_2(\eta) = \bar{\tau}_{\eta z} \quad (7.27) \]

Substituting for independent and dependent variable in equations (7.10)-(7.15) expressions found from equation (7.27). We expect to obtain a set of equations which are ordinary differential equation or very close to ordinary differential equations; specifically we obtain similarity solution flow over a \(90^0\) wedge,

\[
3g_1' + 2f_1 f_1'' - (f_1')^2 + 1 = 0 \quad (7.28)
\]

\[
3g_2' + 2f_1 f_2' - f_1' f_2' + 1 = 0 \quad (7.29)
\]

\[
g_1 = f_1'' \left( \frac{g_2^2 + \mu \gamma}{\gamma + g_2^2} \right) \quad (7.30)
\]

\[
g_2 = f_2' \left( \frac{g_2^2 + \mu \gamma}{\gamma + g_2^2} \right) \quad (7.31)
\]

Subject to the boundary conditions:

\[
\eta = 0 : \quad f_1(0) = 0 ; \quad f_1'(0) = 0 \quad (7.32)
\]

\[
\eta = \infty : \quad f_1'(\infty) = 1 ; \quad f_2(\infty) = 1 \quad (7.33)
\]

### 7.3 DEDUCTION:

For two-dimensional case, the flow will be independent of \(Z\)-direction and hence in this case, equations (7.29) and (7.31) will vanish (as \(g_2 = f_2 = 0\)). Hence equations (7.28) and (7.30) along with boundary conditions (7.32)- (7.33) will be reduced to those derived by Sirohi et al (1984) and Na (1994) which they have independently solved numerically.
7.4 NON-SIMILAR SOLUTION:

For the general case in which the boundary layer over any body shape is to be analyzed general transformations are introduced as follows:

\[ \zeta = \bar{x}; \quad \eta = \sqrt{\frac{\bar{U}_e}{\bar{w}}} \frac{\bar{y}}{\sqrt{\bar{x}}} ; \quad f_1(\zeta, \eta) = \frac{\bar{\psi}}{\sqrt{\bar{x}} \, \bar{U}_e} ; \]

\[ g_1(\zeta, \eta) = \bar{r} \bar{y} \left( \sqrt{\frac{\bar{x}}{\bar{U}_e^3}} \right) ; \quad f_2(\zeta, \eta) = \frac{\bar{w}}{\bar{W}_e} \]

\[ g_2(\zeta, \eta) = \bar{r} \bar{y} \bar{z} \left( \sqrt{\frac{\bar{x}}{\bar{U}_e}} \right) \frac{1}{\bar{W}_e} \]  

(7.34)

Under these transformations, equations (7.10)-(7.15) become,

\[ g_1' + \left\{ \frac{p(\zeta) + 1}{2} \right\} f_1 f_1'' + p(\zeta) \left\{ 1 - (f_1')^2 \right\} = \zeta \left\{ f_1 \frac{\partial f_1'}{\partial \zeta} - f_1'' \frac{\partial f_1}{\partial \zeta} \right\} \]

(7.35)

\[ g_2' + \left\{ \frac{q(\zeta) + 1}{3} \right\} f_1 f_2' + q(\zeta) \left\{ 1 - f_1' f_2 \right\} = \zeta \left\{ f_1 \frac{\partial f_2'}{\partial \zeta} - f_2' \frac{\partial f_1}{\partial \zeta} \right\} \]

(7.36)

\[ g_1 = f_1'' \left( \frac{\mu(\zeta) \gamma + g_1^2 (\bar{U}_e)^3}{\zeta \gamma + g_2^2 (\bar{U}_e)^3} \right) \]

(7.37)

\[ g_2 = f_2' \left( \frac{\mu(\zeta) \gamma \frac{3}{2} + g_2^2 (\bar{W}_e)^2 (\bar{U}_e) (\sqrt{\bar{z}}) \bar{U}_e}{\gamma \zeta \sqrt{\bar{U}_e} + g_2^2 (\bar{W}_e)^2 (\bar{U}_e) \frac{3}{2}} \right) \]

(7.38)

Subject to the same boundary conditions given by equations (7.32)-(7.33).

Where

\[ p(\zeta) = \frac{\bar{x}}{\bar{U}_e} \frac{d\bar{U}_e}{d\zeta} = \frac{\zeta}{\bar{U}_e} \frac{d\bar{W}_e}{d\zeta} \]

(7.39)

\[ q(\zeta) = \frac{\bar{x}}{\bar{U}_e} \frac{d\bar{U}_e}{d\zeta} = \frac{\bar{x}}{\bar{U}_e} \frac{d\bar{W}_e}{d\zeta} \]

(7.40)
Where \( p(\zeta) \) and \( q(\zeta) \) are known as body shape functions and it is easy to verify that for \( \zeta = \bar{x} \); \( \bar{U}_e(\bar{x}) = (\bar{x})^{\frac{1}{3}} \); \( \bar{W}_e(\bar{x}) = (\bar{x})^{\frac{1}{3}} \);

\[
p(\zeta) = \frac{\bar{x}}{(\bar{x})^3} \left( \frac{1}{3} (\bar{x})^{\frac{1}{3}-1} \right) = \frac{1}{3} ;
\]

\[
q(\zeta) = \frac{\bar{x}}{(\bar{x})^3} \left( \frac{1}{3} (\bar{x})^{\frac{1}{3}-1} \right) = \frac{1}{3}
\]

Now for these values of \( p(\zeta) \) and \( q(\zeta) \) above set of equations will reduce to those ordinary differential equations given by (7.28) and (7.29). This also confirms that similarity solutions of present flow problem exist only for the flow past 90° wedge. On the other hand for the body other than 90° wedge, \( \bar{U}_e(\bar{x}) = (\bar{x})^\alpha \) and \( \bar{W}_e(\bar{x}) = (\bar{x})^\beta \) where \( \alpha \) and \( \beta \) are real constants. For the above mentioned case, the related body shape functions will be:

\[
P(\zeta) = \frac{\bar{x}}{(\bar{x})^\alpha} (\alpha (\bar{x})^{\alpha-1}) = \alpha \quad \text{And} \quad q(\zeta) = \frac{\bar{x}}{(\bar{x})^\beta} (\beta (\bar{x})^{\beta-1}) = \beta
\]

And for this case, non-similarity equation (7.35)-(7.38) will become

\[
g_1' + \left( \frac{\alpha+1}{2} \right) f_1 f_1'' + \alpha \left[ 1 - (f_1)^2 \right] = \zeta \left( f_1' \frac{\partial f_1'}{\partial \zeta} - f_1'' \frac{\partial f_1}{\partial \zeta} \right) \quad (7.41)
\]

\[
g_2' + \left( \frac{\beta+1}{3} \right) f_1 f_2' + \beta \left[ 1 - f_1 f_2 \right] = \zeta \left( f_1' \frac{\partial f_2}{\partial \zeta} - f_2' \frac{\partial f_1}{\partial \zeta} \right) \quad (7.42)
\]

\[
g_1 = f_1'' \left[ \frac{\bar{\mu} \zeta \gamma + g_1^2 (\bar{x})^{3\alpha}}{\bar{\mu} \zeta \gamma + g_1^2 (\bar{x})^{3\alpha}} \right] \quad (7.43)
\]

\[
g_2 = f_2' \left( \frac{\bar{\mu} (\zeta)^{\frac{3}{2}} + g_2 (\bar{x})^{2\beta} (\bar{x})^{\alpha} (\sqrt{\zeta})}{(\bar{x})^{2\alpha} \gamma \zeta + g_2^2 (\bar{x})^{2\beta} (\bar{x})^{2\alpha}} \right) \quad (7.44)
\]

With the same boundary conditions are given by equations (7.39)-(7.40).
7.5 DEDUCTION:

For two-dimensional case, the flow will be independent of Z-direction and hence in this case, equations (7.42) and (7.44) will vanish (as $g_2 = f_2 = 0$). Hence equations (7.41) and (7.43) along with boundary conditions (7.32) - (7.33) will be reduced to those derived by Na (1994) which he has solved numerically by Kellar-Box method.

7.6 CONCLUSION:

This similarity analysis of the three dimensional boundary layer flow of non-Newtonian Reiner – Philippoff fluids passed external surface is derived. If it is observed that similarity solutions exist for only the flow past at $90^\circ$ wedge for the flow passed any body shape the same formation can be used and only changes in two functions namely $p(\zeta)$ and $q(\zeta)$ by substituting into these functions the mainstream velocity for that particular geometry. The present analysis provides useful in formulation for the boundary layer flow not only for Reiner – Philippoff fluids but for the other fluids too which are studied by Sirohi et al (1984).