CHAPTER-1

INTRODUCTION

1.1 INTRODUCTION

In day-to-day life, digital images have a key role in the area of computer aided tomography, aerial communications, telecommunication images, synthetic aperture radar, geographical information systems, astronomy etc. In diverse fields, mentioned above, scientists are facing the problem of recovering original images from incomplete, indirect and noisy images [13]. Images get corrupted during acquisition by camera sensors, receivers, environmental conditions, improper lightning, undesirable view angle etc., [159]. A noisy image appears as spotted, granular, hoary image. Therefore, the problem of recovering an original image from noisy image has received an ever increasing attention in recent years [85]. The recovery of image can be accomplished by image denoising, a process of estimating the desired image from a corrupted image [7,110].

Aim of the thesis is to emphasize the problems and solutions in relation to tomographic images which arise in medical field in view of its increasing importance in the present day requirements. In view of this, survey of literature has been done in the area of tomography, wavelets, multiwavelets and various denoising techniques. Mainly, [1] tomography is of two types i) Medical tomography ii) Industrial tomography. [74]Medical image acquisition process and systems introduce noise and artifacts in the images that should be eliminated
by denoising techniques. The denoising process, however, should not destroy anatomical details relevant from a clinical point of view. So, it is very difficult to suggest a robust method for noise removal, which works equally well for different modalities of medical images. A number of researchers have published image denoising literature [10, 12, 13, 14, 15, 17, 22, 27, 28, 36, 39, 43, 60, 61, 105, 120, 134], but still research is going on, for better image quality.

[1] In industrial tomography, high energy X-rays and gamma rays are used. Apart from low energy X-ray imaging, the other imaging modalities in medical tomography are Ultrasound, Magnetic Resonance Imaging (MRI), Nuclear medicine and Microwave techniques. The most familiar form of tomography is X-ray CT. Radon transforms give the mathematical basis for tomographic image reconstruction from projections [39, 49, 144, 89].

Aggregate measurements of the object of interest, need to be processed in tomographic image reconstruction. The measurements consist of line integrals of an object of interest. The most popular method for reconstructing tomographic image is Filtered Back Projection (FBP).

There are some limitations in the filtering area of back projection as follows:[116]

i) If only back projection method is used for reconstruction, it produces an image which has high density in the center. This is due to the fact that many different images are being overlapped in this area. The resulting image is severely blurred. To produce the image exactly, ramp filter is used. This filter, filters out lower frequencies and passes higher frequencies with a linear behavior in between. With
this filter, contrasting features (high frequencies) are accentuated, while blurring (low frequencies) are minimized. The ideal ramp filter works very well for reconstructing projections, but has the undesirable effect of passing and magnifying extraneous noise from projection data.

ii) Most experimental filters work reasonably well for images that had a relatively less complex frequency distribution. For images containing a high degree of complexity, the best trade-off between noise removal and feature preservation did not improve the image radon transform.

iii) In the beginning, the image reconstruction algorithms were developed based on fourier transform techniques. The Convolution Back Projection (CBP) and Direct Fourier Reconstruction (DFR) are two typical examples in this approach. These algorithms were later found to be sensitive to the perturbation of measurements i.e., noise.

One way to reduce noise sensitivity is to model the emission process as a random process and develop the reconstruction algorithms based on the statistical model and estimation techniques.

Another approach to reduce noise sensitivity is to filter measurement data before employing a deterministical model based reconstruction algorithm.

iv) [106] The Radon transform is smoothing transform, any noise in Radon data becomes magnified when inverse Radon transform is applied.

Statistical iterative model based methods, such as Expected Maximum (EM), Maximum-a-Posterior (MAP), Maximum Likelihood (ML), Ordered Subset Expectation Maximization (OSEM), Conjugate gradient methods have been explored by the research community to get better performance of image reconstruction procedures. It is found that the above procedures have provided considerable improvement
compared to FBP. At the same time, the above estimators experience limitations in the area of computational time, theoretical understanding, justification and convergence.

The image denoising methods can be categorized as either spatial or transform domain methods. In spatial domain noise is suppressed directly. In transform domain image is first transformed from the spatial domain to frequency domain or wavelet domain and then noise is suppressed. These are some of the applications in the image processing that require the analysis to be localized in the spatial domain. The classical way to do this is through windowed Fourier transform i.e.; Short Time Fourier Transform (STFT). It conveys the localized frequency component present in 2D spatial image, where it can be determined from windowed transform. In STFT, the window size is fixed. In order to have a variable window for better localization property, wavelet transforms have been developed. It has time-scale variable window. The scale and the translation parameters control the width of the window through dilation, compression or translation. Wavelets have multi-resolution capability. A lot of research is being carried out in the area of wavelets, as it can decompose and denoise the image. It has the property of multiple resolution at different levels, the concentration of energy in the finite regions and localization. Noise is generally associated with small coefficients and desired features are generally associated with large coefficients. By using thresholding techniques, small coefficients where noise is concentrated may be eliminated. The statistical models of wavelet coefficients have recently
been presented for natural images [23, 84]. These models that have shown the dependency among coefficients gave better results compared to the ones using independent assumptions. The Bayesian approach provides a powerful framework for signal estimation. The denoising using multiwavelet techniques has become more effective because of the capability of capturing the signal energy in a very few transformation energy levels. There are some dissimilarities between wavelets and multiwavelets. Wavelets have one scaling function $\phi(t)$ and one wavelet function $\psi(t)$, whereas multiwavelets have multiple scaling and multiple wavelet functions [148,149]. Multiwavelets have an extra degree of freedom, which decreases the limitations on the properties of filters [63]. Multiwavelets using several scaling functions enable to fulfill properties that are impossible in the single wavelet case. A multiwavelet system can simultaneously preserve length, perform well at boundaries and higher order of approximation which is impossible with the scalar wavelet system [64]. It gives perfect reconstruction. [76] The multiwavelet transformation can give good spatial and spectral localization of an image. Hence, it gains more attraction in image denoising.

It is found from various research papers that reduction of noise is a delicate task. A compromise has to be made to reduce the noise and preserve the necessary features in an image. There is a trade-off in achieving the desired image. This motivates us to go for new reconstruction methods that denoise an image effectively and preserve the important features such as edges, corners and local orientations.
1.2 REVIEW OF LITERATURE

Survey of literature and review has been done in the following areas:

1.2.1 Review of tomographic imaging and its reconstruction [172]

In 1917, Radon [68] originally developed mathematical basis for tomography. The first CT scanner was invented in 1972 by G.N.Hounsfield and Alan Mc Cormack [46]. The filtered back projection algorithm was invented by R.H.Bracewell and Riddle [130] and later independently with additional insights by Ramachandran and Lakshminarayanan [47], who have developed convolution back projection algorithm. This algorithm is now used by almost all commercially available CT scanners. The filtered back projection algorithm for fan beam data was developed by A.V. Lakshminarayanan [3]. Several researchers have proposed Algebraic Reconstruction Techniques (ART) for reconstruction namely Gordon, Herman and Budinger [128]. S.Wernecke and D’Addario [141] proposed a maximum entropy method for reconstruction. L.Shepp and Logan [82] have shown that the filtered back projection method is much superior to other methods especially algebraic reconstruction techniques. Hiroyuki kudo [58], has discussed about two important reconstruction problems. (i) Two dimensional Region of Interest(ROI) reconstruction problem, in which he aimed at reconstructing an object $f(x,y)$ on ROI from its projection data $p(r,\theta)$, truncated with respect to radial variable $r$. The methodology used was, super short scan for fan beam tomography and the differentiated back projection using hilbert
transform method. (ii) Cone-beam reconstruction problem in which 3D object \( f(x,y,z) \) has been reconstructed from a set of cone-beam projections, measured by moving an X-ray source along some curve. Jorge Llacer [75], used pseudo-inverse method of reconstruction. Here advantage is taken of the fact that the activity to be measured is localized in a single plane, without overlying and underlying activity. J.D. Sullivan [66] used computationally efficient gridding algorithm which can be used with direct Fourier transformation to achieve arbitrarily small artifact levels. This method can be applied to other measurement geometries such as fan-beam projection, diffraction tomography and NMR imaging. Peyma Oskou-fard and Henry Stark [115] used geometry-free reconstruction algorithm based on the theory of convex projections. Algebraic Reconstruction Technique (ART) is a geometry free reconstruction algorithm. Use of apriori information enhances the quality of the results. Julia F. Barrett and Nicholas Keat [77], has discussed the artifacts in detail. Artifacts degrade the quality of tomographic image. Broadly they classified the artifacts as, physics-based, patient based, scanner based, helical and multi-section technique artifacts. They have also discussed about the avoidance of artifacts. The artifacts can be avoided by, improving design features, partially corrected by scanner software, patient positioning, optimum selection of scan parameters.

Several techniques for improving the speed of the algorithm have been proposed. Paul Edholm et al [114] has used linograms for image reconstruction, which is superior to filtered back projection, from the
point of view of computational speed and accuracy of reconstruction. The implementation is of the order of $N^2 \log N$, as opposed to the less computationally efficient back projection methods which is of the order of $N^3$, where $N$ is proportional to the number of pixels on a side of the reconstruction region. Tanaka [35], Kenue [138], L.Wang [83], Dreike [112] and several others lookup into avenues for the improvement in the algorithms. Improvements in the hardware used for computing the reconstruction (back projection) on the event by event basis with two microseconds per event in position CT was developed by R.Hartz et al [129]. More recently, Agi et al [62] have come up with the VLSI implementation of a dedicated microprocessor for carrying out forward and backward radon transform. Pipelining was used to achieve faster processing. It has been shown that 64 scan images can be obtained from filtered CT data in 70ms.

In order to reconstruct 3D images in real time, parallel implementation of CT algorithms [19, 49, 135] have been used. W.Jones et al [163] proposed a scheme for parallelizing the expectation maximization algorithm. Pixels are circulated through the Processing Elements (PEs), each of which back projects one view of the projection data. J.Lacer et al [67] have proposed a multiprocessor scheme for the Maximum Likelihood Estimator (MLE) based algorithm. Atkin’s et al [92] have implemented the “Stretch” algorithm for use in three dimensional Positron Emission Tomography (PET). The transputers (T800s) were connected in tree types of arrangements with one node working as a master and the others as workers. They used
the same transputers for speeding up the data acquisition, image construction and display. Barresi et al [135] have also implemented the three dimensional filtered back projection algorithm on transputers (T800) using the OCCAM programming language. They tested their algorithms on a five transputer network and they also used parallel FFT algorithms for finding the FFT. They have reported a reconstruction frequency 700MHz with five T800 transputers. Raman P.V.Rao et al [133] implemented the filtered back projection algorithm for two dimension, on two different platforms i.e; a 28 node Intel paragon with the nodes connected as a mesh, and a Connection Machine (CM5). The paragon implementation is fully complete with message passing, controlled by the parallel algorithm. The CM5 software is parallelized by the connection machine compilers. They implemented on Intel paragon, an explicit message passing algorithm using the message passing constructs provided on the machine. Both implementations have been done in high level language using the parallel constructs provided by the language compilers on those machines. The filtered back projection algorithm was first programmed as a serial algorithm using matlab software on a Sun Sparc 10. It took 12 minutes for reconstruction. The same data is parallelized on the intel paragon in about 2.2 seconds and on the CM5 in about 3 seconds.
1.2.2 Review of tomographic image reconstruction from projections using wavelets.

Several researchers have introduced wavelet methods for tomographic problems. Methods based on one dimensional and two dimensional wavelet decomposition of the projected data followed by linear filtered back projection were introduced. Bhatia, Karl and Willsky [15] first introduced a method based on similar principles, and then used a one dimensional wavelet decomposition of the projection data, which was back-projected to find a different basis for the so-called “Natural pixel” formulation of the image reconstruction problem, which yields a sparse matrix problem to solve rather than a full matrix problem with a MAP model of the image. Donoho [31] formulated Wavelet-Vaguelette decomposition for solving non-linear solution of linear inverse problem, which has been studied numerically by E.D.Kolaczyk [37] for tomographic reconstruction and refined by Lee and Lucier [87]. In diagnostic medicine, cone beam CT increases a patient’s exposure dose. The X-ray CT image is degraded by “quantum mottle” noise. In order to remove this noise and reduce the patient’s exposure dose, Yi-Qiang Yang [166] developed a method which separates the image from noise by measuring the lipschitz exponents of the image singularities from the evolution of wavelet transform modulus maxima across scales. They identified the singularities of the 2D projections by computing the wavelet transform modulus sum in the direction indicated by the phase of wavelet transform. Farrokh Rashid-Farrokhi et al [40] has developed an algorithm to reconstruct the wavelet coefficients of an image from the
radon transform data. It used the localization property of wavelets to localize the radon transform and can be used to reconstruct a local region of the cross section of the body, using almost local data that significantly reduces the amount of exposure and computations in X-ray tomography. Stephane Bonnet [146] used non-separable wavelets for tomographic reconstruction. The algorithm computes the quincunx approximation and detail coefficients of a function from projections. Simulation results has shown improvement in reconstruction using non-separable wavelets than compared to separable wavelets. Tatiana Soleski and Gilbert Walter [154] used raised-cosine wavelet to recover an image from its sampled projections in the form of values of the radon transform. Jian Zhou et al [72], has presented a Positron Emission Tomography (PET) reconstruction method using the wavelet-based Maximum-A-Posteriori (MAP), Expectation-Maximization (EM) algorithm. The projection data measured in computed tomography and consequently, the slices reconstructed from these data are noisy. De-Stefano and Olson implemented the first numerical algorithm using wavelets for local reconstruction. Borsdorf et al [12] presented a new wavelet based structure preserving method for noise reduction in CT images that can be used in combination with different reconstruction methods. The approach is based on the assumption that the data can be generated by reconstruction from disjoint subsets of projections. Jerome Kalifa et al [116], studied a new family of regularization method for reconstruction, based on a thresholding procedure in wavelet and
wavelet packet decomposition. This approach is based on the fact that
the decomposition provided a near-diagonalization of the inverse
radon transform and of the prior information on medical images. The
performance of these procedures out performed filtered back
projection and the iterative procedures such as OSEM. Reconstruction
of a low dose computed tomographic image is an unstable inverse
problem, due to the presence of noise. To overcome this problem,
Thavavel and Murugesan [160] proposed a new regularized
reconstruction method that combines the filtered back projection
algorithm and regularization theory. The proposed method exploits the
properties of Dual Tree-Complex Wavelet Transform (DT-CWT) to
remove blurring and noise without the need for assuming a specific
noise model. M. Venu Gopal and S.Vathsal [162] developed two spline
interpolation algorithms and classical interpolation techniques such
as linear interpolation.

1.2.3 Review of denoising of tomographic images using wavelets
and thresholding techniques.

The following literature review discusses denoising using wavelet
transforms in a wide scenario. Donoho and Johnstone [30] proposed a
method for reconstructing an unknown desired function from noisy
data affected with standard Gaussian white noise. Donoho [24] proved
two results about this type of estimator i.e; smooth and adapt. They
have also proposed a very simple thresholding procedure for
recovering functions from noisy data. They applied the interval-
adapted pyramidal filtering algorithm of Cohen, Daubechies, Jawerth
and Vial to the measured data, obtaining empirical wavelet
coefficients. Then applied the soft thresholding non-linearity coordinate wise to the empirical wavelet coefficient. This method is called “Visu shrink”, which gives good visual quality and the estimator has an optimality property with respect to mean square error for estimating functions of unknown smoothness at a point. Inverted the pyramid filtering to recover original data. They have developed a wavelet-based method “SURE shrink”, which offers minimax rates of convergence over all spaces. Sure shrink is based on adaptively chosen thresholds, selected based on the use of Stein’s Unbiased Risk Estimator (SURE). They also developed a practical spatially adaptive method, risk shrink, which works by shrinkage of empirical wavelet coefficients. Yakov Hel-Or and Doron shaked [165], suggested a discriminative approach for wavelet denoising, where a set of mapping functions are applied to the transform coefficients to produce noise free image. Ming Zhang and Bahadir K.Gunturk [104] used bilateral filter for image denoising. It is a non-linear filter that does spatial averaging without smoothing edges. Bilateral filtering is applied to the approximation (low frequency) sub-bands of a signal decomposed using a wavelet filterbank. This multiresolution bilateral filter with wavelet, is a new image denoising framework, which is very effective in eliminating noise in real noisy images. Bart Goossens et al [14] designed a new denoising method for the removal of correlated noise, by modeling the significance of the noise free wavelet coefficients in a local window using a new significance measure that defines the “signal of interest” and that is applicable to correlated noise.
Mario.T.Figueiredo et al [101] exploited, the sparseness and decorrelation properties of the DWT to develop powerful denoising methods. Their approach uses empirical bayes estimation based on a Jeffrey’s non-informative prior. It is a step towards objective bayesian wavelet based denoising. It results in a simple fixed non-linear shrinkage rule which performs better than other more computationally demanding methods. Byung-Jun Yoon and P.P.Vaidyanathan [167] proposed the custom thresholding scheme and demonstrated that it outperformed the traditional soft and hard thresholding schemes, since the custom thresholding function adapted well to the characteristics of the given signal, resulting in a smaller estimator error. L.Sendur and Selesnick [88] proposed new non-Gaussian bivariate distributions and corresponding non-linear threshold functions are derived from the models using bayesian estimation theory. The new shrinkage functions are called bivariate shrinkage functions and do not assume the independence of wavelet coefficients. Gabriel Huerta [50] proposed a hierarchical prior that proposes a multivariate normal distribution with a covariance matrix that allows correlation among wavelet coefficients corresponding to the same level of detail. Hancheng Yu et al [57], proposed an efficient algorithm for removing Gaussian noise from corrupted image by incorporating a wavelet-based trivariate shrinkage filter with a spatial-based joint bilateral filter. In wavelet domain, the wavelet coefficients are modeled as trivariate Gaussian distribution, taking into account the statistical dependencies among intrascale wavelet coefficients and
then a trivariate shrinkage function is derived by using Maximum-a-Posteriori (MAP) estimator. Venu Gopal and S.Vathsal [97,163-165], proposed a new wavelet domain, structure driven denoising technique called steered complex shrinkage which preserves the edges, corners and orientation features of medical images. They also presented an effective and low-complexity image denoising algorithm using the joint statistics of the wavelet coefficients of medical images. For this, a new heavy-tailed bivariate laplacian probability density function (PDF) has been proposed to model the statistics of wavelet coefficients, and a simple nonlinear shrinkage function is derived. They also designed efficient deconvolution algorithm to obtain blur free and noisy free images after reconstruction. This algorithm is comprised of two steps, a global blur compensation using generalized wiener filter followed by a pure denoising algorithm using modified Dual Tree-Complex Wavelet Transform (DT-CWT) named as Complex Fourier Wavelet Regularized Deconvolution (ComForWaRD). They also proposed a new hybrid Bivariate Complex Fourier Wavelet Regularized Deconvolution (Bi-ComForWaRD), that is an extension to ComForWaRD algorithm, for medical imaging. This new algorithm is a two step process, a global compensation using generalized wiener filter and followed by a denoising algorithm using local adaptive bivariate shrinkage function.

It is a low complexity denoising algorithm using the joint statistics of the wavelet coefficients and considers the statistical dependencies between the coefficients. Vasily Strela [94] analyzed statistical properties of the wavelet coefficients of natural images. Damon M.
Chandler and Sheila S. Hemami [22] presented an efficient visual signal-to-noise ratio for quantifying the visual fidelity of natural images based on near-threshold and supra-threshold properties of human vision. Sudha, Suresh and Sukanesh [152] proposed a wavelet based thresholding scheme for speckle noise suppression in ultrasound images. Detlev Marpe et al [26] proposed a spatially adaptive wavelet thresholding method using a context model. Hossein Rabbani et al [61] have proposed noise reduction algorithms that could be used to enhance image quality in various medical imaging modalities such as magnetic resonance and multi-detector CT.

H. Rabbani [60] has presented an image denoising algorithm based on the modeling of coefficients in each sub-band of steerable pyramid with a laplacian probability density function with local variance. Javier Portilla et al [69], described a method for removing noise from digital images based on a statistical model of the coefficients of an over-complete multiscale oriented basis. Felix Abramovich [40], constructed a thresholding procedure based on the False Discovery Rate (FDR) approach. The suggested procedure controls the expected proportion of incorrectly included coefficients among those chosen for the wavelet reconstruction. S.Grace Chang et al [134, 149, 148], proposed an adaptive data driven threshold for image denoising via wavelet soft thresholding. The threshold is derived in a Bayesian frame work. The prior used in the wavelet coefficients is the Generalised Gaussian Distribution (GGD), which is widely used in image processing applications. The proposed Bayes shrink out
performs Donoho and Johnstone’s sure shrink most of the time. They also proposed a spatially adaptive wavelet thresholding method based on context modeling, a common technique used in image compression to adapt the coder to changing image characteristics. Each wavelet coefficient is modeled as a random variable of a GGD with an unknown parameter. They modeled wavelet image coefficients as zero mean Gaussian random variable with high local correlation. They assumed a marginal prior distribution on wavelet coefficient variances and estimated them using an approximation MAP rule. Then they applied an approximation MMSE procedure to restore the noisy wavelet image coefficients. D.Gnanadurai et al [22] proposed a framework for an adaptive threshold estimation method for image denoising in wavelet domain based on Generalized Gaussian Distribution (GGD) modeling of subband coefficients. Pizurica et al [8,118], analyzed two types of filters for noise suppression in magnitude MRI images and denoising Blood Oxygen Level Dependent (BOLD) response in functional MRI images (fMRI). Mathew.S.Crouse et al [101], developed a new framework for statistical signal processing based on wavelet domain Hidden Markov Models, that concisely models the statistical dependencies and non-Gaussian statistics encountered in real world signals. Tang Jingtian et al [156], have put forward an application of Independent Component Analysis (ICA) method to remove the noise from a CT image. The objective of Jin Li et al [72] was to reduce the noise and artifacts in the industrial CT image by anisotropic diffusion. Skiadopoulos et al [123] have carried out a comparative study between
a multi-scale platelet denoising method and the well-established butterworth filter, which was employed as a pre- and post-processing step on image reconstruction. Guangming Zhang et al [55] have proposed an extended model for CT medical image denoising, which have been using ICA and dynamic fuzzy theory. Lanzolla et al [90], have evaluated the effect of different noise reduction filters on Computed Tomography (CT) images. Bing-gang Ye and Xiao-ming Wu [17] work has reduced the figures and eliminated the irrelated coefficient of noise of Small Hepato-Cellular Carcinoma (SHCC) CT image, and finally removed the noise. Jessie Q Xia et al [71] have employed the Partial Differential Equation (PDE) based denoising techniques particularly for breast CT at various steps along the reconstruction process. Zhao Yu-qian et al [170], have introduced a basic mathematical morphological theory and operations, and a novel mathematical morphological edge detection algorithms have been proposed to detect the edge of lungs CT image, with salt-and-pepper noise. F.E.Ali et al [38], have presented a Curvelet based approach for the fusion of Magnetic Resonance (MR) and CT images. The simulation results show the superiority of the curvelet transform to the wavelet transform in the fusion of MR and CT images from both the visual quality and the Peak Signal-to-Noise Ratio (PSNR) point of view. Derek LG Hill and David J Hawkes [26], have presented a new algorithm based on surface fitting that makes use of anatomical knowledge of adjacency of identified anatomical structures to solve the 3D rigid body registration problem. Yousef Hawwar and Ali Reza [168]
presented a new image denoising technique in the wavelet transform domain for multiplicative noise. Mallat [99], has shown how wavelets can be used in a multiresolution representation using orthogonal basis functions and has described an efficient pyramidal algorithm using quadrature mirror filters to compute it. Unser et al [93], has extended these ideas to the case of non-orthogonal basis functions using splines. Dimitrios charalampidis [28], proposed a Steerable Weighted Median Filter (SWMF). Steerability is important for numerous image processing applications. A filter is steerable, if transformed versions of its impulse response can be expressed as linear combinations of a fixed set of basis functions. The SMWFs are applied for edge detection and orientation analysis. Priyam Chatterjee et al [120], estimated a lower bound on the mean square error of the denoised result and compared the performance of current state-of-the art denoising methods with this bound. The limitations of traditional multistage representation such as wavelets are overcome by curvelet transform. G.Y.Chen and B.Kegl [48], proposed a novel image denoising method by incorporating the dual-tree complex wavelets into the ordinary ridgelet transform. The approximate shift invariant property of the dual-tree complex wavelet and the high directional sensitivity of the ridgelet transform make the new method a very good choice for image denoising. Thierry Blu has proposed a new approach to denoise an image based on the image-domain minimization of an estimate of the mean square error-Stein’s Unbiased Risk Estimator (SURE).
1.2.4 Review of denoising of tomographic images using multiwavelets

Downie et al [34], proposed, multiwavelet thresholding, in which they considered the vector coefficients overall instead of thresholding individual elements. Jo Yew Tham et al [76], established the concept of an equivalent scalar wavelet filter bank system in which they presented an equivalent and sufficient representation of a multiwavelet system of multiplicity r in terms of a set of r equivalent scalar filter banks. Ivan W. Selesnick [63, 64], proposed the construction of compactly supported orthogonal multiscaling functions that are continuously differentiable and cardinal. They also, proposed, the properties and design of orthogonal multiwavelet bases, with approximation order >1, that possess those properties that are normally absent in wavelets. Vasily Strela et al [149], proposed that multiwavelets realizable as matrix-valued filter banks leads to wavelet basis which offer simultaneous orthogonality, symmetry and short support. Mariontonia Cotronei [100], proposed that, multiwavelets can be seen as vector valued wavelets. Donovan et al [32], demonstrated that it is possible to construct orthogonal (real-valued) basis for which the scaling functions have compact support, approximation order >1 and symmetry, which is not possible with traditional wavelet bases. Xiang-Gen Xia [164], proposed that, in conventional wavelet transform, prefiltering is not necessary due to the low pass property of the scaling function, where as in multiwavelet transforms, prefiltering is necessary. Kwok-Wai Cheung [80], proposed a low complexity two dimensional approximation based preprocessing scheme for GHM multiwavelet transform. Gilbert Strang et al [51], proposed that,
scaling functions and orthogonal wavelets are created from the coefficients of low pass filter and high pass filter. For multifilters those coefficients are matrices. Geronimo, Hardin and Massopust constructed two scaling functions that have extra properties not previously achieved. Tai-Chiu Hsung et al [152], improved the traditional wavelet method by applying multivariate shrinkage on multiwavelet transform coefficients. Then threshold selections were studied using SURE for each resolution level, provided the noise structure is known. Erdem Bala and Aysin Ertuzun [36], proposed a multivariate thresholding technique for image denoising with multiwavelets. The proposed technique is based on the idea of restoring the spatial dependence of the pixels on the noisy image that has undergone a multiwavelet decomposition. The multivariate thresholding is applied to the whole coefficient vector. T.R.Downie et al [151], proposed, a multivariate universal thresholding for multiwavelet considering coefficient vectors as a whole rather than the individual elements. Strela.V et al [148], proposed new vector filter banks, for discrete multiwavelet transform, thresholding and inverse discrete multiwavelet transform. Zhang et al [169], proposed that noisy image can be decomposed by multiwavelet thresholding and coefficients are processed with different semi soft thresholds according to coefficients energy distribution. Strela.V and Andrew T. Walden [148], presented new vector filter bank, in particular biorthogonal hermite cubic multiwavelets with short, smooth duals. V.Strela et al [148], investigated the emerging notion of multiwavelets in the context of multirate filter banks and applied a multiwavelet system to image
coding and signal denoising. Jerome Lebrun and Martin Vetterli [70], dealt with multiwavelets in the context of multirate filter banks and their applications to signal processing and compression.

There is no end to the list. A number of researchers are still working in many areas apart from wavelets and multiwavelets to find out better solution for image denoising and quality enhancement of tomographic images.

1.3 FORMULATION OF TECHNICAL PROBLEMS

In view of the limitations that exist in the literature to improve PSNR and quality enhancement of an image, this research has focused on the following novel techniques which are described in detail.

1.3.1 Window based multiwavelet transformation, thresholding and reconstruction techniques. [121]

The block diagram of window based multiwavelet transformation and thresholding is given in Fig.1.1 and reconstruction technique is given in Fig.1.2 below.

Fig: 1.1 Block diagram of window based multiwavelet transformation and Thresholding
Generate two zero matrices
Count matrix $Z_1$ and weight matrix $Z_2$

Extract window from $Z_1$ and $Z_2$
i.e.: $w_{c_1}$ & $w_{c_2}$

Determine the value of each window $w_{c_1}$ & $w_{c_2}$

Reconstruct window using kaiser window $w_1$

Replace $n_w$ windows in their corresponding position of $I_{AWGN}$ to obtain reconstructed image

**Fig: 1.2** Block Diagram of a tomographic image reconstruction.

Tomographic original images such as human abdomen and thorax, of size 256*256, are considered. Add additive white Gaussian noise with noise levels, $\sigma = 10, 20, 30, 40$ and $50$, to it. In this window based technique, as shown in Fig.1.1, choose a window of pixel size 8*8 from noisy image. With a sliding step size of 4, extract
windows from noisy image and duplicate noisy image. Therefore, 63*63 windows of pixels are extracted from the noisy image. The obtained windows of pixels from noisy and duplicate noisy image are subjected to GHM multiwavelet transformation. For each duplicate noisy image window compute the distance measure from every noisy image window using L2 Norm distance. Then sort distance measure computation in ascending order based on L2 Norm distance threshold. Select a set of closer number of duplicate noisy image windows for every noisy image window and skip remaining based on L2 norm threshold value. For every duplicate noisy image window elements, that corresponds to noisy image window elements of similar position, must be subjected to CL multiwavelets transformation. Then apply hard thresholding to eliminate the insignificant pixel elements as noise is generally concentrated in the smaller coefficients. Then apply inverse multiwavelet transforms, to get back in spatial domain.

In order to reconstruct a denoised image, generate two zero matrices that is count matrix and weight matrix each of size 256*256. Extract windows from these matrices of size 8*8 as shown in fig 1.3 above. Then determine the values of each of these windows using equation 2.45 & 2.46 i.e., \( \text{w}^{(i)}_{z1} \) and \( \text{w}^{(i)}_{z2} \) and reconstructed window, \( \text{w}^{*}_{i} \).

Reconstruct windows using Kaiser window. Kaiser window is used for smoothing purpose, to minimize the speckles. Mathematically it is given as equation 2.47 on page 52. Replace the reconstructed windows in their corresponding position of noisy image to obtain reconstructed original image.
1.3.2 Modified Genetic algorithm aided window based multiwavelet transformation, thresholding and reconstruction techniques.[122]

The block diagram of modified Genetic algorithm aided window based multiwavelet transformation, thresholding and reconstruction techniques are given in Fig.5.1 and Fig. 5.2 respectively.

![Block diagram of modified Genetic algorithm aided window based multiwavelet transformation and thresholding techniques.](image)

**Fig: 1.4** Block diagram of modified Genetic algorithm aided window based multiwavelet transformation and thresholding techniques.

**Modified Genetic algorithm for optimal window selection:**

In order to carry out the computation in less time, the selection of optimal (closer) windows were missed out. This problem motivated us to modify and implement genetic algorithm for optimum (closer) window selection.
The proposed methodology is comprised of five functional steps, namely, 1) Generation of initial chromosomes 2) Determination of fitness function 3) Crossover and mutation 4) Selection of optimal windows and 5) Termination criteria.

The search space comprises of solutions which are identified by a string as a chromosome. Each chromosome is composed of an objective function called fitness function. A collection of chromosomes together with their associated fitness is termed as the population. The population, at a particular iteration of the Modified Genetic Algorithm (MGA), is known as a generation [25, 53, 59].
MGA begins to function with numerous possible solutions that are obtained from the randomly generated initial population. Then, it tries to find optimum solutions by employing genetic operators namely selection, crossover and mutation [53]. Selection is a process of selecting a pair of organisms to reproduce. Crossover is a process of swapping the genes between the two individuals that are reproducing. Mutation is the process of randomly modifying the chromosomes [119]. The main aim of mutation is to re-establish lost data and explore variety of data. By changing some bit values of chromosomes, different breeds can be provided. New chromosomes may be better or poorer than old chromosomes. If they are poor than old chromosome, then they are removed from selection step [112]. The process continues till the termination criterion is satisfied and hence the modified genetic algorithm can converge to an optimal solution.

1.3.3 Modified quality enhancement techniques using morphological operation [121]

The block diagram of modified quality enhancement technique for tomographic denoised images using morphological operation is given in Fig.1.6.

![Block diagram of modified quality enhancement technique using morphological operation.](image-url)
In this proposed methodology, strengthen the denoised image, obtain the gradient image from the strengthened denoised tomographic image, convert gradient image to binary image, then apply morphological thinning operation, resize, preserve the edges, sharpen and finally preserve the edge, to obtain the final quality of enhanced tomographic image.

1.3.4 Modified quality enhancement techniques using Canny edge detection algorithm.[123]

The block diagram of modified quality enhancement technique for denoised tomographic images using Canny edge detection algorithm is given in Fig.1.7.

Fig: 1.7 Block diagram of modified quality enhancement of a tomographic image using Canny edge detection algorithm

In this proposed methodology, obtain gradient image from the denoised tomographic image, then apply Canny edge detection algorithm to detect edges. Generate a pair of intermediate matrices from X-Y co-ordinates. Generate microblocks from the edge detected tomographic image, which improves the visibility of organs present in the image. The microblock set generated should be subjected to
sharpening using unsharp filter and smoothing using Gaussian filter. This method has the advantage of enhancing the quality of the denoised image and preserving important features and organ surfaces well. By implementing this technique, the noise can be removed efficiently and the particular features can be well preserved and the whole visual quality of an image can be improved.

Procedure for detecting the edges using Canny algorithm:

i) Smoothen the image.
ii) Find gradient of an image.
iii) Spot the local maxima as edge i.e., non-maximum suppression.
iv) Determine potential edge by thresholding.

Determine final edges by suppressing all edges that are not connected to strong edges.

1.3.5 Construction of multi-filter coefficients for multiwavelet transformation.

In search of better multiwavelet transforms, we have proposed and constructed multi-filter coefficients satisfying the conditions, and the properties such as orthogonality, symmetry, short support, higher order of approximation, which can be simultaneously applied to multiwavelet transformation. These multi-filter coefficients are incorporated in the multiwavelet transforms. The multiwavelet transforms along with thresholding techniques can be implemented for denoising a tomographic image.
1.3.6 Noise variance estimation and denoising of patient’s tomographic image.[21, 117]

Take patient’s DICOM tomographic image, normalize, then apply transformation techniques to decompose an image, then estimate the noise variance by using Mean Absolute Deviation (MAD) method. Calculate the threshold value and then reconstruct an image to obtain the denoised image. By this method the naturally available noise can be eliminated.

1.4 OUTLINE OF THE THESIS:

Chapter-1: Presents the introduction, review of the literature, formulation of the technical problem and outline of the thesis.

Chapter-2: Provides the mathematical derivations of multiwavelets and its transformations, multi-filter coefficients, window based multiwavelet transformation and thresholding, and modified Genetic algorithm.

Chapter-3: Presents two modified techniques for the quality enhancement of the denoised tomographic images. This chapter also covers the technique for segmentation analysis.

Chapter-4: Discusses about the implementation of window based multiwavelet transformation, thresholding and its reconstruction techniques.

Chapter-5: Discusses about the implementation of modified genetic algorithm for optimal window selection in window based multiwavelet transformation.
Chapter-6: Presents the implementation of multiwavelet transforms to denoise tomographic images. It also includes the implementation of modified quality enhancement techniques using morphological operation and canny edge detection algorithm.

Chapter-7: Presents the implementation of noise variance estimation technique using mean absolute deviation method, thresholding and denoising the patient’s tomographic images.

Chapter-8: Conclusion and further research.

Appendix-A: Provides the fundamentals of tomography.

Appendix-B: Provides the fundamentals of Multiwavelets and Contourlet transforms.

Appendix-C: Provides the fundamentals of denoising techniques.

Appendix-D: Provides the fundamentals of noise and its models.