10. MIX DESIGN

10.1 GUIDE LINES FOR INITIAL MIX COMPOSITION

Self consolidating concrete, SCC in short, should flow into tight and constricted spaces without segregation and without requiring vibration. The key to creating SCC is a mixture that is almost fluid, but at the same time stable, to avoid segregation. Evidently these two are opposing requirements. To achieve the first requirement, fluidity, a new generation superplasticisers is specified as it gives better water reduction and slower slump loss. The second requirement, stability of a fluid mix, is achieved either by using high fines content or by using viscosity modifying agent. Sometimes a combination of increased fines content and a viscosity modifying agent is used to stabilize the mixture.

To achieve fluidity, homogeneity Okamura focused on three different aspects:

- The influence of coarse aggregate on the flowability of fresh concrete largely depends on the size of the spacing of the obstacle.
- Sufficient deformability of the mortar phase in concrete is required so that concrete can be compacted into structures by its self-weight without need for external vibration.
- Moderate viscosity as well as deformability of the mortar phase was also required to reduce the segregation.
Self-compactability had to be tested by Slump flow, V-flow and Funnel tests for adjusting the water to powder ratio (w/p) and Superplasticizer dosage. Indicative typical ranges of proportions and quantities in order to obtain self-compactability as per EFNARC specifications are given below. Further modifications will be necessary to meet strength and other performance requirements.

- Water/powder ratio by volume of 0.80 to 1.10. Total powder content - 160 to 240 litres (400-600 kg) per cubic meter.
- Coarse aggregate content normally 28 to 35 per cent by volume of the mix.
- Water cement ratio is selected based on strength and workability of SCC. Typically water content does not exceed 200 litres /m$^3$.
- The sand content balances the volume of the other constituent materials.
- The typical content of cement is 350-450 kg/m$^3$. More than 500 kg/m$^3$ cement can be dangerous and increase the shrinkage. Less than 350 kg/m$^3$ may only be suitable with the inclusion of other fine filler, such as fly ash, pozzolanas etc.

Generally, it is advisable to design conservatively to ensure that the concrete is capable of maintaining its specified fresh properties despite anticipated variations in raw material quality. Some variation in aggregate moisture content should also be expected and allowed for at mix design stage. Normally, viscosity-modifying admixtures are a useful tool for compensating the fluctuations.
10.1.1 VARIOUS METHODS OF MIX DESIGN

One of the significant limitations in the ready adoption of self compacting concrete in India is the lack of availability of appropriate mixture proportioning methods. SCC requires some special considerations in mixture proportioning, since the required flowing ability cannot be achieved by just increasing the water content of the mixture. In the mixture proportioning of SCC the quantities to be determined are air, water, cement, filler, fine and coarse aggregate apart from the dosages of superplasticizer and viscosity modifying agent. There are few methods available for determining these quantities. All the methods developed are based on the guidelines given by the scholars in the field of SCC along with number of trials in the laboratory and in the field. All the mixture proportioning methods given below involve trial castings and corrections. Different methods available for getting the trial mixes of SCC are

1. The Japanese Method
2. Sedran et al Method
3. Method proposed by Gomes, Ravindra Gettu et al
4. Nan-Su et al Method
5. Method proposed by Jagadish Vengala
6. European practice and specifications
10.1.2 MIX DESIGN AND TRIAL MIX PROPORTIONS OF SELF-COMPACTING CONCRETE.

An SCC mix which gives a minimum compressive strength of 30 Mpa was aimed in the beginning. In the mixture proportioning of SCC the quantities to be determined are air, water, cement, filler, fine and coarse aggregate apart from the dosages of superplasticiser and viscosity modifying agent. There are few methods available for determining these quantities. All the methods developed are based on the guidelines given by the scholars in the field of SCC along with number of trials in the laboratory and in the field.

In the present study the initial mix proportion is determined by Nan-Su et al method of mix design and fine tuned by using different guidelines to get the mix which satisfies the required fresh and hardened properties of SCC.

10.1.3 MIX DESIGN

SCC mixes of three different grades are designed. In the mixture proportioning of SCC the quantities to be determined are air, water, cement, filler, fine and coarse aggregate apart from the dosages of superplasticiser and viscosity modifying agent. There are few methods available for determining these quantities. All the methods developed are based on the guidelines given by the scholars in the field of SCC along with number of trials in the laboratory and in the field.

10.1.4 MIX DESIGN BY EFNARC

The initial mix proportions were calculated using mix design method of conventional concrete given by Indian Standard method
instead of simply assuming the contents of Cement, Coarse Aggregate and Fine Aggregate as given by the literature. These contents were compared with the EFNARC Specifications and small modifications were done to the contents that arrived in the Indian Standard method to satisfy the SCC mix composition criteria. The mix design of M20, M40 and M60 grade SCC were summarized below.

10.1.4.1 Mix design of M20 grade SCC:

**Design Stipulations:**

Characteristic strength of concrete at 28-days in the field = 20 N/mm²

Maximum size of aggregate = 12mm

Compaction Factor (CF) = 0.8

Specific Gravity of FA = 2.59

Specific Gravity of CA = 2.61

Specific Gravity of Cement = 2.91

Target mean strength: $F_{ck} = f_{ck} + 1.65S$

For M20 grade standard deviation, $S=4.6$ (from table-1 of IS-10262)

$F_{ck} = 20 + 1.65 \times 4.6 = 27.59$ N/mm²

Selection of W/C ratio: (i) From Fig.1 of IS: 10262-1982, for 27.59 N/mm² it is 0.48

(ii) From durability aspect maximum w/c ratio for moderate exposure is 0.6
(From IS 456-2000)
The lesser of the above two is to be adopted, Hence 0.48 is taken.

From table 4 of IS: 10262-1982 the water content per cubic meter of concrete,

\[
\text{Water content} \quad = \quad 203.6 \text{ kg/m}^3
\]

As per EFNARC specifications water content is limited to 200 kg/m\(^3\)

\[
\begin{align*}
\text{FA/TA} & = 39.0 \\
\text{W/C} & = 0.48 \\
\text{Workability} & = 0.8\text{CF}
\end{align*}
\]

For Corrections

<table>
<thead>
<tr>
<th>Change in Condition</th>
<th>Water</th>
<th>% (FA/TA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/C</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>CF</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Zone II</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Correction</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

Water  = 200 kg/m\(^3\)

Cement  = 200/0.48 = 416.67 kg/m\(^3\)

As per durability minimum cement content for moderate exposure is 272 kg/m\(^3\). (From IS 456-2000)

P  = 39\% - 2\% = 37\%

Entrapped air: 2.8\% of volume of concrete as per table 3 of IS 10262-1982.
**Determination of FA content:**

\[ V = \frac{(W+C/S_C+(1/P)*FA/S_{FA})*1}{1000} \]

\[ 0.97 = \frac{(200+417/2.91+(1/0.37)*FA/2.59)*1}{1000} \]

\[ FA = (970-200-143.3)*2.59*0.37 \]

\[ FA = 601 \text{ kg} \]

\[ TA = 601/0.37 = 1623 \text{ kg} \]

\[ CA = 1623-601 = 1022 \text{ kg} \]

<table>
<thead>
<tr>
<th>W</th>
<th>C</th>
<th>FA</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>417</td>
<td>601</td>
<td>1022</td>
</tr>
</tbody>
</table>

\[ 0.48 : 1 : 1.44 : 2.45 \]

Converting into SCC proportions

The normal concrete mix proportions are modified as per EFNARC Specifications and different trial mixes were casted. By considering the fresh properties we finally arrive at the following SCC mix proportions.

The W/C ratio is limited to 0.45

Taking water = 180kg

Cement = \( 180/0.45 = 400 \text{ kg/m}^3 \)

FA = 601 kg

TA = 1623 kg

Taking 52% of TA as FA
FA = 844 kg/m³
CA = 780 kg/m³

The modified SCC proportion is

W : C : FA : CA
180 : 400 : 844 : 780

| 0.45 | 1   | 2.11 | 1.95 |

10.1.4.2 Mix design of M40 grade SCC:

Design Stipulations:

Characteristic strength of concrete at 28-days in the field = 40 N/mm²

Maximum size of aggregate = 12mm

Compaction Factor (CF) = 0.8

Specific Gravity of FA = 2.59

Specific Gravity of CA = 2.61

Specific Gravity of Cement = 2.91

Target mean strength: \( F_{ck} = f_{ck} + 1.65S \)

For M40 grade standard deviation, \( S = 6.6 \) (from table-1 of IS-10262)

\( F_{ck} = 40 + 1.65 \times 6.6 = 50.89 \text{N/mm}^2 \)
Selection of W/C ratio:

(i) From Fig. 1 of IS:10262-1982, for 50.89N/mm² it is 0.31

(ii) From durability aspect maximum w/c ratio for moderate exposure is 0.6 (From IS 456-2000)

The lesser of the above two is to be adopted, Hence 0.31 is taken.

From table 5 of IS: 10262-1982 the water content per cubic meter of concrete,

<table>
<thead>
<tr>
<th>Water content</th>
<th>196 kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>FA/TA</td>
<td>28.0</td>
</tr>
<tr>
<td>W/C</td>
<td>0.31</td>
</tr>
<tr>
<td>Workability</td>
<td>0.8CF</td>
</tr>
</tbody>
</table>

For Corrections

<table>
<thead>
<tr>
<th>Change in Condition</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
</tr>
<tr>
<td>W/C</td>
<td>0</td>
</tr>
<tr>
<td>CF</td>
<td>0</td>
</tr>
<tr>
<td>Zone II</td>
<td>0</td>
</tr>
<tr>
<td>Total Correction</td>
<td>0</td>
</tr>
</tbody>
</table>

Water  = 196 kg/m³

Cement  = 196/0.31 = 632 kg/m³
As per durability minimum cement content for moderate exposure is 272 kg/m³. (From IS 456-2000)

As per SCC mix Specifications the cement is limited to 500 kg/m³ for medium grade concretes

\[ P = 28\% \]

Entrapped air: 2.8% of volume of concrete as per table 3 of IS 10262-1982.

**Determination of FA content:**

\[ V = (W+C/Sc+(1/P)*FA/SFa)*1/1000 \]

\[ 0.97 = (196+500/2.91+(1/0.28)*FA/2.59)*1/1000 \]

\[ FA = (970-196-171.82)*2.59*0.28 \]

\[ FA = 437\ kg \approx 440\ kg \]

\[ TA = 440/0.28 = 1571\ kg \approx 1600\ kg \]

\[ CA = 1600-440 = 1160\ kg \]

\[ W : C : FA : CA \]

\[ 196 : 500 : 440 : 1160 \]

\[
\begin{array}{cccc}
0.39 & 1 & 0.88 & 2.32 \\
\end{array}
\]

Converting into SCC proportions

The normal concrete mix proportions are modified as per EFNARC Specifications and different trial mixes were casted. By considering the
fresh properties we finally arrived at the following SCC mix proportions

The W/C ratio is limited to 0.38

Cement taken    = 500 kg/m³

Then water        = 190 kg/m³

Taking 50% of TA as FA

FA = 800 kg/m³

CA = 800 kg/m³

The modified SCC proportion is

W          :       C         :         FA         :         CA
190        :       500       :         800         :        800

0.38       :        1          :         1.6         :         1.6

**10.1.4.3 Design of M60 Grade SCC:**

**Design Stipulations:**

Characteristic strength of concrete at 28-days in the field = 60 N/mm²

Maximum size of aggregate = 12mm

Compaction Factor (CF)    = 0.8

Specific Gravity of FA    = 2.59

Specific Gravity of CA    = 2.61

Specific Gravity of Cement = 2.91
Target mean strength: \( F_{ck} = f_{ck} + 1.65 \times S \)

\( F_{ck} = 65.25 \text{ N/mm}^2 \)

Selection of W/C ratio:

(i) From Fig. 1 of IS: 10262-1982, for 65.25 N/mm\(^2\) it is 0.30

(ii) From durability aspect w/c ratio for modern exposure is 0.6. (From IS 456-2000)

The lesser of the above two is to be adopted, Hence 0.30 is taken.

From table 5 of IS: 10262-1982 the sand and water contents in cubic meter of concrete.

- Water content = 196 kg/m\(^3\)
- FA/TA = 28.0
- W/C = 0.30
- Workability = 0.8CF

**For Corrections**

<table>
<thead>
<tr>
<th>Change in Condition</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
</tr>
<tr>
<td>W/C</td>
<td>0</td>
</tr>
<tr>
<td>CF</td>
<td>0</td>
</tr>
<tr>
<td>Zone II</td>
<td>0</td>
</tr>
<tr>
<td>Total Correction</td>
<td>0</td>
</tr>
</tbody>
</table>
Water = 196 kg/m³

Cement = 196/0.30 = 653.33 kg/m³

As per durability minimum cement content for moderate exposure is 260 kg/m³.

As per the EFNARC Specifications the cement content is limited to 600 kg/m³.

P = 27%

Entrapped air: 2.8% of volume of concrete as per table 3 of IS 10262-1982.

**Determination of FA content:**

\[
V = \frac{(W+C)}{S_c} + \frac{1}{P} \times \frac{FA}{S_{FA}} \times 1/1000
\]

\[
0.97 = \frac{(196+600)}{2.91} + \frac{(1/0.27) \times FA}{2.59} \times 1/1000
\]

FA = (970-196-206.2)*2.59*0.27

FA = 397 kg

TA = 397 / 0.27 = 1471 kg

CA = 1471-397 = 1074 kg

<table>
<thead>
<tr>
<th>W</th>
<th>C</th>
<th>FA</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>196</td>
<td>600</td>
<td>397</td>
<td>1074</td>
</tr>
</tbody>
</table>

| P    | 0.33 | 1    | 0.66 | 1.79 |
Converting into SCC proportions

The normal concrete mix proportions are modified as per EFNARC Specifications.

The W/C ratio is limited to 0.3

Cement taken = 600 kg/m$^3$

Then water = 180 kg/m$^3$

Taking 55% of TA as FA

FA = 809 kg/m$^3$ ≈ 810 kg

CA = 662 kg/m$^3$ ≈ 660

The modified proportion is

\[
\begin{align*}
W : C : FA : CA \\
180 : 600 : 810 : 660 \\
0.32 : 1 : 1.35 : 1.1
\end{align*}
\]
According to the concept of reinforced concrete the following design philosophies have been evolved for design of R.C. structures.


2) Ultimate load method.

3) Limit state method.

**Working stress method:** - This has been the traditional method used for reinforced concrete design. It is assumed that concrete is elastic, steel and concrete act together elastically, and the relationship between loads and stresses is linear right up to the collapse of the structure. The sections are designed in accordance with the elastic theory of bending assuming that both materials obey the Hooke’s law. The elastic theory assumes linear variations of strain and stress from zero at the neutral axis to a maximum at the extreme fibre. Typical stress strain distribution in rectangular sections is shown below.

STRESS - STRAIN CURVE IN WORKING STRESS DESIGN
Where \( A_t \) = Area of tension steel, \( b \) = Width of the section
\( C \) = Total force of compression. \( T \) = total force of tension.
\( D \) = Overall depth of the section \( d \) = effective depth
\( j_d \) = lever arm \( n_d \) = depth of neutral axis
\( \epsilon_c \) = compressive strain concrete. \( \epsilon_{st} \) = tensile strain in steel.
\( \sigma_{st} \) = permissible tensile stress in steel
\( \sigma_{cbc} \) = permissible compressive stress in concrete

**Ultimate load method** In this method the working loads are increased by suitable factors called load factors to obtain ultimate loads. The structure is then designed to resist the ultimate loads. The term safety factor has been used in the working stress method to denote the ratio between the yield stress and permissible stress. The term load factor has been used to denote the collapse or ultimate load to the working load. Whitney’s theory is based on the assumptions that an ultimate strain in concrete is 0.3%. Whitney replaced the actual parabolic stress diagram by a rectangular stress diagram such that the centre of gravity of both the diagrams lies at the same point and areas are also equal. The major advantage of this method over the working stress method is that total safety factor the structure thus found is nearer to its actual value. The structure designed by Ultimate load method requires less reinforcement than those designed by working stress method.

\( a \) = depth of rectangular stress block,
\( \epsilon_{sy} \) = Yield strain in concrete
\( \sigma_{cb} \) = Ultimate compressive strength of concrete
\( \varepsilon_{cu} \) = Ultimate strain in concrete

**Limit state method**: - Limit state design has originated from Ultimate load or plastic design. The object of design based on the limit state concept is to achieve an acceptable probability that the structure will not become unserviceable in its lifetime for the use for which it is intended, that is, it will not reach limit state.

Where \( \sigma_y \) = characteristic strength of steel.

\( \varepsilon_s \) = strain in steel at failure ; \( E_s \) = modulus of elastic of steel
**Modes of failure:**

If the ratio of steel to concrete in a beam is such that the maximum strain in the two materials reach simultaneously, a sudden failure would occur with less alarming deflection. Such a beam is referred as a balanced reinforced beam. When the amount of steel is kept less than that in the balanced condition, the neutral axis moves upwards to satisfy the equilibrium condition that force of compression is equal to the force in tension. In this process center of gravity of compressive force also shifts upwards. Under increasing bending moment, steel is strained beyond the yield point and the maximum strain in concrete remains less than 0.35%.

If the beam is further loaded the strain in the section increases. Once steel has yielded, it does not take any additional stress for the additional strain and the total force of tension remains constant. However, compressive stresses in concrete do increase with the additional strain. Thus, neutral axis and the center of gravity of compressive forces further shift upward to maintain equilibrium. This results in an increase in the moment of resistance of the beam. This process of shift in the neutral axis continues until maximum strain in the concrete reaches its ultimate value, that is 0.35% and the concrete crushes. Such a beam is referred to as an under reinforced beam. When the amount of steel is kept more than that in the balanced condition, the neutral axis tend to move downward and strain in steel remains in the elastic region. If the beam is further loaded, the strain and the stress in steel keep on increasing and so the force of tension.
The additional increase in the concrete stress is much slower. Thus to maintain the equilibrium of tension and compression forces, the area of concrete resisting compression has to increase. In this process, neutral axis further shifts downwards until the maximum strain in the concrete reaches its ultimate value, which is a 0.35% and concrete crush the steel is still well within the elastic limit. Such a beam is referred as an over reinforced beam and failure as a compression failure.

**Moment of Resistance:**

Consider a simply supported beam subject to bending under factored load. For equilibrium, total force of compression must be equal to total force of tension. The applied bending moment at collapse, that is, factored bending moment is equal to the resisting moment on the section provided by internal stress, this is called the ultimate moment of resistance.

\[
\text{Total force of compression in concrete } = 0.36 \sigma_{ck} b X_u. \\
\text{Total force of tension in steel } = 0.87 \sigma_y A_{st}. \\
\text{Moment of resistance with respect to concrete } = \text{compressive force } \times \text{lever arm.} \\
= 0.36 \sigma_{ck} b X_u (d-0.42 X_u) \\
\text{Moment of resistance with respect to steel } = \text{tensile force } \times \text{lever arm } = 0.87 \sigma_y A_{st} (d-0.42 X_u) 
\]
**Design of Beam Using Standard Grade Concrete - M20**

Beam dimensions : 100 X 150 X 1200 mm.

Grade of concrete : M20

Grade of steel : Fe 415

\[ X_{u\text{ max}} = 0.48d = 0.48 \times 125 = 60.0 \text{ mm} \]

**For balanced section,**

\[ 0.36 \times f_{ck} \times b \times X_{u\text{ max}} = 0.87 \times F_y \times A_{st}. \]

\[ 0.36 \times 20 \times 100 \times 60.0 = 0.87 \times 415 \times A_{st}. \implies A_{st} = 119.65 \text{ mm}^2 \]

**For under reinforced section**

Providing 2-8 mm dia. tor steel bars:

\[ A_{st} = 2 \left( \frac{\pi}{4} \right) 8^2 = 100.57 \text{ mm}^2 < 119.65 \text{ mm}^2 \]

\[ 0.36 \times 20 \times 100 \times X_{u} = 0.87 \times 415 \times 100.57. \implies X_{u} = 50.43 \text{ mm.} \]

M.R = 0.87 \times 415 \times 100.57(125-0.42 \times 50.43) = 3.77 \text{ kN-m.} 

Self Wt. of the beam = 0.1 \times 1.5 \times 25 = 0.375 \text{ Kn/m} 

M.R=Max.B.M = Wl^2/8+Wl/3

\[ 3.77=0.375(1.2)^2/8+W (1.2)/3 \implies W = 9.26 \text{ kN.} \]

Shear force: \[ V = W/2 = 9.26/2 = 4.63 \text{ kN.} \implies V_u = 4.63 \text{ kN.} \]

Normal shear stress \[ \tau_v = V_u / (b.d) = 4.63 \times 10^{-3} / (100 \times 125) \]

\[ = 0.37 \text{ N/mm}^2 \]

Percentage of steel \[ p = 100 \times 100.57 / (100 \times 125) = 0.80 \]

\[ \tau_c = 0.62 \text{ N/mm}^2 \text{ (from IS456:2000)} \implies \tau_v < \tau_c \]

Normal shear reinforcement is required.

Spacing for 6mm dia-2legged stirrups

\[ A_{sv} = \left( \frac{\pi}{4} \right) 6^2 = 56.57 \]

Maximum spacing = 0.75d = 0.75 \times 125 = 90 \text{ mm.}
**For over reinforced section**

Providing 2-12 mm dia. tor steel bars

\[
\text{Ast} = 2 \left( \frac{\pi}{4} \right) \left( \frac{12^2}{2} \right) = 226.20 \text{ mm}^2 > 119.65 \text{ mm}^2
\]

\[
= 0.36\sigma_{ck} b \ X_u (d-0.42 \times u) \\
= 0.36 \times 20 \times 100 \times 60 \ (125-0.42 \times 60) \implies 4.32 \text{ Kn}
\]

Self Wt. of the beam = 0.1 \times 0.15 \times 25 = 0.375 \text{ Kn/m}

\[
\text{Max.B.M} = Wl^2/8+Wl/3=M.R \\
4.32 = 0.375X \ (1.2)^2/8 +w(1.2)/3 \implies W = 10.63 \text{ kN.}
\]

Shear force: \( V = 10.63/2 = 5.32 \text{ kN.} \)

\( Vu = 5.32 \text{ kN.} \)

Normal shear stress \( \tau_v = \frac{Vu}{b \times d} = \frac{5.32\times 10^3}{(100\times 125)} \)

\( = 0.43 \text{ N/mm}^2 \)

Percentage of steel \( P = \frac{100 \times 226.8}{(100 \times 125)} = 1.81 \)

\( \tau_c = 0.79 \text{ N/mm}^2 \) (from IS456:2000) \implies \tau_v < \tau_c

**Design of shear reinforcement**

Spacing for 6mm dia-2legged stirrups

\( A_{sv} = 56.57 \)

\( Sv \leq A_{sv} \times F_y /0.4b = 56.57 \times 250 /0.4 \times 100 \implies Sv = 353.5 \text{ mm c/c.} \)

Maximum spacing = 0.75 \times 125 = 93.75 \text{ mm.}

A spacing of 90 mm c/c is provided for both under and over reinforced beams.
**Design of Beam Using Standard Grade Concrete - M40**

Beam dimensions: 100 X 150 X 1200 mm.

Grade of concrete: M40

Grade of steel: Fe 415

\[ X_u \text{ max} = 0.48d. = 0.48 \times 125 = 60.0 \text{ mm} \]

**For balanced section,**

\[ 0.36 \times f_{ck} \times b \times X_u \text{ max} = 0.87 \times F_y \times A_{st}. \]

\[ 0.36 \times 40 \times 100 \times 60.0 = 0.87 \times 415 \times A_{st}. \Rightarrow A_{st} = 239.30 \text{ mm}^2 \]

**For under reinforced section**

Providing 2-12 mm dia. tor steel bars:

\[ A_{st} = 2 \left( \frac{\pi}{4} \right) 12^2 = 226.2 \text{ mm}^2 < 239.30 \text{ mm}^2 \]

\[ 0.36 \times 40 \times 100 \times X_u = 0.87 \times 415 \times 226.20. \Rightarrow X_u = 56.72 \text{ mm}. \]

M.R = 0.87 x 415 x 226.2(125-0.42 x 56.72) = 8.26 kN-m.

Self Wt. of the beam=0.1x.15x25=0.375 KN/m

M.R=Max.B.M = Wl^2/8+Wl/3

\[ 8.26 = 0.375(1.2)^2 / 8+W (1.2)/3 \Rightarrow W = 20.48 \text{ kN}. \]

Shear force: \( V = W/2 =20.48/2 = 10.24 \text{ KN}. \Rightarrow Vu \)

Normal shear stress \( \tau_v = Vu / (b.d) =10.24\times10^{-3} / (100x125) \)

\[ = 0.82 \text{ N/mm}^2 \]

Percentage of steel \( p = 100 \times 226.2/ (100 \times 125) = 1.81 \)

\( \tau_c = 0.88 \text{ N/mm}^2 \text{ (from IS456:2000)} \Rightarrow \tau_v < \tau_c \)

Provide Normal shear reinforcement.

Spacing for 6mm dia-2legged stirrups

\[ A_{sv} = (\pi/4) 6^2 =56.57 \]

Maximum spacing = 0.75d = 0.75x125 =90mm.
For over reinforced section

Providing 2-16 mm dia. tor steel bars

\[ Ast = 2 \times \left( \frac{\pi}{4} \right) 16^2 = 402.29 \text{ mm}^2 > 239.30 \text{ mm}^2 \]

\[ 0.36 \times 40 \times 100 \times X_u = 0.87 \times 415 \times 402.29 \implies X_u = 100.87 \text{ mm.} \]

\[ = 0.36\sigma_{ck} b \times X_u \times (d - 0.42 \times X_u) \]

\[ = 0.36 \times 40 \times 100 \times 100.87 (125 - 0.42 \times 100.87) = \implies 12.00 \text{ Kn} \]

Self Wt. of the beam = 0.1 x 0.15 x 25 = 0.375 Kn/m

Max.B.M = Wl^2/8 + Wl/3 = M.R

\[ 12.00 = 0.375 \times X_u^2/8 + w(1.2)/3 \implies W = 29.83 \text{ kN.} \]

Shear force: V = 29.83/2 = 14.92 kN.

\[ Vu = 14.92 \text{ kN.} \]

Normal shear stress \( \tau_v \) = \( Vu / b \cdot d = 14.92 \times 10^3 / (100 \times 125) \)

\[ = 1.19 \text{ N/mm}^2 \]

Percentage of steel \( P \) = 100 x 402.29/100 x 125 = 3.22

\[ \beta = 0.8 \times f_{ck} / 6.89 \times Pt = 0.8 \times 40 / 6.89 \times 3.22 = 1.44 \text{ (from IS456:2000)} \]

\[ \tau_c = 0.85 \sqrt{0.8 \times f_{ck} \times (\sqrt{(1 + 5 \times \beta - 1) / 6 \times \beta}} \text{ (from IS456:2000)} \]

\[ = 0.85 \sqrt{0.8 \times 40 \times (\sqrt{(1 + 5 \times 1.44 - 1) / 6 \times 1.44}} = 1.037 \]

\[ \tau_c = 1.037 \text{ N/mm}^2 \implies \tau_v > \tau_c \]

\[ V_c = \tau_c \times b \times d = 1.037 \times 100 \times 125 = 12.962 \text{ Kn.} \]

\[ V_{us} = Vu - V_c = 14.920 - 12.962 = 1.958 \]

**Design of shear reinforcement**

Spacing for 6mm dia-2legged stirrups

\[ . A_{sv} = 56.57 \]

\[ V_{us} = 0.87 \times F_y \times A_{st} \times d / S_v \]

\[ S_v = 0.87 \times 250 \times 56.57 \times 125 / (1.958 \times 10^3) = 785 \text{ mm} \]
Maximum spacing = 0.75 d = 0.75 x 125 = 93.75 mm.

A spacing of 90 mm c/c is provided for both under and over reinforced beams.

**Design of Beam Using Standard Grade Concrete M-60**

Beam dimensions : 100 X 150 X 1200 mm.

Grade of concrete : M60

Grade of steel : Fe 415

\[ X_{u \text{ max}} = 0.48d = 0.48 \times 125 = 60.0 \text{ mm} \]

**For balanced section,**

\[ 0.36 \times f_{ck} \times b \times X_{u \text{ max}} = 0.87 \times F_y \times A_{st}. \]

\[ 0.36 \times 60 \times 100 \times 60.0 = 0.87 \times 415 \times A_{st}. \quad \Rightarrow \quad A_{st} = 358.95 \text{ mm}^2 \]

**For under reinforced section**

Providing 2-12 mm dia. tor steel bars:

\[ A_{st} = 2 \left( \pi /4 \right) 12^2 = 226.2 \text{ mm}^2 < 239.30 \text{ mm}^2 \]

\[ 0.36 \times 60 \times 100 \times X_{u} = 0.87 \times 415 \times 226.20. \quad \Rightarrow \quad X_{u} = 37.81 \text{ mm}. \]

Moment of resistance = \[ 0.87 \times 415 \times 226.2(125-0.42 \times 37.81) \]

\[ = 8.91 \text{ kN-m}. \]

Self Wt. of the beam = 0.1 x 1.5 x 25 = 0.375 KN/m

M.R = Max.B.M = \[ Wl^2/8 + Wl/3 \]

\[ 8.91 = 0.375(1.2)^2 /8+W(1.2)/3 \Rightarrow W = 22.11 \text{ KN}. \]

Shear force: \[ V = W/2 = 22.11/2 = 11.05 \text{ kN}. \Rightarrow Vu \]

Normal shear stress \[ \tau_v = V_u / (b.d) = 11.05 \times 10^3 / (100 \times 125) \]

\[ = 0.88 \text{ N/mm}^2 \]

Percentage of steel \[ p = 100 \times 226.2/ (100 \times 125) = 1.81 \]
\[ \beta = 0.8 \frac{f_{ck}}{6.89Pt} \text{ (from IS456:2000)} \]

\[ = 0.8 \times 60 / 6.89 \times 3.22 = 3.85 \]

\[ \tau_c = 0.85 \sqrt{0.8f_{ck}(\sqrt{(1+5 \beta - 1)/6} \beta)} \text{ (from IS456:2000)} \]

\[ = 0.85 \sqrt{0.8 \times 60 (\sqrt{(1+5 \times 3.85 - 1)/6} \times 3.85)} = 0.89 \]

\[ \tau_c = 0.89 \text{ N/mm}^2 \]

\[ \Rightarrow \tau_v < \tau_c \]

Provide Normal shear reinforcement.

Spacing for 6mm dia-2legged stirrups

\[ A_{sv} = (\pi /4) \times 6^2 = 56.57 \]

Maximum spacing = 0.75d = 0.75 x 125 = 90mm.

**For over reinforced section**

Providing 2-16 mm dia. tor steel bars

\[ A_{st} = 2 (\pi /4) \times 16^2 = 402.29 \text{ mm}^2 > 358.95 \text{ mm}^2 \]

\[ 0.36 \times 60 \times 100 \times X_u = 0.87 \times 415 \times 402.29 \Rightarrow X_u = 100.87 \text{ mm.} \]

\[ = 0.36 \sigma_{ck} b \times X_u (d-0.42 X_u) \]

\[ = 0.36 \times 60 \times 100 \times 100.87 \times (125-0.42 \times 100.87) = \Rightarrow 18.00 \text{ Kn} \]

Self Wt. of the beam = 0.1 x 0.15 x 25 = 0.375 Kn/m

\[ \text{Max.B.M} = Wl^2/8 + Wl/3 = M.R \]

\[ 18.00 = 0.375 \times (1.2)^2/8 + w(1.2)/3 \Rightarrow W = 44.83 \text{ kN} \]

Shear force: \[ V = 44.83/2 = 22.42 \text{ kN}. \]

\[ \nu = 22.42 \text{ kN.} \]

Normal shear stress \( \tau_v \) = \[ \frac{V}{b \times d} = 22.42 \times 10^3 / (100 \times 125) \]

\[ = 1.8 \text{ N/mm}^2 \]

Percentage of steel \( P = \frac{100 \times 402.29}{(100 \times 125)} = 3.22 \]

\[ \beta = 0.8 \frac{f_{ck}}{6.89Pt} = 0.8 \times 60 / 6.89 \times 3.22 = 2.16 \text{ (from IS456:2000)} \]
\[ \tau_c = 0.85 \sqrt{0.85 f_{ck}} \left( \sqrt{1 + 5 \beta - 1}/6 \right) \] (from IS456:2000)

\[ = 0.85 \sqrt{0.85 \times 60} \left( \sqrt{1 + 5 \times 1.16 - 1}/6 \times 1.16 \right) = 1.106 \]

\[ \tau_c = 1.106 \text{ N/mm}^2 \quad \Rightarrow \quad \tau_v > \tau_c \]

\[ V_c = \tau_c b d = 1.106 \times 100 \times 125 = 13.875 \text{ Kn.} \]

\[ V_{us} = V_u - V_c = 22.42 - 13.87 = 8.57 \]

**Design of shear reinforcement**

Spacing for 6mm dia-2legged stirrups

\[ A_{sv} = 56.57 \]

\[ V_{us} = 0.87 \times F_y \times A_{st} \times d / S_v \]

\[ S_v = 0.87 \times 250 \times 56.57 \times 125 / (8.57 \times 10^3) = 179 \text{ mm} \]

Maximum spacing = 0.75 \( d = 0.75 \times 125 = 93.75 \text{ mm} \).

A spacing of 90 mm c/c is provided for both under and over reinforced beams.