CHAPTER – 6

ANALYSIS

6.0 INTRODUCTION TO STRESS-STRAIN BEHAVIOUR

Graph obtained by drawing a curve for the values of stresses and strains obtained during testing a material specimen of materials is called a stress-strain curve. By testing cylinders of standard size made with concrete, under uni-axial compression values of stresses and strains are obtained and the stress-strain curves are plotted. Even though the stress strain relation for cement paste and aggregate when tested individually is practically linear, it is observed from the stress-strain plots of concrete that, no portion of the curves is in the form of a straight line. In concrete the rate of increase of stress is less than that of increase in strain because of the formation of micro cracks, between the interfaces of the aggregate and the cement paste. Thus the stress strain curve is not linear. In conventional concrete the value of stress is maximum corresponding to a strain of about 0.002 and further goes on decreasing with the increasing strain, giving a dropping curve till it terminates at ultimate crushing strain.

A total number of one hundred and forty four cylinders made with eight selected SCC mix proportions with and without the addition of admixtures with different percentages of steel were tested for stress-strain behavior under uni-axial compression under strain control. Three cylinders for each mix were cast, tested under uni-axial compression and the average of three cylinders were taken for
obtaining the stress-strain behaviour of each SCC mix. Thus stress-strain curves for eight SCC mixes with different percentage of steel with combinations of GGBS and RHA and chemical admixtures were plotted.

6.1 MATHEMATICAL MODELING FOR STRESS-STRAIN BEHAVIOUR OF SCC WITH AND WITHOUT STEEL CONFINEMENT

After obtaining the stress-strain behaviour of SCC mixes with combination of GGBS and RHA experimentally, an attempt was made to get the analytical stress-strain curves for SCC mixes.

To represent uni-axial stress-strain behaviour of conventional concrete, number of empirical equations has been proposed but most of them can be used for only ascending portion of the curve. Carriera and Chu (14) extended the empirical equation proposed by Popovics in 1985, which includes both ascending and descending portions of complete stress-strain curve. T.Suresh Babu (99) recently proposed an empirical equation which includes the effect of glass fibres in concrete. NRD Murthy et al (56) also proposed an empirical equation which includes the effect of steel fibres in concrete. Most of the equations proposed were for conventional concrete. Only few researchers worked on modeling of stress-strain behaviour of SCC made with GGBS and RHA.

Considering this gap in existing literature an attempt has been made to develop empirical equations for SCC with and without steel confinement with Ground Granulated Blast Furnace Slag, Rice Husk Ash as mineral admixtures.
6.1.1 Non-Dimensional Stress-Strain Curves

The stress-strain curves indicate that, the behaviour is similar for all the specimens with and without steel confinement. But on close examination it is observed that the SCC mixes with steel confinement have shown improved stress values for the same strain levels compared to that of SCC mixes without steel confinement. The similarity leads to the conclusion that there is only a unique shape of the stress-strain diagram, if expressed in a non dimensional form, along both the axes. The said form can be obtained by dividing the stress at any level by peak stress and the strain at any level by peak strain. Thus all the stress-strain curves will have same point (1,1) at peak stress. By non- dimensionalising the stresses and strains as above the behaviour can be represented as a general behaviour.

The stress- strain curves obtained experimentally for SCC were normalized as specified above and normalized stress-strain values were calculated for all SCC mixes with and without steel confinement.

The eight SCC mixes with GGBS and RHA, taken for investigation are of M20, M40, M60 grade mixes. A single normalized stress-strain curve is developed for the combination of three M20 SCC mixes, three M40 SCC mixes and two M60 SCC mixes taking the average values of normalized stresses and strains.
6.1.2 Models Available for Stress-Strain Curves of Conventional Concrete

Many researchers developed various models for the prediction of stress-strain behavior of concrete. Some of the models are given below.

1) Desay’s and Krishnan’s model [84].

For Normal strength concrete, stress-strain relationship is given by

\[ f = \frac{A x}{1 + B x^2} \]

Where \( f \) = The Normalised stress

\( X \) = Normalised strain and

\( A, B \) are the constants and they can be find out by using boundary conditions. This model is valid only up to ascending branch of stress-strain curve.

2) Saenz Model. [84].

With reference to Desay’s model Saenz proposed a model by taking into account both the ascending and descending portions of the stress-strain curve. This model is in the form of

\[ Y = \frac{A x}{1 + B x^2 + C x^3} \]

Where \( Y = (\sigma / \sigma_u) \) and \( X = (\varepsilon / \varepsilon_u) \)

3) Hognestad’s Model [32].

For Normal strength concrete up to ascending portion:

The stress-strain model is \( f_c = f_c' \{2 (\varepsilon / \varepsilon_o) - (\varepsilon / \varepsilon_o)^2\} \)

\( f_c' = \) compressive strength N/mm²

\( \varepsilon_o = \) strain at peak stress.

\[ = 0.0078 \ (f_c')^{1/4} \]
4) **Wang et.al. Model**

The model used by Wang et.al. is in the form of

\[
f_c = f'_c \left\{ \frac{A(\varepsilon_c/E_o) + B (\varepsilon_c/E_o)^2}{1+c(\varepsilon_c/E_o)+D(\varepsilon_c/E_o)^2} \right\}
\]

However instead of using one set of the coefficients A, B, C, and D to generate the complete curve, Wang et.al, used two sets of coefficients – one for the ascending branch and the other for the descending branch. The respective coefficients being obtained from the relevant boundary conditions assigned to each part of the curve.

5) **Carreria and Chu’s Model** \cite{14}.

This model is in the form of

\[
f_c = f'_c \left\{ \frac{\beta (\varepsilon_c/E_o)}{\beta - 1 + (\varepsilon_c/E_o)} \right\}
\]

In which \( \beta = 1 - (f_c' / \varepsilon_o E_{it}) \)

Where \( f_c' = \) cylinder ultimate compressive strength and

\( \varepsilon_o = \) strain at ultimate stress & \( E_{it} = \) initial tangent modulus.”

**6.1.3 Proposed Model for Stress-Strain Behaviour of SCC With and Without Steel confinement**

Equations in different forms were tried to get the complete stress-strain behaviour of SCC. Out of different possible trials, the developed normalized stress-strain curves were fitted with analytical equations using Seanz’s model.
The developed equation is in the form of

\[ Y = \frac{Ax}{1+Bx^2} \]

where

\( x \) - normalized strain,
\( Y \) - normalized stress

\( A, B \) - are constants for ascending portion and
\( A_1, B_1 \) - for descending portion.

\( A, B \) and \( A_1, B_1 \) are a set of constants for M20, M40 and M60 SCC mixes. Constants are determined based on the boundary conditions of normalized stress-strain curves for SCC.

Boundary conditions for ascending and descending portions of stress-strain curves are,

i. At the origin the ratio of stresses and strains are zero
   i.e. at origin \( \frac{\varepsilon}{\varepsilon_0} = 0, \frac{\sigma}{\sigma_0} = 0 \)
   \( \varepsilon_0 \) - strain at peak stress, \( \sigma_0 \) - peak stress

ii. The strain ratio and stress ratio at the peak of the non-dimensional stress-strain curve is unity.
   i.e. at \( \frac{\varepsilon}{\varepsilon_0} = 1, \frac{\sigma}{\sigma_0} = 1 \)

iii. The slope of non-dimensional stress-strain curve at the peak is zero
   i.e. at \( \frac{\varepsilon}{\varepsilon_0} = 1.0 \frac{d(\sigma/\sigma_0)}{d(\varepsilon/\varepsilon_0)} = 0 \)

iv. At 85% stress ratio the corresponding values of strain ratio is 1.3
   i.e. at \( \frac{\sigma}{\sigma_0} = 0.85 \frac{\varepsilon}{\varepsilon_0} = 1.3 \)

Where \( \sigma_0 \) - corresponds to peak stress and
\( \varepsilon_0 \) - corresponds to strain at peak stress
\( \sigma \) and \( \varepsilon \) corresponds to stress and strain values at any other point
Boundary conditions i and ii are for determining the constants in the ascending portion of the normalized stress-strain curve and ii, iii and iv are for determining the constants in the descending portion of the curve.

Using the boundary conditions in the non dimensional stress-strain curves, constants for different SCC mixes are determined and from that the equations are developed. Ultimately analytical equations giving the complete stress-strain behaviour are developed for M20, M40 and M60 grade SCC with GGBS and RHA and different percentage of steel confinement. For different SCC mixes, the proposed equation is in the form $Y = \frac{Ax}{1+Bx^2}$

**The constants for SCC mixes are**

<table>
<thead>
<tr>
<th>Mix</th>
<th>Constants for ascending portion</th>
<th>Constants for descending portion</th>
</tr>
</thead>
<tbody>
<tr>
<td>M20SCC mix</td>
<td>$A = 1.44, \quad B = 0.53$</td>
<td>$A_1 = 1.49, \quad B_1=0.71$</td>
</tr>
<tr>
<td>M20 GGBS mix</td>
<td>$A = 1.53, \quad B = 0.37$</td>
<td>$A_1=2.0, \quad B_1= 1.20$</td>
</tr>
<tr>
<td>M20 GGBS-RHA mix</td>
<td>$A=1.20, \quad B = 0.37$</td>
<td>$A_1=1.29, \quad B_1=1.22$</td>
</tr>
<tr>
<td>M40 SCC mix</td>
<td>$A=1.25, \quad B = 0.58$</td>
<td>$A_1=2.56, \quad B_1=1.77$</td>
</tr>
<tr>
<td>M40 GGBS mix</td>
<td>$A=1.25, \quad B = 0.27$</td>
<td>$A_1=2.42, \quad B_1= 1.62$</td>
</tr>
<tr>
<td>M40 GGBS-RHA mix</td>
<td>$A=1.09, \quad B = 0.22$</td>
<td>$A_1=2.08, \quad B_1=1.30$</td>
</tr>
<tr>
<td>M60 SCC mix</td>
<td>$A=1.41, \quad B = 1.0$</td>
<td>$A_1=2.27, \quad B_1=1.47$</td>
</tr>
<tr>
<td>M60 GGBS-RHA mix</td>
<td>$A=1.64, \quad B = 0.33$</td>
<td>$A_1=2.59, \quad B_1= 1.80$</td>
</tr>
</tbody>
</table>
Thus the equations for ascending and descending portions of SCC mixes are

<table>
<thead>
<tr>
<th>Mix</th>
<th>Equations for ascending portion</th>
<th>Equations for descending portion</th>
</tr>
</thead>
<tbody>
<tr>
<td>M20SCC mix</td>
<td>( Y = 1.44 \frac{x}{(1 + 0.53 x^2)} )</td>
<td>( Y = 1.49 \frac{x}{(1 + 0.71 x^2)} )</td>
</tr>
<tr>
<td>M20 GGBS mix</td>
<td>( Y = 1.53 \frac{x}{(1 + 0.37 x^2)} )</td>
<td>( Y = 2.0 \frac{x}{(1 + 1.20 x^2)} )</td>
</tr>
<tr>
<td>M20 GGBS-RHA mix</td>
<td>( Y = 1.20 \frac{x}{(1 + 0.37 x^2)} )</td>
<td>( Y = 1.29 \frac{x}{(1 + 1.22 x^2)} )</td>
</tr>
<tr>
<td>M40 SCC mix</td>
<td>( Y = 1.25 \frac{x}{(1 + 0.58 x^2)} )</td>
<td>( Y = 2.56 \frac{x}{(1 + 1.77 x^2)} )</td>
</tr>
<tr>
<td>M40 GGBS mix</td>
<td>( Y = 1.25 \frac{x}{(1 + 0.27 x^2)} )</td>
<td>( Y = 2.42 \frac{x}{(1 + 1.62 x^2)} )</td>
</tr>
<tr>
<td>M40 GGBS-RHA mix</td>
<td>( Y = 1.09 \frac{x}{(1 + 0.22 x^2)} )</td>
<td>( Y = 2.08 \frac{x}{(1 + 1.30 x^2)} )</td>
</tr>
<tr>
<td>M60 SCC mix</td>
<td>( Y = 1.41 \frac{x}{(1 + 1.0 x^2)} )</td>
<td>( Y = 2.27 \frac{x}{(1 + 1.47 x^2)} )</td>
</tr>
<tr>
<td>M60 GGBS-RHA mix</td>
<td>( Y = 1.64 \frac{x}{(1 + 0.33 x^2)} )</td>
<td>( Y = 2.59 \frac{x}{(1 + 1.80 x^2)} )</td>
</tr>
</tbody>
</table>

Where \( Y = \frac{\sigma}{\sigma_0} \) and \( x = \frac{\varepsilon}{\varepsilon_0} \)

These normalized stress-strain curves are used in further investigations. The proposed empirical equations can be used as stress block in analyzing the flexural behaviour of sections of SCC structural elements. The proposed equations have shown good correlation with experimental values.

### 6.2 Calculation of Theoretical Stresses Using Proposed Analytical Equations

Theoretical stresses have been calculated using proposed empirical equations for SCC with and without steel which are derived from Seanz’s model in the form of
\[ Y = \frac{AX}{1+BX^2} \]

Where \( Y = \frac{\sigma}{\sigma_0} \) and \( X = \frac{\varepsilon}{\varepsilon_0} \)

Substituting

\[ \sigma = \frac{A \varepsilon / \varepsilon_0}{1+B \varepsilon / \varepsilon_0} \]

\[ \Rightarrow \sigma = \frac{\sigma_0 A \varepsilon / \varepsilon_0}{1+B \varepsilon / \varepsilon_0^2} \]

Let \((A \sigma_0) / \varepsilon_0 = A^1\) and \(B / \varepsilon_0^2 = B^1\)

Then \( \sigma = \frac{A^1 \varepsilon}{1+B^1 \varepsilon^2} \)

Where \( \varepsilon_0 \) - is the strain corresponding to peak stress \( \sigma_0 \)

\( \sigma \) - is the stress corresponding to any strain \( \varepsilon \)

A & B - are constants for normalized stress-strain curves.

\( \sigma_0 \) - Corresponds to cylinder strength (taken as) = 0.8 \( f_{ck} \)

\( \varepsilon_{0.85} \) - is strain corresponding to 85% peak stress on the descending portion of stress-strain curve

From the experimental stress-strain curves, value of \( \varepsilon_0 \) is observed to be 0.0074 for M 20 SCC mixes, 0.0076 for M 40 mixes and for M 60 mixes this value is 0.0064.

If \( A, B, \sigma_0 \) and \( \varepsilon_0 \) values are known, the constants \( A^1 \) and \( B^1 \) are determined using the relationships \( A^1 = (A \sigma_0) / \varepsilon_0 \) and \( B^1 = B / \varepsilon_0^2 \)

Substituting the values of \( \varepsilon \) i.e. strain at extreme fibre of concrete, theoretical stress values at different values of \( \varepsilon \) are determined using the relationship

\[ \sigma = \frac{A^1 \varepsilon}{1+B^1 \varepsilon^2} \]
6.2.1 Development of Theoretical Stress-Strain Curves for SCC with and without steel.

After developing empirical equations for stress-strain curves of SCC, theoretical values of stresses are calculated at different values of strains in concrete based on the developed empirical equations as given above and theoretical stress-strain curves are plotted. These theoretical stress-strain curves are compared with experimental stress-strain curves and found that, theoretical stress-strain curves have shown good correlation with experimental stress-strain curves for all SCC mixes.

6.3 THEORETICAL COMPUTATION OF MOMENTS AND CURVATURES

Beams of under-reinforced and over-reinforced are tested under third point loading and its load-deflection, moment-curvature behaviour were investigated.

The experimental results of moments have been analysed by developing procedures for obtaining the complete theoretical moment-curvature diagrams. The models proposed for stress-strain behaviour of SCC mixes are used as the basis for prediction of the analytical behaviour of moment-curvature and in deriving the expressions of the resisting moments and curvatures. For obtaining the complete moment curvature relationship for any cross-section, discrete values of concrete strains (€) were selected such that even distribution of points on the plot, both before and after maximum was obtained.
The procedure used in the computation is given below.

i) Calculation of neutral axis depth for given values of concrete strains (€)

ii) Calculation of moment carrying capacities (M)

iii) Calculation of theoretical moment curvature values

**6.3.1 Calculation of neutral axis depth**

The value of “nd” is calculated by equating force in tension and force in compression

\[
\frac{b \cdot nd}{C} \int_0^\varepsilon \sigma \, d\varepsilon
\]

Where

\[
\sigma = \frac{A_1 \cdot C}{1 + B_1 \cdot C^2}
\]

\[
\int \sigma \, d\varepsilon = \frac{A_1}{2B_1} \log (1 + B_1 \cdot C^2)
\]

As two separate equations are proposed for ascending and descending portions of the stress-strain curve
\[ C_c = \frac{1}{b} \frac{\sigma_1}{\varepsilon} \left\{ \int_{\varepsilon_0}^{\sigma_1} \varepsilon_1 \, d\varepsilon + \int_{\sigma_1}^{\sigma_2} \varepsilon_2 \, d\varepsilon \right\} \]

\( \sigma_1 \) - Corresponds to ascending portion of stress-Strain curve

\( \sigma_2 \) - Corresponds to descending portion of stress-strain curve

Force in tension \( (T_s) \) can be obtained by \( T_s = A_{st} \cdot \sigma_{st} \)

Where \( \sigma_{st} \) - is stress in steel and \( A_{st} \) - area of tensile reinforcement

The strains in tension steel are calculated based on the strain compatibility and \( \sigma_{st} \) is arrived by taking the corresponding stresses from the stress-strain diagram of steel.

Knowing the values of \( A^1 \) and \( B^1 \) for ascending and descending portions of stress-strain curves of SCC mixes, for known values of strain in extreme fibre of concrete \( \varepsilon \) (say 0.0002, 0.0004 ---) the values of \( C_c \) and \( T_s \) can be determined.

“Initially assuming \( n = 0.5d \) and \( \varepsilon = 0.0002 \), \( C_c \) and \( T_s \) can be determined. For equilibrium \( C_c = T_s \) should be satisfied. Hence for a given value of \( \varepsilon \), by changing the values of \( n \) till \( C_c = T_s \) is satisfied, the neutral axis depth coefficient for that given value of \( \varepsilon \) can be determined. By repeating the process, values of \( n \) for different values of \( \varepsilon \) can be determined. This is continued till \( \varepsilon = \varepsilon_{0.85} \) or \( 1.9\varepsilon_0 \).

Using the above procedure, for \( \varepsilon = 0.0002 \) the value of ‘n’ which satisfies \( C_c = T_s \) is calculated. By changing the values of \( \varepsilon \) with increments of 0.0002 corresponding values of n are calculated.
6.3.2 Moment of Resistance Offered by the Materials

From the basics stress-strain compatibility of steel and concrete the theoretical moment-curvatures have been developed. The moment of resistance offered by the concrete \( (M_c) \) can be determined by

\[
M_c = b \left( \frac{nd}{\varepsilon} \right)^2 \int \sigma \varepsilon \, d\varepsilon
\]

Where

\[
\sigma = \frac{A^1 \varepsilon}{1 + B^1 \varepsilon^2}
\]

\[
\int \sigma \varepsilon \, d\varepsilon = \frac{A^1}{B^1} \varepsilon - \frac{A^1}{B^1} \tan^{-1} \frac{1}{B^1 \varepsilon}
\]

As two separate equations are proposed for ascending and descending portions of the stress-strain curve

\[
M_c = b \left( \frac{nd}{\varepsilon} \right)^2 \left\{ \int_0^{C_0} \sigma_1 \varepsilon \, d\varepsilon + \int_0^{C_{0.85}} \sigma_2 \varepsilon \, d\varepsilon \right\}
\]

\[
= bd^2 \left( \frac{n}{\varepsilon} \right)^2 \left\{ \int_0^{C_0} \sigma_1 \varepsilon \, d\varepsilon + \int_0^{C_{0.85}} \sigma_2 \varepsilon \, d\varepsilon \right\}
\]

For the given values of \( b, d \) and \( \varepsilon \), the value of \( M_c \) for different values of \( n \) can be determined using above relationship.

The moment of resistance offered by steel \( (M_s) \) can be determined by

\[
M_s = T_s (d - nd)
\]

\[
= \sigma_{st} \times A_{st} (d - nd)
\]

As the values of \( b, d, A_{st}, A^1, B^1 \), neutral axis depth and \( T_s \) for different values of \( \varepsilon \) are already known, just by substituting these values in the expressions of \( M_c \) and \( M_s \), the moment of resistance offered by concrete and steel can be determined.
6.3.3 Calculation of theoretical moment-curvature values

The moment of resistance of the section can be determined by moment of resistance offered by concrete or steel. Thus

\[ M \]  - Moment of resistance of the section

**Calculation of theoretical curvature:**

Knowing the values of \( \varepsilon \) and \( n \), theoretical curvature \( (\phi) \) can be determined from strain distribution diagram. The curvature is given by the relationship \( \phi = (\varepsilon / nd) \). The values of \( M \) are the theoretical moment values, knowing the values of \( \varepsilon \) and \( n \) theoretical curvatures are calculated using the relation \( \phi = (\varepsilon / nd) \) and the results are tabulated.

Thus theoretical moment curvature values are calculated for all sixteen SCC beams and results are shown in tables 8.6.1 to 8.6.8. From these results theoretical M-\( \phi \) diagrams are plotted for all SCC beams and are shown in figures 8.6.1 to 8.6.8.

![Moment Curvature Diagram](image)

**Fig. 6.3.4.  STRESS BLOCK PARAMETERS**

The area under the normalized stress-strain diagrams for SCC is calculated to evaluate the stress-block parameter. The coefficient of stress block parameter is obtained as 0.46 for M20, 0.49 for M40 and
0.44 for M60. This coefficient is used in estimating the flexural strength SCC beams.

The section of the beam, strain distribution and stress blocks are shown below.

Fig. 6.3.5 Cross section of beam, strain distribution and stress block for SCC M20, M40 & M60