SYNOPSIS

This is the synopsis of the entitled “THE COMPARATIVE STUDY OF COLLOCATION METHOD-SPLINE, FINITE DIFFERENCE AND FINITE ELEMENT METHOD FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS” to be submitted to the Veer Narmad South Gujarat University, Surat for the award of Ph.D. degree.

The present study emphasizes the use of spline collocation, finite difference and finite element method to various types of Partial Differential equations (PDE) under different set of boundary conditions. We can decide which method is more appropriate from concerned aspect by comparing the results. The PDEs can be transformed to ordinary differential equations (ODE) using similarity transformations and then the solution of the physical problem can be obtained with due compromises with the original problems in many cases. In order to avoid such a situation, we must solve the given problem keeping it as a PDE only. Here attempts are made to solve the problems in the form of PDE only and it is successfully applied to
various types of physical problems. The thesis demonstrates a method to solve linear PDE.

The thesis comprises of five chapters. The first chapter represents a general discussion about approximate and numerical method for solving PDE. The rapid development of high speed digital computer and the advanced technology fulfill demands of numerical methods. As the majority of the differential equations in science and engineering are quite difficult to solve analytically, we have to be careful while selecting the method when analytical solution are not available. Generally the method is selected which requires minimum number of steps, consumes the shortest computing time and does not produce excessive errors. In this chapter we also discuss the finite difference approximation method for solving PDE and classification of PDEs.

The second chapter gives a survey of spline functions. The spline functions were used in the field of differential equation initially by W.G.Bickly in 1969. The cubic splines were expressed in the terms of infinite power series. Ahlberg et al suggested the use of cardinal splines
for solving non-linear boundary value problems (BVP) in 1969. Also **J. L. Blue** in 1969 used the cubic spline consisting of the moments to solve second order non-linear differential equation over a restricted domain merely the unit interval [0, 1]. The simplicity of the method of spline collocations is that a problem of differential equations remains no more problem involving clumsy integrations of different orders but it translates it to an algebraic techniques. By comparing finite difference and spline collocation it shows that spline collocation method has one type of analogy with finite difference technique.

This chapter deals with finite element method. In 1941 **Hrenikoff** introduced the so-called frame work method, in which a plane elastic-medium was represented as a collocation of bars and beams. In 1943 the use of piecewise continuous function defined over a sub-domain to approximate the unknown function back to the work of **Courent**, who used an assemblage of triangular elements and principle of minimum potential energy to study the St-Venant torsion problem. The term finite element was first used by **Clough** in 1960. The basic idea behind the finite
element method is to replace a continuous function by piecewise polynomials. Two important variational method Rayleigh Ritz and Galerkin technique, temporal approximation of first and second order of time derivatives and finite element method for solving PDE with one as well as two dimensional are discussed. Here we use linear rectangular element for solving PDE with two space variables.

Chapter three demonstrates an application of spline function to solve parabolic PDEs with one as well as two space variables. The flow of electricity in a transmission lines, heat conduction in a rod of finite length and heat flow in a rectangular thin plate are discussed and solved by cubic spline explicit and implicit methods. It also discusses an application of spline collocation technique to solve hyperbolic partial differential equation with one and two space variables. The flow of electricity in a transmission lines, vibrating string and the vibrating membranes of finite length as well as width are discussed as the case study. The solutions are obtained by explicit and implicit cubic spline methods and compared with the exact solution. They are
quite satisfactory. The solutions of the problems are presented in tabular as well as graphical forms, proving the reliability of spline collocation method. The general conclusion is that spline implicit method gives better results than the explicit method. The computation can be carried out by a desk calculator even for an accuracy of five digits. The cumbersome integrations and tedious computations are avoided by this spline collocation technique and lot of computer time as well as labour is saved.

The fourth chapter deals with an application of finite difference technique to solve parabolic and hyperbolic PDEs with one and two space variables. The problems discussed in previous chapter are solved by finite difference explicit and implicit schemes with same parameters. The results are compared with exact as well as spline solutions, which indicate the accuracy of spline solutions. The solutions of the problem are presented in tabular and graphical form exhibit the reliability of spline collocation method. In all the cases, results are accurate up to five decimal places. It is concluded that reduction of the length of sub-interval give more closed results.
Finally, the fifth chapter deals with finite element models to solve parabolic and hyperbolic PDE with one as well as two space variables. The problems which we have discussed in previous chapter are solved by finite element method. The solutions obtained by finite element method are compared with spline explicit and implicit as well as exact solution. The solutions of the problem are presented in tabular form along with the graphs establishing the reliability of spline collocation method. If the step size is decreased, then spline solutions converge to the analytical solutions. When the length of sub-interval reduces, calculation in finite element method becomes quite lengthy and difficult. So it is seen that spline solutions are quite accurate and reliable.

The present work justifies the application of spline collocation and reliability of the method. We conclude that spline explicit and implicit schemes are more effectives and quite encouraging than finite difference as well as finite element method. It will be possible that same technique can be applied for higher order partial differential equation.
Thus the wide applicability of spline collocation is sought in this thesis.

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