CHAPTER 3

GPS RECEIVER SIGNAL PROCESSING

3.1 INTRODUCTION

The basic principle of operation of the GPS is the measurement of distance between the satellites and the receiver. GPS utilises the concept of Time-of-Arrival (ToA) ranging to determine the user position. The timing information embedded within the satellite ranging signal allows the receiver to calculate when the signal has left the satellite antenna. From this information, along with the received signal time, the receiver measures the time taken by the signal transmitted by the satellite at a known location in the space to reach the user. This time interval is called signal propagation time or travel time. For exact measurement of the true propagation time of a signal from the satellite to the receiver, the clocks in the satellite and the receivers must be perfectly synchronised. The propagation time is then multiplied by the velocity of the signal (3×10⁻⁸ m/sec) to obtain the satellite to receiver distance. The receiver estimates an apparent range or pseudorange to each satellite (SV) by measuring the transit time of the signal. By making TOA measurements to multiple satellites, three dimensional (3D) positioning is achieved. The important steps followed in the GPS receiver signal processing are signal conditioning, acquisition, tracking, navigation data recovery and computation of user position. In this chapter, the processing of the signal in a GPS receiver, the navigation message format
and the observation data format which are required for processing the data are explained.

3.2 GPS RECEIVER

The GPS receiver captures the satellite transmitted signals in view, separates the signals of the visible satellites, performs the measurement of the Doppler shift and signal transit time, decodes the navigation message and determines the satellite position, velocity and clock parameters. The simple generic GPS receiver block diagram is given in Fig. 3.1 below.

![Fig. 3.1 Simplified generic GPS receiver](image)

The GPS receiver consists of a suitable antenna. This antenna receives extremely weak satellite signals on frequencies, L1 (1575.42MHz) and L2 (1227.60MHz). The signal output is around 163dBW. Some GPS receiver antennas have a 3-dB gain. The antenna is followed by an RF front end, where the Low Noise Amplifier (LNA) amplifies the signal by approximately 15dB to 20dB. The RF front end also consists of a High Frequency (HF) filter that reduces the effect of signal interference. The amplified GPS signal is
mixed with the frequency of the local oscillator (LO) to generate Intermediate Frequency (IF) signals. The filtered IF signal is maintained at a constant amplitude level via amplitude gain control (AGC). Since most receivers are implemented in software, A/D convertors are introduced before the acquisition and tracking blocks. The purpose of acquisition is to identify all the satellites visible to the user and determine the frequency and phase of the signal. Once the signal from the satellite is acquired, tracking is done using parallel channels. The main purpose of tracking is to refine the coarse values of code phase and frequency and to keep track of these as the signal properties change with time. When the signals are properly tracked, the C/A code and the carrier wave can be removed from the signal, leaving only the navigation data bits. The final task of the receiver is to compute the user position which is computed from pseudoranges and satellite positions found from ephemeris data extracted from the navigation data.

The schematic functional block diagram of a generic GPS receiver is shown in Fig. 3.2. The generic receiver consists of the following blocks:

(i) Antenna
(ii) Preamplifier
(iii) Reference oscillator
(iv) Frequency synthesizer
(v) Down converter
(vi) Intermediate frequency (IF) amplifier
(vii) Signal processing section and
(ii) Application processing section

Fig. 3.2 Schematic functional block diagram of the generic GPS receiver

In order to cover maximum number of satellites at elevation angles greater than 50°, the antenna should have wide beam width. But wide beam width usually results in jamming and interference. Therefore, in selecting a GPS antenna, a trade-off between the maximum number of receiving satellites and interference must be evaluated. Also, it should have uniform gain to distinguish the strong signals from the weak. The preamplifier generally consists of burnout protection, a filter section and a low noise amplifier (LNA). Its primary function is to set the receiver's noise figure and to reject the out-of-band interference. The reference oscillator provides time and frequency reference to the receiver. Because GPS receiver measurements are based on the time-of-arrival of pseudo random noise (PRN) code phase, received carrier phase and frequency information, the reference oscillator is a key functional component of the receiver. The reference oscillator output is
used in the frequency synthesizer from which it derives the local oscillator and clock frequencies required by the receiver. One or more of these local oscillator frequencies are used by the down converter to convert the radio frequency (RF) inputs to IFs that are easier to process in the IF section of the receiver.

The purpose of the IF section is to provide further filtering of the out-of-band noise and interference and to increase the amplitude of the signal-plus-noise to a workable signal processing level. The IF section may also contain automatic gain control (AGC) circuits to control the workable level in order to provide adequate dynamic range and to suppress the pulse type interference (Enge and Van Dierendonck, 1996). The signal processing function of the receiver is the core of a GPS receiver, performing the following steps:

(i) The signal is split into multiple channels for signal processing of multiple satellites simultaneously

(ii) Generating reference PRN codes of the signals

(iii) Acquiring the satellite signals

(iv) Tracking the code and carrier of the signals

(v) Demodulating the signal data from satellite signals

(vi) Extracting the code phase (pseudorange) measurements from the PRN code of the satellite signals
Extracting the carrier frequency (pseudorange rate) and carrier phase (delta pseudorange) measurements from the carrier of the satellite signals

Extracting the signal to noise ratio (SNR) information from the satellite signals

Estimating a relationship to the GPS system time

The application processing function controls the signal processing function and uses its outputs to satisfy the application requirements. These requirements vary with the kind of application. Although, GPS is primarily a satellite based navigation system, the applications of a GPS receiver are diverse. Some other applications with significantly differing processing requirements are as follows:

Time and frequency transfer

Static and kinematic surveying

Ionospheric total electron content (TEC) and amplitude and phase scintillation monitoring

Differential GPS (DGPS) reference station receivers

GPS satellite signal integrity monitoring

The common link between these diverse applications is that they all use similar signal-processing measurements in one form or another. However, because of bandwidth and accuracy requirements imposed by various applications, the requirements on signal processing function also differ. In general, GPS receivers do not meet the signal and application
processing requirements for all the applications. Special processing is required for some applications.

The GPS L1 signal has the C/A signal in phase and the P signal in quadrature phase. The C/A signal is stronger than the P signal by 3dB. It has the form

\[ S(t) = \sqrt{2P_{C/A}}C_{C/A}(t)d(t)\cos(\phi + 2\pi f_{L1}t) + \sqrt{2P_p}C_{P}(t)d(t)\sin(\phi + 2\pi f_{L1}t) \]  

(3.1)

where, \( P_{C/A} \) and \( P_p \) are the C/A and P signal powers respectively,

\( C_{C/A} \) and \( C_P \) are the C/A and P signal codes respectively,

\( d \) is the navigation data,

\( f_{L1} \) is the L1 carrier frequency and

\( \phi \) is an initial phase.

The frequency down conversion procedure in a typical GPS receiver is given below. The received signal goes through several stages in the receiver before it is processed to acquire and track the visible satellites. The RF signal is received by a circularly polarized Omni directional antenna. The signal then goes through the receiver front end i.e. Low-Noise Amplifier (LNA) and some filters to reject multipath and interference signals. Following from (3.1), the received L1 C/A code signal for one satellite has the form as shown in (3.2).

\[ r_{RF}(t) = \sqrt{2P_{C/A}}C_{C/A}(t)d(t)\cos(\phi_0 + 2\pi(f_{L1} + f_d)t) + n_{RF}(t) \]  

(3.2)

where, \( \phi_0 \) is the initial received phase,
$f_d$ is the Doppler shift and
$n_{RF}(t)$ is a Gaussian noise with zero mean and $\sigma_n^2$ variance.

The received RF signal is converted into an intermediate frequency (IF) signal by mixing the RF signal with a local oscillator. The local oscillator has the form

$$LO(t) = 2 \cos(2\pi f_{LO} t)$$

(3.3)

where, $f_{LO}$ is the frequency of the local oscillator.

The mixing process will result in a signal with some harmonics along with the upper and lower sidebands (i.e., sidebands with centre frequencies of $(f_{L1}+f_{LO})$ and $(f_{L1}-f_{LO})$ respectively). A band pass filter is used to remove the harmonics and a low pass filter is used to remove the upper sideband. The centre frequency of the lower sideband is the IF carrier frequency, $f_{IF}$. The resulting signal will have the form

$$r_{IF}(t) = \sqrt{2P_{C/A}C_{C/A}(t)d(t)\cos(\phi_0 + 2\pi(f_{IF} + f_d)t)} + n_{IF}(t)$$

(3.4)

where $f_{IF} = f_{L1} - f_{LO}$

Example calculation:

In general practice, all GPS receivers are built using a triple down conversion technique. Fig. 3.3 shows the block diagram of a single front-end with its three stage mixers. This allows the rejection of large out-of-band jamming signals. As observed from the figure, it contains three mixers at 1.4GHz, 140MHz and 31.11MHz.
The satellite transmitted signal is received by the RF front-end section i.e., the Low Noise Amplifier (LNA) via the GPS antenna. In the first stage, the signal is down converted to 175.42MHz and filtered with a double couple resonator that allows rejection of the image frequency and large out-of-band jamming signals. The second stage mixer uses a 140MHz frequency generated by the local oscillator to take the signal down to 35.42MHz. At this instant, selective filtering of the L1 signal is done. The output of this filter then feeds the main IF amplifier and arrives at the third stage mixer. A local oscillator signal of 31.11MHz provides a final IF at 4.309MHz. The output is filtered by a filter centered at 4.309MHz.

3.3 DATA FORMATS

3.3.1 Navigation message format

The navigation message transmitted by each GPS satellite contains the ephemeris of that satellite and the almanac data of all the satellites in
the GPS satellite constellation. The ephemeris is the satellite orbital parameters that include the satellite’s position, velocity, and clock bias parameters. The almanac includes reduced precision information about all the GPS satellites in the constellation. Navigation message is a continuous data stream transmitted at the rate of 50bps (bits per second). The navigation message is required to calculate the current position of the satellites and to determine signal travel times. The navigation message from each satellite carries the following information to earth (Rao 2010):

(i) Satellite time of transmission
(ii) Satellite position (determined from the broadcast orbital data (ephemeris))
(iii) Satellite health
(iv) Satellite clock correction
(v) Propagation delay effects
(vi) Time transfer to UTC
(vii) Constellation status (approximate orbital data for all other satellites (almanac))

Navigation data has the following characteristics:

(i) Data is a continuous stream of 50bps
(ii) It is modulated onto the carrier wave of each individual satellite
(iii) It is transmitted in logically grouped units known as frames or pages
(iv) A complete navigation message consists of 25 frames (pages)
(v) Each frame is 1500 bits long and takes 30 seconds for transmission
(vi) The frames are divided into 5 subframes

(vii) Each subframe is in turn divided into 10 words, each containing 30 bits
- Each subframe is 300 bits long and takes 6 seconds for transmission
- Each subframe begins with a telemetry (TLM) word and a handover word (HOW)

(viii) Transmission time for the entire data is therefore 12.5 minutes.

The structure of the navigation message is illustrated in Fig. 3.4.

---

**Fig. 3.4 Structure of the navigation message**

A frame is divided into five subframes, each subframe is transmitting different information.

(i) Subframe 1 contains the time values of the transmitting satellite including the parameters for correcting signal transit delay, onboard
clock time as well as information on satellite health and an estimate of the positional accuracy of the satellite. It transmits the 10-bit week number. The GPS time began on Sunday, 6th January, 1980 at 00:00:00 hours. The week number restarts at 0 for every 1024 weeks, called a ‘week rollover’ (Rao 2010). The first rollover of the GPS week number occurred on August 22, 1999 and the next will occur in April 2019.

(ii) Subframes 2 and 3 contain the ephemeris data of the transmitting satellite. This data provides extremely accurate information on the orbit of the satellite.

(iii) Subframe 4 contains the almanac data on satellite numbers 25 to 32 (each subframe can transmit data from one satellite only), the difference between GPS and Universal Coordinated Time (UTC) (leap seconds or UTC offset) and information regarding any measurement errors that are caused by the ionosphere.

(iv) Subframe 5 contains the almanac data on satellite numbers 1 to 24 (each subframe can transmit data from one satellite only). All 25 pages are transmitted along with the information on the health of satellite numbers 1 to 24.
### 3.3.2 The Navigation data format

The navigation data format of the GPS receiver is as shown in Table 3.1.

<table>
<thead>
<tr>
<th>Navigation record</th>
<th>Description</th>
<th>Format</th>
<th>Example value with Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRN/Epoch</td>
<td>Satellite PRN number</td>
<td>I2</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Time of clock: Year</td>
<td>I2.2</td>
<td>10 (Year 2010)</td>
</tr>
<tr>
<td></td>
<td>Month</td>
<td>I2</td>
<td>2 (February Month)</td>
</tr>
<tr>
<td></td>
<td>Day</td>
<td>I2</td>
<td>19 (Day)</td>
</tr>
<tr>
<td></td>
<td>Hour</td>
<td>I2</td>
<td>2 (Hour)</td>
</tr>
<tr>
<td></td>
<td>Minute</td>
<td>I2</td>
<td>0 (Minute)</td>
</tr>
<tr>
<td></td>
<td>Second</td>
<td>F5.1</td>
<td>0.0 sec</td>
</tr>
<tr>
<td>Satellite clock</td>
<td>$a_0$ (Satellite clock bias)</td>
<td>D19.12</td>
<td>1.30936503410D-03 sec</td>
</tr>
<tr>
<td></td>
<td>$a_1$ (Satellite clock drift)</td>
<td>D19.12</td>
<td>0.284217094304D-11 sec/sec</td>
</tr>
<tr>
<td></td>
<td>$a_2$ (Satellite clock drift rate)</td>
<td>D19.12</td>
<td>0.000000000000D+00 sec/sec</td>
</tr>
<tr>
<td>Broadcast orbit - 1</td>
<td>IODE (Issue of data ephemeris)</td>
<td>D19.12</td>
<td>0.520000000000D+02</td>
</tr>
<tr>
<td></td>
<td>$C_{rs}$</td>
<td>D19.12</td>
<td>-0.458750000000D+02 meters</td>
</tr>
<tr>
<td></td>
<td>$\Delta n$ (Mean motion difference from computed value)</td>
<td>D19.12</td>
<td>0.459804867004D-08 rad/sec</td>
</tr>
<tr>
<td></td>
<td>$M_0$ (Mean anomaly at reference time)</td>
<td>D19.12</td>
<td>-0.828793474803D+00 radians</td>
</tr>
<tr>
<td>Broadcast orbit - 2</td>
<td>$C_{uc}$ (Eccentricity (e))</td>
<td>D19.12</td>
<td>-0.242143869400D-05 radians</td>
</tr>
<tr>
<td></td>
<td>$C_{us}$</td>
<td>D19.12</td>
<td>0.517541018780D-02</td>
</tr>
<tr>
<td></td>
<td>Square root of the semi-major axis ($\sqrt{a}$)</td>
<td>D19.12</td>
<td>0.899285078049D-05 radians</td>
</tr>
<tr>
<td></td>
<td>$\Omega_0$ (Mean of daily motion at weekly epoch)</td>
<td>D19.12</td>
<td>0.515356405830D+04 (\sqrt{\text{meter}})</td>
</tr>
<tr>
<td>Broadcast orbit - 3</td>
<td>TOE (Time of ephemeris)</td>
<td>D19.12</td>
<td>0.439200000000D+06 sec</td>
</tr>
<tr>
<td></td>
<td>$C_{ic}$</td>
<td>D19.12</td>
<td>0.163912773123D-06 radians</td>
</tr>
<tr>
<td></td>
<td>$\Omega$ (Longitude of ascending node of orbit plane at weekly epoch)</td>
<td>D19.12</td>
<td>0.96679653185D-01 radians</td>
</tr>
<tr>
<td></td>
<td>$C_{is}$</td>
<td>D19.12</td>
<td>0.391155481339D-07 radians</td>
</tr>
<tr>
<td>Broadcast orbit - 4</td>
<td>$i_0$ (Inclination angle at reference time)</td>
<td>D19.12</td>
<td>0.960514963909D+00 radians</td>
</tr>
<tr>
<td></td>
<td>$C_{ic}$</td>
<td>D19.12</td>
<td>0.207750000000D+03 meters</td>
</tr>
<tr>
<td></td>
<td>$\omega$ (Argument of perigee)</td>
<td>D19.12</td>
<td>-0.257473721952D+01 radians</td>
</tr>
<tr>
<td></td>
<td>$\Omega$ (Rate of right ascension)</td>
<td>D19.12</td>
<td>-0.805997858760D-08 rad/sec</td>
</tr>
<tr>
<td>Broadcast orbit - 5</td>
<td>$i$ (Rate of inclination angle)</td>
<td>D19.12</td>
<td>-0.412517182996D-09 rad/sec</td>
</tr>
<tr>
<td></td>
<td>Codes on L2 channel</td>
<td>D19.12</td>
<td>0.100000000000D+01</td>
</tr>
<tr>
<td></td>
<td>GPS week (to go with TOE)</td>
<td>D19.12</td>
<td>0.157100000000D+04 week</td>
</tr>
<tr>
<td></td>
<td>L2 P data flag</td>
<td>D19.12</td>
<td>0.000000000000D+00</td>
</tr>
<tr>
<td>Broadcast orbit - 6</td>
<td>SV accuracy</td>
<td>D19.12</td>
<td>0.000000000000D+00 meters</td>
</tr>
<tr>
<td></td>
<td>SV health</td>
<td>D19.12</td>
<td>0.000000000000D+00</td>
</tr>
<tr>
<td></td>
<td>$T_{GD}$ (Issue of data clock)</td>
<td>D19.12</td>
<td>-1.02445483208D-07 sec</td>
</tr>
<tr>
<td></td>
<td>IOEDC (Issue of data clock)</td>
<td>D19.12</td>
<td>0.520000000000D+02</td>
</tr>
<tr>
<td>Broadcast orbit - 7</td>
<td>Transmission time of message 3 - spares</td>
<td>D19.12</td>
<td>0.432006000000D+06 sec</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D19.12</td>
<td>---</td>
</tr>
</tbody>
</table>
The navigation data corresponding to 19th February, 2010 is collected from a dual frequency GPS receiver located at Andhra University College of Engineering, Visakhapatnam (17.73°N/83.319°E). The navigation data for a particular epoch i.e., at 02:00:00 hours along with the header section is as given below. The navigation data is available for every two hours only.

### Sample navigation data

<table>
<thead>
<tr>
<th>2.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>N: GPS NAV DATA</td>
</tr>
<tr>
<td>Convert</td>
</tr>
<tr>
<td>Ionosphere parameters A0-A3 unknown</td>
</tr>
<tr>
<td>Ionosphere parameters B0-B3 unknown</td>
</tr>
<tr>
<td>Delta-UTC unknown</td>
</tr>
<tr>
<td>Leap Seconds unknown</td>
</tr>
</tbody>
</table>

END OF HEADER

| 17 10 02 19 02 00 00.0 | .130936503410D-03 | .284127094304D-11 | .000000000000D+00 |
| 5200000000000D+02 | .4587500000000D+02 | .459804676004D-08 | .828793474803D+00 |
| -.242143869400D-05 | .517541018780D-02 | .899285078049D-05 | .515356405830D+04 |
| .4392000000000D+06 | .163912773132D-06 | .391155481339D-07 | .391155481339D-07 |
| .960514963909D+00 | .218761462616D-04 | .432066000000D+06 | .000000000000D+00 |

| 28 10 02 19 02 00 00.0 | .568434188608D-12 | .000000000000D+00 |
| .2700000000000D+02 | .8312500000000D+01 | .47709130811D-05 |
| -.32968819141D-06 | .158369115088D-01 | .514276325703D-05 |
| .4392000000000D+06 | -.106170773506D-06 | .157100000000D+01 |
| .969087489767D+00 | .286656250000D+00 | .849821112746D-08 |
| .84646383045D-10 | .432066000000D+06 | .000000000000D+00 |

| 09 10 02 19 02 00 00.0 | .106879044324D-03 | .216004991671D-11 |
| .144000000000D+03 | .428125000000D+01 | .390873424304D-08 |
| -.558793544769D-07 | .206922631478D-01 | .943802297115D-05 |
| .4392000000000D+06 | .27753127235D-06 | .204943917144D+01 |
| .979316982472D+00 | .202750000000D+03 | .153062182557D+01 |
| .262153776900D-09 | .100000000000D+01 | .157100000000D+01 |
| .100000000000D+01 | .000000000000D+00 | .558793544769D-08 |
| .432066000000D+06 | .000000000000D+00 |

| 08 10 02 19 02 00 00.0 | .341841951013D-05 | .34106513165D-12 |
| .630000000000D+02 | .175000000000D+01 | .376908556897D-08 |
| .30174851417D-06 | .10943625351D-01 | .1008994324304D-08 |
| .4392000000000D-05 | -.353902578354D-07 | -.19513996288D+01 |
| .991762305989D+00 | .203718750000D+03 | .30882960888D+01 |
| .261796619166D-09 | .100000000000D+01 | .157100000000D+01 |
| .100000000000D+01 | .000000000000D+00 | .41905158577D-08 |
| .432066000000D+06 | .000000000000D+00 |

| 04 10 02 19 02 00 00.0 | .183694064617D-04 | .909494701773D-11 |
| .6000000000000D+02 | .650625000000D+02 | .479591405479D-08 |
| .107959529012D+01 |
**Steps of satellite position computation:**

Using the real time navigation data corresponding to 19\textsuperscript{th} February, 2010, the satellite position is found using the following steps.

Step 1: Find the semi-major axis of elliptical orbit

\[ a = \left( \sqrt{a} \right) \text{ (meter)} \]

Example value: 26559222.5030016m

Step 2: The GPS time of transmission, \( t \), corrected for transit time

\[ t = t_{sv} - \Delta t_{sv} \]

Example value: 439200sec

Step 3: Calculate the time difference \( (t_k) \) between the time \( (t) \) and the epoch time \( (t_{oe}) \) and must account for the beginning or end of the week

\[ t_k = t - t_{oe}^* \]

\[
\begin{align*}
\text{If } t_k = t - t_{oe} > 302400 \text{ then } t_k &= t_k - 604800 \\
\text{If } t_k = t - t_{oe} < -302400 \text{ then } t_k &= t_k + 604800
\end{align*}
\]

where, \( t \) represent the coarse GPS system time, and the value can be obtained from Time Of Week (TOW). In addition, \( t_{oe} \) can be obtained from the ephemeris data, 302,400 is the time of half a week in seconds.

Example value: 0sec

Step 4: Calculate the mean motion,

\[ n = n_0 + \Delta n = \sqrt{\frac{\mu}{a^3}} + \Delta n \text{ (rad/ sec)} \]

Example value: 0.000145867847274776rad/sec
Step 5: From this value, the mean anomaly can be found as,

\[ M_k = M_0 + nt_k \]

Example value: 5.45439183237659deg

Step 6: The eccentric anomaly \( E_k \) can be found from Kepler’s equation \((M = E - e \sin E)\) through iteration as

\[ E_k = M_k + e \sin E_k \quad \text{with, } E_0 = M_k \]

Example value: 5.45056359095357deg

Step 7: Once \( E_k \) is obtained, the value of concise expression of the orbital radius, \( r_k \) can be found from,

\[ r_k = a(1 - e \cos E_k) \]

Example value: 26466723.7877529m

Step 8: Now let us find the true anomaly \( \nu \). The value can be found from,

\[ \nu_k = \tan^{-1}\left(\frac{\sin \nu_k}{\cos \nu_k}\right) = \tan^{-1}\left(\frac{\sqrt{1 - e^2 \sin^2 E_k}}{\cos E_k - e / (1 - e \cos E_k)}\right) \]

Example value: -0.836456656676363deg

Step 9: The argument of perigee \( (\omega) \) can be found from the ephemeris data.

Using the definition of argument of latitude, the value of

\[ \phi_k = \nu_k + \omega \]

Example value: -3.41119387619636deg

Step 10: Calculate the correction terms of argument of latitude, orbital radius and inclination angle;
\[
\begin{align*}
\delta u_k &= C_{us} \sin 2\phi_k + C_{uc} \cos 2\phi_k \\
\delta r_k &= C_{rs} \sin 2\phi_k + C_{rc} \cos 2\phi_k \\
\delta i_k &= C_{is} \sin 2\phi_k + C_{ic} \cos 2\phi_k
\end{align*}
\]

Example: \(\delta u_k = -6.69527629098947e^{-06}\) deg, \(\delta r_k = 201.828709519285\) m, \(\delta i_k = 1.20572629503287e^{-007}\) deg.

Step 11: Calculate the correction terms as follows,

\[
\begin{align*}
\nu_k &= \phi_k + \delta u_k \\
\rho_k &= r_k + \delta r_k \\
\iota_k &= i_k + \delta i_k + \dot{i}_k
\end{align*}
\]

Example value: \(u_k = -3.41120057147265\) deg, \(r_k = 26466925.6164624\) m and \(i_k = 0.96051508448163\) deg

Step 12: Compute the longitude of the ascending node \(\Omega_k\) by adding the right ascension parameter, \(\Omega_0\) and the mean earth rotation rate,

\[
\Omega_k = \Omega_0 + (\dot{\Omega} - \Omega) t_k - \dot{\Omega}_{oc}
\]

Example value: \(-0.514363534889964\) deg

Step 13: Once all the necessary parameters are obtained, the position of the satellite can be found by applying the three rotations (through \(\phi\), \(i\) and \(\Omega\)) described previously. The satellite position calculated in this equation is in the ECEF frame.

\[
\begin{bmatrix}
x_k \\
y_k \\
z_k
\end{bmatrix} =
\begin{bmatrix}
r_k \cos u_k \cos \Omega_k - r_k \sin u_k \cos i_k \sin \Omega_k \\
r_k \cos u_k \sin \Omega_r + r_k \sin u_k \cos i_k \cos \Omega_k \\
r_k \sin u_k \sin i_k
\end{bmatrix}
\]

Example value: \(x_k = -20222226.5285829\) m, \(y_k = 16068158.9113958\) m, \(z_k = 5777021.29234133\) m
3.3.3 Observation data format

The observation data corresponding to 19\textsuperscript{th} February, 2010 is collected from a dual frequency GPS receiver located at Andhra University College of Engineering, Visakhapatnam (17.73\degree N/83.319\degree E). The observation data along with the header section for a particular epoch i.e., at 00:00:00 hours is as given below. The observation data file given below is available for every 30 seconds. The observation data file consists of the following information.

Table 3.2 Observation data format

<table>
<thead>
<tr>
<th>Observation record</th>
<th>Description</th>
<th>Format</th>
<th>Example value with Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPES OF OBSERVATIONS</td>
<td>Number of different observation types stored in the file Observation types</td>
<td>I6, 4X, A2</td>
<td>8 C1, L1, D1, S1, P2, L2, D2, S2</td>
</tr>
<tr>
<td>EPOCH / SAT or EVENT FLAG</td>
<td>Year (2 digits)</td>
<td>I2.2</td>
<td>10 (Year 2010)</td>
</tr>
<tr>
<td></td>
<td>Month</td>
<td>I2</td>
<td>2 (February month)</td>
</tr>
<tr>
<td></td>
<td>Day</td>
<td>I2</td>
<td>19 (Day)</td>
</tr>
<tr>
<td></td>
<td>Hour</td>
<td>I2</td>
<td>00 (Hour)</td>
</tr>
<tr>
<td></td>
<td>Minute</td>
<td>I2</td>
<td>00 (Minute)</td>
</tr>
<tr>
<td></td>
<td>Second</td>
<td>F11.7</td>
<td>0.0000000 seconds</td>
</tr>
<tr>
<td></td>
<td>Epoch flag 0: OK</td>
<td>I1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1: Power failure between previous and current epoch</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>&gt;1: Event flag</td>
<td>I3</td>
<td>9 G04G05G02G27G08G10G28G17G09 (G indicates GPS system)</td>
</tr>
<tr>
<td></td>
<td>Number of satellites in current epoch</td>
<td>3X, A1, I2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>List of PRNs (Satellite numbers with system identifier) in current epoch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBSERVATIONS</td>
<td>Observation represented within record for each observation type</td>
<td>F14.3</td>
<td>20081000.43849 meters 105526401.06649 Full cycles -882.14149 Hz 51.000 dBHz 20080999.25047 meters 82228365.97347 Full cycles -687.38347 Hz 44.000 dBHz</td>
</tr>
</tbody>
</table>
Sample observation data

2.10 OBSERVATION DATA             G (GPS)       RINEX VERSION / TYPE
Convert AUCE DEPT. OF ECE       21-Aug-2010 03:27 PGM / RUN BY / DATE
Signal Strength values S1,S2 are in dBHz       COMMENT
MARKER NAME
MARKER NUMBER
OBSERVER / AGENCY
REC # / TYPE / VERS
ANT # / TYPE
0.0000  0.0000  0.0000 APPROX POSITION XYZ
0.0000  0.0000  0.0000 ANTENNA: DELTA H/E/N
1  1 WAVELENGTH FACT L1/2
1  1 7 G02 G03 G04 G05 G06 G07 G08 WAVELENGTH FACT L1/2
1  1 7 G09 G10 G11 G12 G13 G14 G15 WAVELENGTH FACT L1/2
1  1 7 G16 G17 G18 G19 G20 G21 G22 WAVELENGTH FACT L1/2
1  1 7 G23 G24 G26 G27 G28 G29 G30 WAVELENGTH FACT L1/2
1  1 2 G31 G32 WAVELENGTH FACT L1/2
COMMENT
8 C1 L1 D1 S1 P2 L2 D2 S2 # / TYPES OF OBSERV
30.000 INTERVAL
2010  02  19  00  00  0.0000000 GPS TIME OF FIRST OBS
2010  02  19  23  59  30.0000000 GPS TIME OF LAST OBS
Leap Seconds Unknown COMMENT
END OF HEADER

10 02 19 00 00  0.0000000  0  9G04G05G02G27G08G10G28G17G09
20081000.43849 105526401.06649 -882.14199  51.000 20080999.25047
82228365.97347 -687.38347  44.000
24972234.40643 131229976.33243 3092.35943  35.000 24972231.68041
102257116.69941 2409.62141  28.000
21435073.07849 112642092.57449 1663.50049  49.000 21435069.10946
87773057.30546 1296.23446  43.000
22909907.46948 120392292.7748 1217.69148  47.000 22909905.74244
93882241.87544 948.84844  38.000
24452022.86746 128496275.96146 -3805.96946  42.000 24452021.79741
100126951.56241 -2965.69941  31.000
23707514.61747 124583848.03947 3170.71547  49.000 237075116.48053
97078237.02042 2470.68442  33.000
21714955.69549 114112917.73049 -1196.12549  49.000 21714953.62544
88919176.94944 -932.04744  38.000
22292551.7849 117148083.02349 -838.20349  48.000 22292529.37545
91248237.53145 653.14845  41.000
22319467.03948 117289603.19948 939.98448  47.000 22319465.71945
91344991.47745 732.45745  39.000
10 02 19 00 00  0.0000000  0  9G04G05G02G27G08G10G28G17G09
20086084.31249 105553116.80149 -899.41049  51.000 20086083.11747
82249183.42647 -700.84447  44.000
24954564.13343 131137116.48053 3098.01243  35.000 24954564.18041
102184747.37151 2414.03541  30.000
21425613.94549 112592384.53549 1649.83649  49.000 21425610.07846
87734323.76646 1285.58246  42.000
22902998.50849 120356086.10248 1202.16048  46.000 22902997.08645
93783951.36345 936.75045  39.000


3.4 CONCLUSIONS

The GPS receiver processes the received satellite signals and records the extracted parameters of all the satellites (orbital parameters) and the pseudoranges computed to each satellite in a binary file. From the recorded binary file, the navigation message is written into the navigation data file and the observation information is written into observation data file. The navigation data file, observation data file and the parameters contained in these files are explained with example values. The parameters in the two files are required for determining the navigation solution. The satellite position is estimated from the navigation message file and the 5 types of pseudoranges (C/A, P1, L1, P2 and L2) are used in forming the navigation equations. The example calculation of satellite position from the ephemeris recorded in the GPS receiver is also presented in this chapter. The navigation and observation data formats explained in this chapter will be helpful in analyzing the data and for estimating the user position using the proposed navigation solutions presented in Chapter 4.