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# LIST OF SYMBOLS AND ABBREVIATIONS

## Symbols

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<tr>
<td>A</td>
<td>General system matrix</td>
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<td>$A_i$, $A_j$, $A_{ij}$</td>
<td>System matrices</td>
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<tr>
<td>$A_p$</td>
<td>Number of auxiliary polynomials</td>
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<td>$A(P^n)$</td>
<td>Peak amplitude value of unit step response of $Y(z)$ corresponding to $P_n$ for $n$-th composition.</td>
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<td>$A_{n,3}(s)$</td>
<td>Auxiliary Polynomials</td>
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<tr>
<td>$b$</td>
<td>Friction of the cart</td>
</tr>
<tr>
<td>$B$</td>
<td>Positive definite matrix</td>
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<tr>
<td>$c_1$, $c_2$</td>
<td>Learning factors</td>
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<tr>
<td>$C(z)$</td>
<td>Characteristic equation in $z$-domain</td>
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<td>$D$</td>
<td>Common denominator of the transfer function of a higher order multi input multi output system</td>
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<td>$D^2$</td>
<td>Common denominator of the transfer function of a second order multi input multi output system</td>
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<td>$D(P^n)$</td>
<td>Determinant of relational matrix $P$ for $n$-th composition</td>
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<tr>
<td>$e$</td>
<td>Error</td>
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<tr>
<td>$\dot{e}$</td>
<td>Change in error</td>
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<tr>
<td>$E$</td>
<td>Integral square error</td>
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<tr>
<td>$E(P^n)$</td>
<td>Energy of relational matrix $P$ for $n$-th composition</td>
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<tr>
<td>$Et$</td>
<td>Total number of elements computed in Routh table</td>
</tr>
<tr>
<td>$F$</td>
<td>Force applied to the cart</td>
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<tr>
<td>$F(\cdot)$</td>
<td>Family of fuzzy sets</td>
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<td>$F(s)$</td>
<td>Characteristic equation of Linear Time Invariant Continuous System represented by the Laplace variable $s$</td>
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<td>$F(S)$</td>
<td>Transformed characteristic equation in ‘$S$’ domain</td>
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f - Frequency of oscillations
f(v) - Activation function
\( f_{ij}(v_{ij}) \) - Outputs calculated from the hidden and output layer neurons
F(z) - Characteristic equation in z-domain
g - Gravitational acceleration
g_1 - Minimum value of f(v)
g_2 - Maximum value of f(v)
g_{best} - Best fitness value obtained so far by any particle in the entire population
G(p) - Transfer function of a single input single output system and Transfer function matrix of multi input multi output system in continuous domain represented by p, which is equivalent to the Laplace variable ‘s’
G_{ij}(p) - Transfer function between \( i^{th} \) input and \( j^{th} \) output of a multi input multi output system in continuous domain represented by p, which is equivalent to the Laplace variable ‘s’
G(s) - Transfer function of a single input single output system and Transfer function matrix of multi input multi output system in continuous domain represented by the Laplace variable ‘s’
G_{ij}(s) - Transfer function between \( i^{th} \) input and \( j^{th} \) output of a multi input multi output system in continuous domain
G(z) - Transfer function of a single input single output system and Transfer function matrix of multi input multi output system in discrete domain represented by the complex variable z
G_{ij}(z) - Transfer function between \( i^{th} \) input and \( j^{th} \) output of a multi input multi output system in discrete domain
h(k) - Weighting parameter
H - Maximum number of columns to be computed in Routh table
I - Unit identity matrix
I_p - Inertia of the pendulum
k - Number of rows to be computed
K - Gain parameter to be designed for system stability
K' - Second gain parameter to be designed for system stability
K_1 - Lowest real value of gain parameter
K_2 - Highest real value of gain parameter
K_3 - Mid point between the parameters K_1 and K_2
K_H - Sharpened higher limit of gain parameter K
K'_H - Sharpened higher limit of gain parameter K'
K_L - Sharpened lower limit of gain parameter K
K'_L - Sharpened lower limit of gain parameter K'
l - Length to pendulum centre of mass
L - Minimum number of columns to be computed in Routh table
m - Order of the lower order model
m_p - Mass of the pendulum
M_p - Mass of the cart
M_{ij} - Fuzzy sets
M_1, M_2 - Membership functions
n - Order of the system under consideration represented either by characteristic equation or transfer function
n_i - Number of neurons in the input layer
n_h - Number of neurons in the hidden layer
$N_a$ - Total number of elements computed in Routh table using auxiliary polynomial approach

$N_P$ - Total number of elements computed in Routh table using Pseudo Routh column polynomial approach

$N_{pa}$ - Total number of elements computed in Routh table using auxiliary polynomials extracted from Pseudo Routh column polynomial

$P$ - Relational matrix of the fuzzy system

$p_i$ - Position of particle $i$

$p_{best}$ - Position with the ‘best’ fitness value found so far by particle $i$

$P_m(s)$ - Pseudo Routh Column Polynomial constructed for $m$-th column

$P_R$ - Number of Pseudo Routh Column Polynomials

$P_s$ - Scaled (Normalized) relational matrix

$P^*_s$ - Compositional matrix computed using max-min composition for $n$-th composition

$q$ - Slope parameter

$Q$ - Output Relational matrix

$Q_s$ - Scaled output relational matrix $Q$

$r$ - Number of IF-THEN rules

$R$ - Fuzzy relation describing the fuzzy system

$R_1$, $R_2$ - Random variables in the range $[0, 1]$

$R^2(p)$ - Second order transfer function of a single input single output system and second order transfer function matrix of multi input multi output system in continuous domain represented by $p$, which is equivalent to the Laplace variable $s$
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<td>Second order transfer function of a single input single output system and second order transfer function matrix of multi input multi output system in discrete domain represented by the complex variable $z$</td>
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<td>$S$</td>
<td>Variable substituted parameter including the damping ratio</td>
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<td>$SSG$</td>
<td>Steady state gain</td>
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<tr>
<td>$TG$</td>
<td>Transient gain</td>
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<td>$T(P^n)$</td>
<td>Trace of relational matrix $P$ for $n$-th composition</td>
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<td>$u$</td>
<td>Input to the pendulum</td>
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<td>$U(k)$</td>
<td>Input signal</td>
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<td>$U_k$</td>
<td>Input at the $k$-th instant</td>
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<td>$v_i$</td>
<td>Velocity of particle $i$</td>
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<tr>
<td>$v_{ij}$</td>
<td>Net inputs calculated for the hidden and output layer neurons</td>
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<tr>
<td>$w_i(k)$</td>
<td>Product of grade of memberships</td>
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<td>$w_{ijs}$</td>
<td>Connection weights between input, hidden and output layer neurons</td>
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<td>$x$</td>
<td>Cart position coordinate</td>
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<td>$\dot{x}$</td>
<td>Velocity of the cart</td>
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<td>$X(k-1)$</td>
<td>Input at $(k-1)$-th instant</td>
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<tr>
<td>$X(k)$</td>
<td>Input at $k$-th instant</td>
</tr>
<tr>
<td>$X(k+1)$</td>
<td>Output at $(k+1)$-th instant</td>
</tr>
<tr>
<td>$X_k$</td>
<td>Fuzzy set of the states at the $k$-th time instant</td>
</tr>
<tr>
<td>$X_{k+1}$</td>
<td>Fuzzy set of the states at the $(k+1)$-th time instant</td>
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$y_t$ - Unit step time response of the lower order system at the $t^{th}$ time instant

$Y_t$ - Unit step time response of the given higher order system at the $t^{th}$ instant in the time interval $0 \leq t \leq \tau$

$Y(z)$ - Open loop all pole fuzzy system

$z$ - Discrete variable

$\xi$ - Damping ratio for system with oscillations

$\alpha_i, \alpha_k$ - Roots of the characteristic equation $F(s)$

$\sigma_k$ - Real part of the root $\alpha_k$

$\omega_k$ - Imaginary part of the root $\alpha_k$

$\lambda$ - Gain parameter

$\tau$ - Time period

$\theta$ - Pendulum angle from vertical position

$\dot{\theta}$ - Angular velocity of the pendulum

$\mu_e$ - Membership values of $e$

$\mu_{\dot{e}}$ - Membership values of $\dot{e}$

$\mu_Q$ - Membership values of $Q$

$\circ$ - Composition operator

**Abbreviations**

AMSE - Association for the advancement of Modelling and Simulation techniques in Enterprises

ANN - Artificial Neural Network

AVR - Automatic Voltage Regulator

BAM - Bidirectional Associative Memory

BPN - Back Propagation Network

CPU - Central Processing Unit

FAM - Fuzzy Associative Memory

FLC - Fuzzy Logic Controller
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<td>Inverted Pendulum</td>
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<tr>
<td>ISE</td>
<td>Integral Square Error</td>
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<tr>
<td>LDI</td>
<td>Linear Differential Inclusions</td>
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<tr>
<td>LHS</td>
<td>Left Half Side</td>
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<td>LMI</td>
<td>Linear Matrix Inequality</td>
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<tr>
<td>LT</td>
<td>Linear Transformation</td>
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<td>LTIS</td>
<td>Linear Time Invariant Systems</td>
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<td>LTICS</td>
<td>Linear Time Invariant Continuous Systems</td>
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<tr>
<td>LTIDS</td>
<td>Linear Time Invariant Discrete Systems</td>
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<tr>
<td>MIMO</td>
<td>Multi Input Multi Output</td>
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<td>NL</td>
<td>Negative Large</td>
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<td>NM</td>
<td>Negative Medium</td>
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<td>NS</td>
<td>Negative Small</td>
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<tr>
<td>PL</td>
<td>Positive Large</td>
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<td>Positive Medium</td>
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<td>PS</td>
<td>Positive Small</td>
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<td>Steady State Gain</td>
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<td>Transfer Function</td>
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