CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

The phenomenal growth of the internet along with the ubiquitous use of digital cameras, scanners and camera phones have made the capture, display, storage and transmission of images, a routine experience. In addition, imaging is extensively used in medicine, law enforcement, internet gaming and data collected by satellites. Inspired by the enhanced computing powers of the modern day computer and the ubiquitous presence of sensing devices, scientists are collecting and analyzing data at an ever increasing pace. Owing to natural human proclivity toward visual or pictorial representation of data, more and more of the scientific data being generated is in the form of images.

In many fields such as astronomy, medical imaging and computer vision, the data that is collected is often noisy as a result of data acquisition processes or due to natural phenomena such as atmospheric disturbances. Even acquiring an image with the use of a digital camera corrupts the image of the scene with the noise generated by the capturing media (such as CCD sensors). Furthermore, noise is added to the data when it is transmitted over transmission channels.

Noise is common to the image data generated as the data itself. The corrupting noise might result in degradation of the visual quality of the images and may also mask important image information. Even if the
perceived image do not show noise degradation due to the masking effects of the human visual systems, many image analysis tasks such as segmentation might suffer in the presence of noise. Thus, it becomes imperative that the level of the noise present in the digital images be reduced prior to any further processing.

Another aspect associated with this ever increasing generation and transmission of digital images is the storage media. Despite rapid improvements in data storage, processing speeds and digital communication system performance, this proliferation of digital media often outstrips the amount of data storage and transmission capacities. Thus the compression of such signals has assumed great importance in the use, storage and transmission of digital images. However, this compression of input images, perform in order to reduce the amount of storage space and transmission bandwidth required, also suffers in the presence of corrupting noise. There is a need for compression algorithms that can adaptively remove any noise that occurs in an input image.

Removing noise data can be considered as the process of constructing optimal estimates of the unknown signal or image from the available noisy data. Spatial filters have long been used as the traditional means of removing noise from the images and signals (Weeks 1996). These filters usually smooth the data to reduce the noise, but, in the process, also blur the data. In general, image denoising imposes a compromise between noise reduction and preserving significant image details. To achieve a good performance in this respect, a denoising algorithm has to adapt to image discontinuities. In the last decade, several new techniques have been developed that improve on spatial filters by removing the noise more effectively while preserving the edges in the image. A different class of methods exploits the decomposition of the image data into the wavelet
domain (De vore and Lucier 1992, Donoho and Johnstone 1994, Donoho and Johnstone 1995, Bruce and Gao 1996, Ogden 1997, Vidakovic 1999, Weaver et al 1999, Chang et al 2000). The wavelet representation naturally provides a useful tool in the construction of spatially adaptive algorithms that can preserve edges in an image. It compresses the essential information in an image into a few large coefficients which represent image details at different resolution scales and facilitates the removal of the corrupting noise. This sparse representation of the data in the wavelet domain also makes them ideal for compression applications. In addition, the human visual system (HVS) also employs multi resolution decomposition to process the visual images. It is this ability of the wavelet transforms to form a bridge between theory and applications, and provide a suitable representation for processing the image data, that has enabled it to emerge as an important tool for performing simultaneous compression and noise reduction in digital images.

In the wavelet domain there are two basic approaches to image denoising and they are:

i) Coefficient thresholding methods

ii) Signal coefficient estimation methods

Among these, the non linear coefficient thresholding is the most investigated. This procedure exploits sparsity property of the wavelet transform and the fact that the wavelet transform maps white noise in the signal domain to white noise in the transform domain. Thus, while signal energy becomes more concentrated into fewer coefficients in the transform domain, noise energy does not. It is this important principle that enables the separation of signal from noise. The procedure in which small coefficients are removed while others are left untouched is called hard thresholding (Donoho 1995). As some of the coefficients are removed, analytical compression is
inherently performed along with thresholding denoising. But the hard thresholding method generates spurious blips, better known as artifacts, in the images as a result of unsuccessful attempts of removing moderately large noise coefficients. To overcome the demerits of hard thresholding, soft thresholding was introduced (Donoho 1995). In this scheme, coefficients above the threshold are shrunk by the absolute value of the threshold itself. Here slightly more number of coefficients are retained after thresholding than the earlier hard thresholding procedure. The main issue in thresholding is how to choose the optimal threshold which can be adaptive or non adaptive to the image.

Considering the persistence property of the wavelet transform, the inherent analytical compression present in the thresholding operation could be improved further by accommodating the inter scale relationship. As the large/small wavelet coefficients tend to propagate through scales of the quad trees, if a wavelet coefficient at a coarse scale is insignificant with respect to the proposed threshold then all of its descendants are also insignificant and they are removed.

Another popular approach for denoising is based on constructing probabilistic models using the statistical properties of the wavelet coefficients. This approach seemed to outperform the earlier thresholding techniques and gained ground. Denoising using wavelet coefficient model approach focuses on exploiting the multi resolution properties of wavelet transform. This technique identifies close correlation of signal at different resolutions by observing the signal across multiple resolutions. This method produces excellent output but is computationally much more complex and expensive. The modeling of the wavelet coefficients can either be deterministic or statistical. Finding the choice of an accurate statistical image model is critical. But identifying an apt model for the wavelet coefficients will definitely
improve the performance. A number of researchers have developed homogeneous local probability models for images in the wavelet domain. Specifically, the marginal distributions of wavelet coefficients are highly kurtotic, and usually have a marked peak at zero and heavy tails. The Gaussian mixture model (GMM) (Chipman et al 1997) and the generalized Gaussian distribution (GGD) (Moulin and Liu 1999) are commonly used to model the wavelet coefficients distribution.

In the literature (Mihcak et al 1999) authors proposed a methodology in which the wavelet coefficients are assumed to be conditionally independent zero mean Gaussian random variables, with their variance modeled as identically distributed, highly correlated random variables. An approximate Maximum A Posteriori (MAP) probability rule is used to estimate marginal prior distribution of wavelet coefficients’ variance. From the estimated signal variance of noisy wavelet coefficients the noise free signal coefficients are estimated. Thus only if the signal variance estimate is above zero, then the corresponding signal coefficient estimate is utilized in denoised signal reconstruction. Otherwise that wavelet coefficient is identified as noisy and hence dropped. Thus here also the analytical compression is inherently carried out along with denoising. The experimental results show that both the analytical compression and denoising performance are better for this approach than the thresholding denoising.

Performance of denoising algorithms is measured using quantitative performance measures such as peak signal to noise ratio (PSNR)/mean squared error (MSE), as well as in terms of visual quality of the images. As the wavelet based denoising algorithms provide a means of analytical compression along with denoising, the proportion of retained coefficients is also included as one such performance measure. An ideal denoising procedure requires a priori knowledge of the noise, whereas a practical procedure may
not have the required information about the variance of the noise or the noise model. Thus, most of the algorithms assume known variance of the noise and the noise model to compare the performance with different algorithms. Different levels of Gaussian noise are added in the natural images to test the performance of the various denoising algorithms mentioned in this thesis.

The denoising performance of the thresholding denoising procedure may be improved by deriving an optimal threshold through stochastic modeling of wavelet coefficients. The *Bayes shrink* wavelet threshold method (Chang et al 1997) adopts Bayesian approach which assumes the knowledge of the probability distribution of the original signal and seeks to optimize the threshold for the purpose of minimizing the expected risk. The *Bayes shrink* yields results that are consistent with the human visual system where extra denoising is performed in smooth regions of the image and less denoising is performed near edges to preserve the sharpness of the image. Thus the denoised image using *Bayes shrink* is smoother and more visually appealing, besides retaining less number of significant coefficients than the conventional thresholding denoising procedures.

Jiecheng et al (2004) extended the denoising approach in a different way and improved both the analytical compression and denoising performance by adopting Minimum Description Length (MDL) principle. Assuming the Gaussian distribution as the model for the wavelet coefficients, Jiecheng considered the denoising problem as a model selection problem - one of capturing as much of the *signal* as possible, while leaving out as much of the *noise* as possible. Here the MDL principle is used to search for a threshold that reflects an optimal model. The denoising performance is comparable with the Bayes thresholding method as the MDL threshold is a sort of Bayes threshold, but improves the analytical compression than Bayes method by retaining only smaller quantity of significant coefficients. In this
MDL based hard thresholding, as the signal coefficients are identified better, only less number of significant coefficients are retained and thus the analytical compression is improved.

It is required to further improve the analytical compression of the denoising algorithms through better extraction of signal coefficients without sacrificing the denoising performance. To achieve this, new techniques need to be developed for simultaneous noise reduction and analytical compression in natural images.

In this thesis three new approaches are proposed and they are listed below.

i) Improved MDL denoising with Normalized Maximum Likelihood (NML) density
ii) Indirect MDL soft thresholding
iii) Efficient MDL with NML density soft thresholding

The first proposed algorithm namely Improved MDL denoising with NML density is based on the normalized maximum likelihood density incorporated MDL principle. The MDL principle with NML density identifies the better optimal model than conventional MDL principle, thus slightly better PSNR compared to the Jiecheng Xie et al (2004) thresholding is obtained. As the best possible model is selected the proposed threshold achieves smaller proportion of significant coefficients than simple MDL denoising.

The second proposed approach, Indirect MDL soft thresholding is the modified version of the Jiecheng Xie MDL based hard thresholding denoising algorithm. It interprets separate Gaussian densities for the non informative and informative coefficients and finds out the proportion of
informative part in each coefficient. Thus the noisy wavelet coefficients are modified such that the proportion of informative part in each wavelet coefficient is revealed. These modified coefficients are then subjected to Jiecheng Xie MDL hard thresholding. Due to this two fold procedure the noise in the image is effectively removed, of course by retaining still reduced proportion of significant coefficients than Jiecheng Xie MDL hard thresholding. This approach is data dependent and since it is completely characterized by the properties of the MDL hard thresholding solution, it does not require any additional parameters to be estimated.

In the third proposed algorithm, Efficient MDL with NML density soft thresholding, MDL with NML density denoising is extended with the soft thresholding modification. This results in the finer extraction of the informative portion from the noisy coefficients than the second proposed approach and thus strongly influences further reduction in the proportion of retained coefficients used for denoised image reconstruction while maintaining the denoising performance same.

All of the above three proposed algorithms are briefly accounted in the forthcoming sections.

1.2 IMPROVED MDL DENOISING WITH NML DENSITY

The NML probability plays a prominent role in the MDL approach to statistical inference. The NML distribution involves a normalizing sum over all the possible data samples of a fixed size. As the NML distribution has several theoretical optimality properties, Rissanen (1997) quoted that it is a very attractive candidate for performing model class selection. MDL denoising is a model selection problem and aims at minimizing the stochastic complexity of the data. The NML code identifies the more optimal model than MDL principle. As the best possible model is selected, the regularities of
the data is better identified and learnt (compressed), hence the MDL principle incorporated with NML density gives the shortest description of the data achievable with a given model class.

Here the Gaussian distribution is assumed as the model for the wavelet coefficients and the signal variance is estimated. A new spatially adaptive threshold in terms of the estimated variance is proposed and derived using the MDL principle with NML density. It is observed that the proposed threshold is more than the Jiecheng Xie (2004) threshold. As the value of the threshold increases, still more coefficients are likely to be removed resulting in still lesser number of significant coefficients. This results in a greater compression gain (reduced proportion of retained significant coefficients) than the Jiecheng Xie (2004) MDL hard thresholding procedure without any degradation in the PSNR values.

The performance of the proposed technique is compared with those of four major conventional hard thresholding approaches- Jiecheng Xie’s MDL hard thresholding, Bayes hard thresholding, Universal hard thresholding and Sure hard thresholding. Universal hard thresholding and the Sure hard thresholding procedures results in moderate PSNR values and they retained comparatively more number of significant coefficients as they do not involve probabilistic models for the noisy wavelet coefficients. Both MDL principle based methods yield comparable PSNR values with the Bayes method as the MDL threshold is a sort of Bayes threshold. For example, in the LENA image (added noise variance $\sigma^2 = 0.01$) with two levels of wavelet decomposition, Bayes hard thresholding retains around 46%, Jiecheng Xie’s MDL hard thresholding retains 33% and the proposed method retains only 10% of the total wavelet coefficients to produce the equivalent denoising performance. Figure1.1 shows the visual comparison of the LENA image for the various denoising methods. The edge features are better revealed in the proposed
method than other conventional approaches including the MDL (Jiecheng Xie et al 2004) method. Despite the fact that these edges have high frequency content, they are not regarded as noise and can be simply compressed by the use of the retained coefficients.

Figure 1.1 Visual comparison of LENA image in the proposed Improved MDL denoising with NML density (a) Noisy ($\sigma^2 = 0.01$) (b) SURE hard (c) Universal hard (d) Bayes hard (e) MDL hard (f) Improved MDL with NML density hard (proposed)

Rather using the square windows in all the detailed subbands for signal variance estimation, the proposed method is also tested with directional edge adaptive rectangular windows in the relevant detailed subbands. It is revealed that by using the rectangular windows the proportion of the retained coefficients is still reduced without degrading the PSNR.

The proposed MDL based procedure is checked up with more levels of wavelet decomposition. It is observed that as the level of decomposition is increased by one, PSNR is improved (at the expense of computations) except
for low added noise variance level. The significant coefficients are better identified in both levels of decomposition which improves the analytical compression. But this may not be true for all higher levels of wavelet decomposition as the compression may get saturated at one point.

The proposed procedure’s performance is also checked by incorporating the interscale dependencies along with thresholding. If a wavelet coefficient at a coarse scale is insignificant with respect to the proposed threshold then all of its descendants are also insignificant. The high frequency edge information is scattered among a large number of insignificant coefficients. However, the MDL principle with NML density and the structure of the wavelet decomposition tree is exploited here in order to locate those significant coefficients better and thus reduced ratio of retained coefficients is obtained without affecting the PSNR values much.

1.3 INDIRECT MDL SOFT THRESHOLDING

In hard thresholding all the coefficients with greater magnitudes than the threshold are retained as they are thought to comprise the informative part of data while the rest of coefficients are considered to represent noise and set to zero. However, it is reasonable to assume that coefficients are of neither purely noise nor information, but mixture of both. To cope with this, soft thresholding approaches have been proposed.

In the hard thresholding methods the observed wavelet coefficients shall correspond to one of the two categories, either informative signal or noise. Typically in image data the smaller wavelet coefficients consist not only noise but also important image details such as edges. The soft thresholding methods are based on the idea that the coefficients have contributions on both the informative signal and noise, so that shrinking the retained coefficients attempts to attenuate the effects of noise. By retaining a
slightly larger amount of coefficients and shrinking them, the soft thresholding methods usually give better results than the hard thresholding methods.

The existing Jiecheng Xie MDL denoising method is based on selecting a subset of wavelet coefficients to represent the informative signal, which is equivalent to hard thresholding. As soft thresholding has been found in some cases superior to hard thresholding, the existing Jiecheng Xie MDL based hard thresholding denoising method is extended with the soft thresholding like modification. However, it has proven to be difficult to combine soft thresholding with a selection of model class according to shortest description length in a theoretically rigorous ways. By interpreting separate Gaussian densities for the non informative and informative coefficients, the proportion of informative part in each coefficient is found out. The weight (informative proportion) is defined for each coefficient as a ratio of the value of the informative density function to the sum of values of both densities. The typical behavior of these weighted coefficients follows the conventional soft thresholding operation - when the noise density function has significantly larger values than the informative signal density function the weight tends to zero, and in the opposite case the weight tends to one. Thus the noisy wavelet coefficients are modified such that the proportion of informative part in each wavelet coefficient is revealed. These modified (soft thresholded) coefficients are subjected to MDL hard thresholding then. This again provides an optimal division of noise and noise free coefficients. Thus due to this two fold procedure the noise in the image is effectively removed.

This second proposed technique is compared with the widely used Bayes hard thresholding, Bayes soft thresholding and with the MDL hard thresholding (Jiecheng Xie et al 2004). It is observed that of all the compared methods, MDL soft thresholding retains lesser number of significant
coefficients to reconstruct the denoised image without degrading the PSNR values. Conventional thresholding approaches like Bayes methods retain more number of wavelet coefficients in the soft thresholding than hard thresholding. Unlike the conventional approaches, experimental results show that Indirect MDL soft thresholding retains only lesser number of coefficients than Jiecheng Xie MDL hard thresholding. This is because in the proposed soft thresholding approach, soft thresholding like modification is first applied to the noisy wavelet coefficients and MDL hard thresholding is followed then. This aids the suitable modification in the adaptive threshold for each wavelet coefficient and therefore aids better in model selection thus shows the reduction in the number of retained coefficients.

The experimental numerical results reveal that with two levels of wavelet decomposition, Bayes hard thresholding retains 46% of total coefficients and Bayes soft thresholding retains 52% of total wavelet coefficients for almost all the simulated noisy test images with an added noise variance of 0.01. For the same testing environment Jiecheng Xie MDL hard thresholding retains 33% of total coefficients whereas the proposed Indirect MDL soft thresholding retains just 16% of total wavelet coefficients for the equivalent denoising performance. Also it is inferred that in the Bayes hard/soft thresholding procedures the ratio of retained coefficients increases as the added noise variance increases but in the MDL hard/soft thresholding schemes it is just the reverse.

Figure 1.2 shows the visual perception of the HOUSE image for the considered soft and hard thresholding schemes. It is certain that MDL based methods yield better visual perception than the Bayesian procedures. The quality of MDL based soft thresholded images is closer to its corresponding hard thresholded image.
1.4 EFFICIENT MDL WITH NML DENSITY SOFT THRESHOLDING

As the first proposed technique namely Improved MDL denoising with NML density is considered to be the most rigorous MDL denoising approach, this procedure is also extended with the above said soft thresholding like modification. Here by interpreting separate normalized maximum likelihood Gaussian densities for the non informative and informative coefficients, the proportion of informative part in each coefficient is found out. The weight (informative proportion) for each coefficient is defined as a ratio of the value of the informative density function to the sum of values of both densities. Here the weights are different than previously computed MDL weights for each wavelet coefficient as the normalized maximum likelihood density is incorporated along with MDL principle. The typical behavior of these effectively weighted coefficients (with new weights)
also follows the conventional soft thresholding operation - when the noise density function has significantly larger values than the informative signal density function the weight tends to zero, and in the opposite case the weight tends to one. This effective modification results in the finer extraction of the informative portion from the noisy coefficients. These effectively modified coefficients are then subjected to MDL with NML density hard thresholding, which again identifies the informative coefficients better. This strongly influences the further reduction in the proportion of retained coefficients used for denoised image reconstruction.

This proposed algorithm is tested with number of test images of size 256X256 and the performance is compared with the conventional as well as the other proposed techniques. Of all the compared denoising procedures Efficient MDL with NML density soft thresholding retains the lowest proportion of wavelet coefficients without degrading the denoising performance. With two level wavelet decomposition, to obtain the same denoising performance, Improved MDL with NML density hard thresholding retains 10% of the total coefficients whereas Efficient MDL with NML density soft thresholding retains only 9% of the total coefficients for almost all the simulated noisy test images with an added noise variance of 0.01. For other higher values of added noise variance (0.04, 0.07, 0.1) also the same 1% of reduction in the retained coefficients is maintained (without degrading the respective denoising performance) in the proposed soft thresholding method than the corresponding hard thresholding counterpart.

Figure 1.3 shows the visual comparison of the CAMERAMAN image for the compared denoising procedures. The visual perception of the denoised images for all the MDL based methods is not inferior to Bayesian methods. Thus it is concluded that the Efficient MDL with NML Density soft thresholding provides denoising performance on par with the most rewarded denoising procedures by correctly locating the signal coefficients and exactly identifying the informative portion in them.
Figure 1.3  Visual perception of CAMERAMAN image in the proposed Efficient MDL with NML soft thresholding  (a) Original (b) Noisy ($\sigma^2 = 0.09$) (c) Bayes hard (d) Bayes soft (e) MDL hard (f) Indirect MDL soft (g) Improved MDL with NML density hard (h) Efficient MDL with NML density soft
1.5 SUMMARY

This thesis aims at developing new techniques for simultaneous noise reduction and analytical compression in Gaussian noise added images. The analytical compression of the wavelet based denoising algorithms has to be improved without sacrificing the denoising performance. This can be achieved through better extraction of signal (noise free) coefficients than the conventional techniques. Three different techniques namely Improved MDL with NML density hard thresholding, Indirect MDL soft thresholding and Efficient MDL with NML density soft thresholding are developed to improve the analytical compression along with denoising.

The first technique Improved MDL Denoising with NML Density involves the model selection through MDL principle incorporated with NML density. This technique assumes Gaussian distribution as the model for the wavelet coefficients and derived the optimal and adaptive threshold for denoising. The NML code identifies the more optimal model and the regularities of the data is better identified, hence the MDL principle incorporated with NML density gives the shortest description of the data achievable with a given model class and achieves smaller proportion of significant coefficients than simple MDL denoising.

This proposed technique is tested with the application of directional edge adaptive rectangular windows in the relevant detailed subbands for signal variance estimation. The performance is also individually checked up with more levels of wavelet decomposition and incorporating the interscale dependencies.

The second technique, Indirect MDL soft thresholding involves the soft thresholding like modification of the noisy wavelet coefficients such that the proportion of informative part in each wavelet coefficient is
revealed. These modified (soft thresholded) coefficients are subjected to MDL hard thresholding then. This two fold procedure aids the suitable modification in the adaptive threshold for each wavelet coefficient. This provides an optimal division of noise and noise free coefficients and therefore shows the reduction in the number of retained coefficients than the first technique.

The third technique, Efficient MDL with NML density soft thresholding again involves soft thresholding like modification of the noisy wavelet coefficients such that the proportion of informative part in each wavelet coefficient is better revealed than the second technique. These effectively modified coefficients are then subjected to MDL with NML density hard thresholding, which again identifies the informative coefficients better. This strongly influences the further reduction in the proportion of retained coefficients used for denoised image reconstruction.

Number of test images of size 256X256 is used in the experiments. Additive white Gaussian noise of variance (0.01, 0.04, 0.07 and 0.1) is added to the test images to simulate the noisy images. From the experimental results it is concluded that compared to the more rewarding Bayesian denoising methods, all the three proposed MDL based methods show improvement in analytical compression without compromising the PSNR performance. Among the proposed methods Efficient MDL with NML density soft thresholding optimizes the analytical compression.

1.6 THESIS ORGANIZATION

The remaining part of the thesis is organized as follows. Chapter 2 gives the literature survey of popular image denoising schemes which provides simultaneous analytical compression and describes the tool, the wavelet transform utilized in this particular task. It also describes the wavelet based state of art image denoising approaches such as coefficient
thresholding and the signal coefficients estimation procedure through stochastic modeling. This chapter also introduces the MDL principle and discusses the MDL hard thresholding to improve both the denoising performance and the analytical compression. In chapter 3 the MDL principle is extended with NML density, and the proposed Improved MDL with NML density hard thresholding is analyzed in detail. The Indirect MDL soft thresholding technique is proposed to improve the analytical compression than the MDL hard thresholding, while maintaining the denoising performance the same and it is described in Chapter 4. The third proposed approach called Efficient MDL with NML density soft thresholding to optimize the analytical compression is dealt in detail in Chapter 5. Chapter 6 presents the thesis summary, conclusion and the future extension of this research work.