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Study of Dissipative Intermediate State of a Reentrant Thin-Film Superconductor

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We analyse the current–voltage (I–V) characteristics of a thin-film sample of the ferromagnetic superconductor $ErRh_4B_4$ within a simple heating approximation. The results demonstrate that the salient features of the experimental nonlinear I–V characteristics observed in the dissipative intermediate resistance state can be readily understood within the framework of this approximation. The magnitude of the thermal boundary conductance of the interface between the film and its thermal environment is found to be typically of the same order as that generally observed during Kapitza-like heat transfer between solids, suggesting that the strength of the thermal coupling of the film to the substrate dominates the electrical behavior of a reentrant thin-film superconductor.

1. INTRODUCTION

The study of superconductors driven to states far away from thermal equilibrium has been a subject of high current interest.[1–3] Recently, the electrical behavior of reentrant thin-film superconductors carrying a current in the ferromagnetic normal state has been investigated theoretically,[4] leading to the prediction of several novel features associated with the dissipative intermediate resistance state. However, so far no experimental evidence has become available to quantitatively confirm these predictions and for assessing their significance.

In this paper we present a brief summary of the experimentally observed[5] nonlinear current–voltage (I–V) characteristics of a thin-film sample of the ferromagnetic superconductor $ErRh_4B_4$ at bath temperatures T_b below the lower critical temperature T_{c2}. Based on a systematic analysis

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of these I–V characteristics, we demonstrate that the effects associated with the dissipative intermediate resistance state of the reentrant thin-film superconductor can be adequately accounted for by a simple heating model.[4] Furthermore, we find that the magnitude of the thermal boundary conductance of the film–substrate interface obtained from the present analysis is of the same order as that expected for Kapitza-like heat transfer between solids[1,3] at low temperatures. The implications of these observations are discussed.

2. EXPERIMENTAL DETAILS

2.1. Sample

Rowell *et al.*[5,6] have prepared $ErRh_4B_4$ films by getter sputtering from an arc-melted target onto sapphire substrates held at 1000°C. The film is quite rough, with evidence for small crystallites growing from the surface with diameters 3000 Å. The film is then patterned by photolithography and ion milling to give a bridge (number 68–78 T for example[5]) 216 μm long between the two voltage probes (see upper inset of Fig. 1) and 8.2 μm wide, with separate current contacts (the two squares shown in the upper inset of Fig. 1), voltage contacts, and an extra pair of contacts at the midpoint (not shown in Fig. 1). The average film thickness is ~3000 Å. The resistance at 300 K is 104 Ω, giving a resistivity of 118 $\mu\Omega$-cm. The superconducting transition was at $T_{c1} = 8.5$ K. The resistance at 10 K was 18.3 Ω, yielding a resistance ratio of 5.7.

2.2. Resistance–Temperature Characteristics

The resistance–temperature (R–T) characteristic[5] in the vicinity of the transition to the ferromagnetic normal state is hysteretic, which can be approximatley represented in the form shown by the lower inset of Fig. 1. For the typical bridge 67–78 T, it is observed[5] that $T_{c2} \approx 1.094$ K, $\delta T \approx 0.059$ K, $T'_{c2} \approx 1.013$ K, and $\delta T' \approx 0.051$ K. The resistance in the ferromagnetic state R_0 is 17.3 Ω. The effect of stopping partway through the resistive transition and reversing the temperature sweep leads to different paths (like EA shown in the lower inset of Fig. 1) of the R–T characteristic.

2.3. Current–Voltage Characteristics

The I–V characteristic obtained[5] for both forward and reverse bias at $T_b = 0.3$ K is shown in Fig. 1. Up to a current of about 0.1 mA the I–V curve is quite linear and the voltage reaches a maximum of 1.7 mV at 0.11 mA. For I exceeding 0.11 mA, we note a negative differential resistance

246

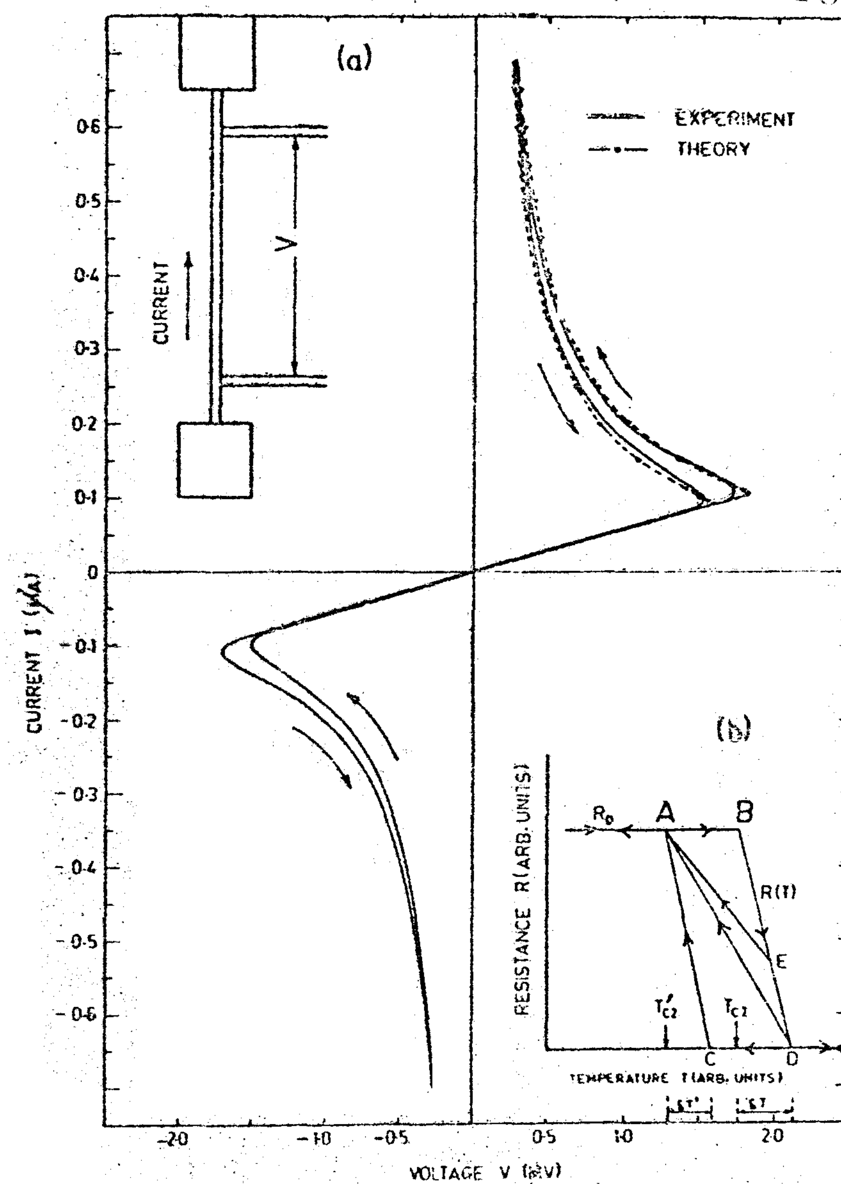

Fig. 1. The I–V characteristics of $ErRh_4B_4$ thin-film bridge[5] 68–78 T at $T_b \sim 0.3$ K, for both forward and reverse bias. The arrows indicate the direction of current sweep. The upper inset (a) depicts the sample geometry and the lower inset (b) represents a typical R–T characteristic, manifesting hysteresis when the temperature sweep is reversed at various stages.[5]

(dV/dI) region in the I–V curve in qualitative agreement with the earlier theoretical predictions.[4] The I–V characteristics at various bath temperatures T_b are shown in Fig. 2, where it is observed that the effect of increasing current is to reduce the voltage, but the sample as a whole is never driven to the zero-voltage superconducting state. At a current of 19.5 mA, Rowell *et al.*[5] observed that the voltage was 0.008 mV. At this current the film generally switches abruptly to the normal state, possibly due to heating of some region of the film above its upper critical temperature T_{c1}. In one-half of the film, a zero-voltage superconducting state was observed for a certain interval of current. The I–V characteristics were unchanged when in one experiment the film was covered entirely with a layer of rubber cement, suggesting that any heat escaping from the top

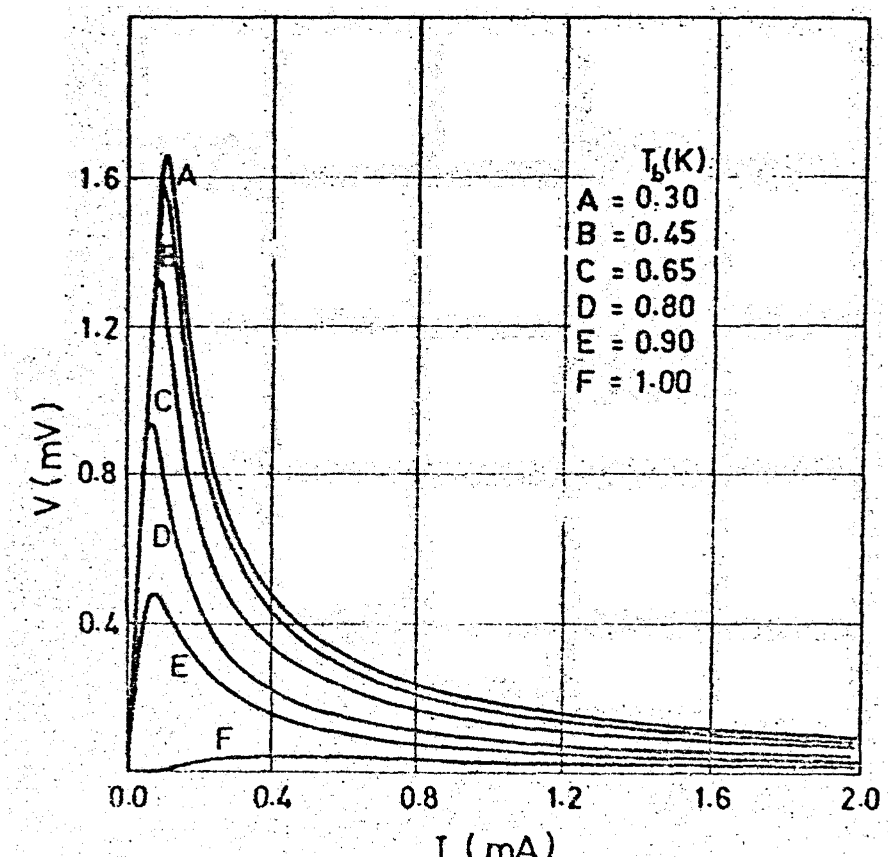

Fig. 2. Typical I–V characteristics of the bridge 68–78 T at various bath temperatures.[5]

247

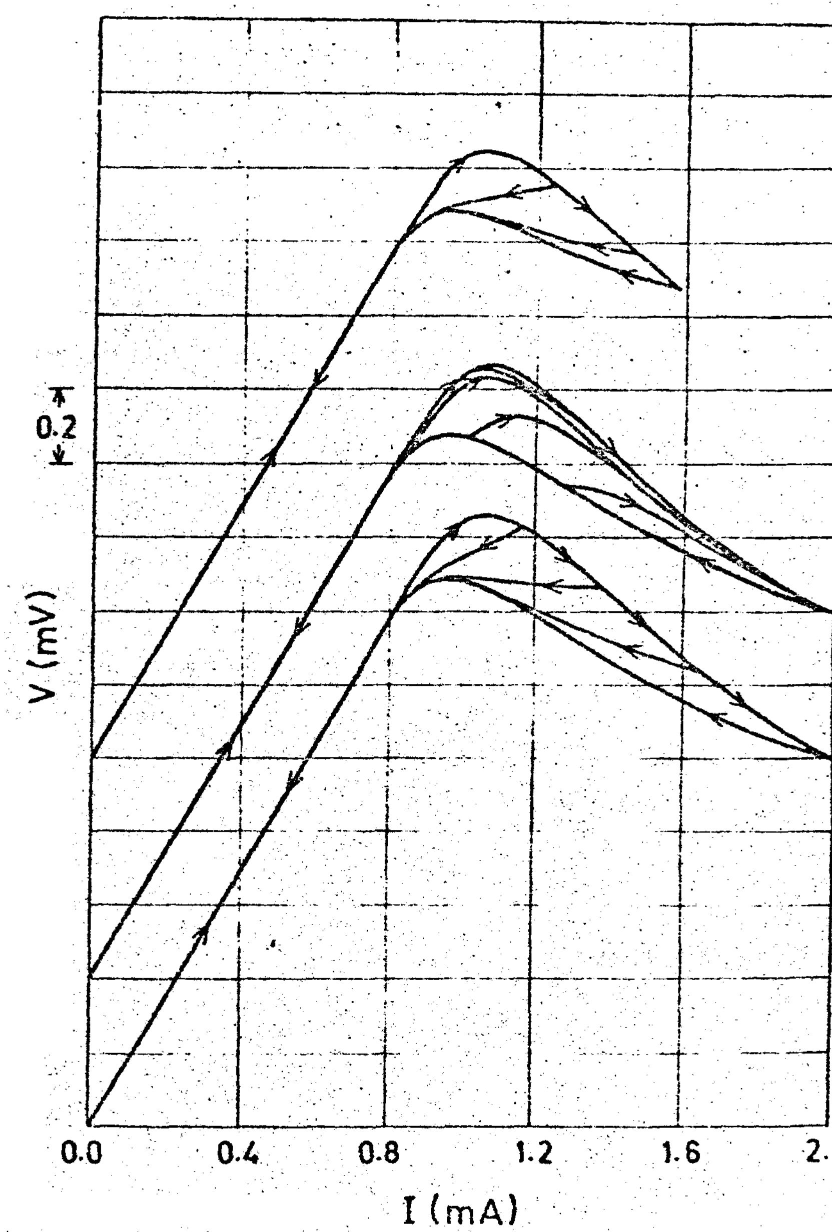

Fig. 3. Hysteretic I–V pattern obtained by reversing the direction of current sweep[5] at various points of the I–V curves at $T_b = 0.3$ K.

surface of the film is apparently negligible. Interesting hysteresis effects can be traced out in the I–V characteristic, as shown in Fig. 3.

3. SIMPLE HEATING APPROXIMATION

The nonlinear I–V characteristics shown in Figs. 1–3 can be readily understood by invoking a simple heating model given in Ref. 4. Since only the surface heat transfer is dominant in long films[2,4] with length L much greater than thermal healing length $\eta = (K_N d/\alpha)^{1/2}$, the heat balance equation for the film carrying a current I in the ferromagnetic normal state can be written as

$$\frac{\alpha(T-T_b)}{d} = \left(\frac{I}{Wd}\right)^2 \rho(T) \tag{1}$$

Here K_N is the thermal conductivity of the film, W is its width, d is its thickness, α is the coefficient of surface heat transfer (or thermal boundary conductance), T is the temperature of the film, which would be higher than the bath temperature T_b due to Joule heating in the dissipative state, and $\rho(T)$ is the normal state resistivity of the film at the elevated temperature T.

Assuming that $\rho(T)$ corresponding to this dissipative state follows the same temperature dependence as the equilibrium $R(T)$ depicted by the lower inset of Fig. 1, then for the increasing mode of temperature we can approximate $\rho(T)$ by

$$\rho(T) = \rho_0 \qquad \text{for} \quad T < T_{c2} \tag{2}$$

$$\rho(T) \approx \rho_0(T_{c2} + \delta T - T)/\delta T, \qquad \text{for} \quad T_{c2} < T \leq T_{c2} + \delta T \tag{3}$$

where $\rho_0 = R_0 Wd/L$ is the normal state resistivity. The voltage across the film can be expressed as

$$V = I\rho(T)L/Wd \tag{4}$$

As long as the dissipation in the ferromagnetic normal state is such that the elevated temperature T is below T_{c2}, Eq. (2) and (4) together would yield a linear I–V characteristic. However, as the current I exceeds a threshold value I_0, the elevated temperature T of the film would surpass T_{c2}, in which case, combining Eqs. (1), (3), and (4), we get

$$V = IR_0 \frac{T_{c2} + \delta T - T_b}{\delta T(1 + I^2\rho_0/\alpha W^2 d\delta T)} \tag{5}$$

The value of the critical current I_0 follows from Eq. (1) since

$$P_0 = V_0 I_0 = \alpha WL(T_{c2} - T_b) \tag{6}$$

where P_0 is the threshold power level at the onset of the metastable state of intermediate resistance $(0 < R(T) < R_0)$. Combining Eqs. (5), (6), we obtain

$$VI = V_0 I_0 \frac{1 + \delta T/(T_{c2} - T_b)}{1 + (I_0/I)^2\, \delta T/(T_{c2} - T_b)} \qquad \text{for} \quad I \geq I_0 \tag{7}$$

From the above expressions, it is evident that the I–V characteristic becomes nonlinear at $I = I_0$ with a negative differential resistance (dV/dI) feature for I exceeding I_0. This negative differential resistance region is sustained by nearly constant power dissipation in the film, which has been driven into an intermediate resistance state, out of equilibrium with the temperature bath. This negative differential resistance could be important for practical device applications.

4. ANALYSIS OF THE EXPERIMENTAL RESULTS

As shown below, the experimental observations[5] given in Figs. 1–3 can be readily understood within the above simple heating approximation with an appropriate choice of the relevant parameters. Considering the forward trace of I–V characteristics given in Fig. 1, we choose $V_0 = 1.8$ mV and $I_0 = 0.11$ mA, corresponding to the extrapolated peak of the experimental I–V curve. Substituting these values and other parameters from Section 2 in Eqs. (6), we obtain $\alpha = 0.014$ W/cm^2 K. The thermal conductivity K_N near T_{c2} can be estimated, using the Wiedeman–Franz law, by

$$K_N = (1/3)\pi^2(K_B/e)T/\rho_0 \tag{8}$$

where K_B is the Boltzmann constant and e is the electronic charge. With $\rho_0 = 20\ \mu\Omega$-cm, $K_N \approx 0.001$ W/cm K at $T \approx 1$ K, and $\eta \approx 16\ \mu$m. The total length ($\sim 260\ \mu$m) of the film bridging the two current contacts (see the upper inset of Fig. 1) is about 16 times larger than the thermal healing length η. Hence we are justified in assuming a uniform temperature distribution[2,4] in the effective length $L = 216\ \mu$m of bridge across which the voltage is monitored.

With the above justification, we use Eq. (7) for describing the forward trace of the I–V characteristic shown in Fig. 1 by a dashed-dot curve, the relevant parameters being taken from Section 2. We note that the agreement between theory and experiment is remarkably good within the accuracy of our computation. For describing the returning trace of the I–V characteristic, we have used $V_0 = 1.575$ mV and $I_0 = 0.09$ mA, corresponding to the extrapolated peak of the relevant experimental I–V curve of Fig. 1, along with T_{c2} of Eq. (7) replaced by T'_{c2} and taking $\delta T = 0.14$ K, the temperature interval between points A and D of the R–T characteristic given in Fig. 1.

This choice of δT is based on the assumption that the $R-T$ curve corresponding to the returning trace of the $I-V$ curve can be closely approximated by the path DA. By virtue of the excellent agreement between theory and experiment for the returning trace of the $I-V$ curve as well, we find that the present simple heating approximation can satisfactorily account for the observed hysteresis in the nonlinear region of the $I-V$ characteristic. Thus it is evident that the hysteretic $I-V$ pattern shown in Fig. 3 can also be readily attributed to the hysteresis manifested by the $R-T$ characteristic when the temperature sweep is reversed in the intermediate resistance region $[0 < R(T) < R_0]$. However, the reasons for the deviation of the linear porton of the $I-V$ curve at $T_b = 0.9$ K (Fig. 2) and for the switching over to a zero-voltage superconducting state in one-half of the film for a certain interval of current[5] are not clearly understood.

5. THERMAL BOUNDARY CONDUCTANCE

Since the $I-V$ characteristics in the above experiment remained unchanged when the film was covered with a layer of rubber cement,[5] the

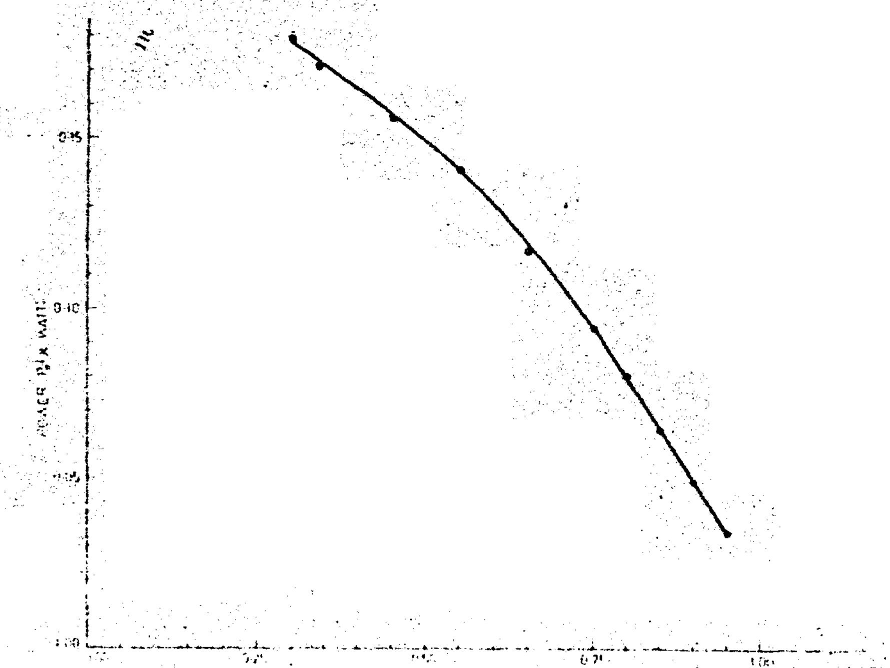

Fig. 4. Typical temperature dependence of the critical power P_0.

film–substrate interface appears to be the essential channel available for phonon escape from the dissipating film. Thus for the present case we can expect the thermal boundary conductance of the film–substrate interface to be dictated by the theory governing Kapitza-like heat transfer between solids at low temerpatures. The values of P_0 deduced by fitting Eq. (7) to the $I-V$ characteristics at various bath temperatures (Fig. 2) are shown in Fig. 4. Near T_{c2}, P_0 varies linearly with T_b and the thermal boundary conductance α for this region as estimated using Eq. (6) turns out to be $\sim 14\ \mathrm{mW/cm^2\,K}$, which is of the same order as that observed in other theoretical estimates.[1] The deviation of P_0 from linear temperature dependence for T_b far below T_{c2} can be attributed to a possible rise in the substrate temperature T_s just below the film due to the large power dumped at such high power levels, in which case $T_s(> T_b)$ may have to be used in Eq. (6) for estimating α.

6. CONCLUSIONS

Thus we conclude that the essential features of the dissipative intermediate resistance states of a current-carrying reentrant thin-film superconductor can be quantitatively understood within a simple heating approximation. Further, experimental study would be desirable for unambiguously resolving such subtle features as the abrupt switching to the normal state at high current levels and switching over to a zero-voltage superconducting state of one-half of the film[5] for a certain interval of current.

The present results clearly indicate that thin-film reentrant superconductors with well-defined $I-V$ characteristics in the dissipative intermediate resistance state can serve as novel candidates for the study of the Kapitza-like heat transfer between solids. The scope for further theoretical study is clear-cut since the present formalism does not consider either the temperature dependence of the thermal boundary conductance or the effect of transport current on the lower critical temperature of the reentrant superconductor.

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REFERENCES

1. J. J. Chang and D. J. Scalapino, *J. Low Temp. Phys.* **31**, 1 (1978).
2. G. Dharmadurai, *Phys. Stat. Sol.* (*a*) 92, 11 (1980); *Ind. J. Cryogen.* **5**, 53 (1980).
3. K. E. Gray, *Nonequilibrium Superconductivity, Phonons and Kapitza Boundaries* (Plenum Press, New York, 1981), and references therein.
4. G. Dharmadurai, *Physica B* **108**, 1233 (1981); *Nuovo Cimento B* **64**, 503 (1981), and references therein.
5. J. M. Rowell and P. H. Schmidt, unpublished.
6. J. M. Rowell, R. C. Dynes, and P. H. Schmidt, *Solid State Commun.* **30**, 191 (1979).

phys. stat. sol. (b) 123, [illegible] (1984)

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Role of Thermal Transport in the Non-Equilibrium Behaviour of a Cylindrical Ferromagnetic Superconductor

By

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The steady state temperature distribution in a long current carrying cylindrical wire of a ferromagnetic superconductor at bath temperatures below the magnetic transition is obtained using some simplifying approximations. It is found that dissipation in this ferromagnetic resistive state would lead to the onset of a non-equilibrium intermediate resistance state almost simultaneously in the entire cross-section of the wire at a critical power level. The results indicate the dominant role of surface thermal transport in deciding the gross electrical behaviour of this non-equilibrium reentrant superconductor under various thermal environments involving gaseous and liquid helium. The significance of these findings is discussed.

Die stationäre Temperaturverteilung in einem langen stromführenden zylindrischen Draht eines ferromagnetischen Supraleiters wird bei Badtemperaturen unterhalb des magnetischen Übergangs mit einigen vereinfachenden Näherungen erhalten. Es wird gefunden, daß die Dissipation in diesem ferromagnetischen Widerstandszustand zum Auftreten eines Nichtgleichgewichts-Zwischenzustandes des Widerstands fast gleichzeitig im gesamten Querschnitt des Drahtes bei einem kritischen Leistungsniveau führt. Die Ergebnisse zeigen die dominierende Rolle des thermischen Oberflächentransports für das elektrische Gesamtverhalten in diesem Nichtgleichgewichts-Wiedereintritts-Supraleiter unter verschiedenen thermischen Umgebungen einschließlich gasförmigen und flüssigen Heliums. Die Bedeutung dieser Entdeckung wird diskutiert.

1. Introduction

Recently, the crucial role of phonon escape from dissipating superconductors in deciding their non-equilibrium properties [1 to 3] has been well recognised. However, the poorly understood phonon transmission properties [3] of the interface between the helium bath and a non-equilibrium superconductor have posed some difficulties in testing the relevant theoretical ideas. Particularly, thin film samples evaporated onto substrates involve thermal boundary transport through both solid–fluid and solid–solid interfaces [4], leaving some uncertainty in the fraction [2] of phonons transmitted through these interfaces.

In this paper, we suggest how a cylindrical wire of a reentrant ferromagnetic superconductor immersed in a helium bath can serve as a better candidate for studying non-equilibrium superconductivity, by virtue of the fact that thermal boundary transport in this case involves only a solid–fluid interface. Since the thermal transport through solid–gas interfaces appears to be well understood in terms of acoustic mismatch theory [5] both at low [6] and high [7] temperatures, we can expect the behaviour of the above equilibrium reentrant superconductor in gaseous helium to be better described by theoretical models. With this in view, we calculate the temperature distribution in a cylindrical sample of the reentrant superconductor $ErRh_4B_4$ carrying a current in the ferromagnetic resistive state, using some simplifying ap-

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proximations. We find that the radial temperature distribution is nearly uniform throughout the cross-section of the wire, leading to a simultaneous onset of a non-equilibrium intermediate resistance state in the entire cross-section of the wire at a critical power level. The present results indicate that surface thermal transport dominates the current–voltage (I–U) characteristics of this non-equilibrium reentrant superconductor. The significance of the findings for gaseous and liquid helium environments is discussed.

2. Radial Temperature Distribution

For the sake of mathematical simplicity we consider an infinitely long cylindrical sample of a reentrant superconductor carrying a current I in its ferromagnetic normal state. If this wire is immersed in a helium bath at temperature T_b below T_{c2}, the magnetic transition temperature, we can safely assume a uniform temperature distribution along the axial direction [8]. Under this approximation, the radial temperature distribution $T(r)$ can be obtained by solving the radial heat flow equation

$$-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(rK\frac{\mathrm{d}T}{\mathrm{d}r}\right)=\frac{I^2\varrho}{(\pi r_0^2)^2}, \tag{1}$$

where ϱ is the specific resistance of the wire, K the thermal conductivity of the material, and r_0 the radius of the sample (see inset Fig. 1). Applying the condition that $\mathrm{d}T/\mathrm{d}r$ is finite (zero) at $r = 0$, we obtain

$$K\frac{\mathrm{d}T}{\mathrm{d}r}=-\frac{I^2\varrho r}{2(\pi r_0^2)^2}. \tag{2}$$

If we assume [8] $K = a_0 T$ and impose the boundary condition that $T(r)|_{r=r_0} = T_0$, we get

$$T^2(r) = T_0^2 + \frac{I^2\varrho}{2\pi^2 a_0 r_0^2}\left(1-\frac{r^2}{r_0^2}\right). \tag{3}$$

The temperature T_0 at the surface can be related to the bath temperature T_b and thermal boundary conductance [5 to 7] h_k of the solid–fluid interface through the expression

$$h_k(T_0 - T_b) = \frac{I^2\varrho}{2\pi^2 r_0^3}. \tag{4}$$

Combining (3) and (4), we arrive at an expression for the radial temperature distribution $T(r)$ in the form

$$T(r) = \left\{\left(T_b + \frac{I^2\varrho}{2\pi^2 r_0^3 h_k}\right)^2 + \frac{I^2\varrho}{2\pi^2 a_0 r_0^2}\left(1-\frac{r^2}{r_0^2}\right)\right\}^{1/2}. \tag{5}$$

3. Calculation for $ErRh_4B_4$

We can have a feel for the radial distribution in a reentrant superconductor by considering $ErRh_4B_4$ as a typical example. The readily available data [8], viz. $a_0 \approx \approx 0.01$ W/cm K^2 and $\varrho \approx 2 \times 10^{-6}$ Ωcm, can be assumed to be valid for a cylindrical wire of 0.1 mm diameter. In this case, with $r_0 = 0.05$ mm, (5) yields

$$T^2 = \{T_b + 0.81\ \Omega\mathrm{cm}^{-2}\ I^2 h_k^{-1}\}^2 + 0.405\ \mathrm{K}^2\ \mathrm{A}^{-2}\ I^2\left(1-\frac{r^2}{r_0^2}\right). \tag{6}$$

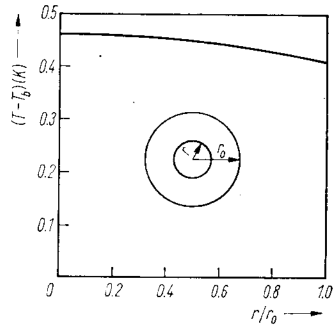

Fig. 1. Difference between the radial temperature T and the bath temperature T_b for various values of normalized radius (r/r_0). The inset shows the sample cross-section

From the above equation, it is evident that for real experimental situations where $h_k \leqq 1$ W/cm² K, the radial distribution will be virtually uniform. In Fig. 1, we have plotted $(T - T_b)$ versus (r/r_0) for a typical case with $T_b = 0.5$ K, $h_k = 0.5$ W/cm² K, and $I = 500$ mA. There is only 50 mK difference between the centre of the wire and its surface, which is just 5% of the centre line temperature = 0.959 K. Thus we can expect the entire cross-section of the wire to reach a temperature above the magnetic transition or lower critical temperature T_{c2} at a certain critical power level $U_0 I_0$, with the corresponding current and voltage being I_0 and U_0, respectively. Above this current level I_0, the wire would be driven into a non-equilibrium intermediate resistance state, with the voltage across it falling [8] below U_0. By monitoring the current I_0 at which downturn in this voltage occurs, the thermal boundary conductance h_k of the interface between the sample and its surroundings can be readily obtained from (5).

4. Estimates of Thermal Boundary Conductance for a Gaseous Environment

The dominant role of surface thermal transport in deciding the onset of a non-equilibrium intermediate resistance state would become more evident if we estimate h_k for the case of various thermal environments. Particularly, in the case of the sample surrounded by gaseous helium, reliable estimates of h_k can be obtained theoretically [6]. At low temperatures, h_k for an interface between gaseous ^{3}He and a solid can be well approximated by [6]

$$h_k = \left(\frac{2k_B P^2}{\pi m T_g}\right)^{1/2}, \tag{7}$$

where P is the (vapour) pressure of the gas, m its molecular mass, k_B the Boltzmann constant, and T_g the temperature of the gas. Using the standard vapour pressure data [9], we obtain $h_k = 3.1$, 0.124, 0.002 W/cm² K at $T_g = 0.9$, 0.5, and 0.3 K, respectively. In this case we can infer from (6) that at temperatures below 0.5 K the surface thermal resistance h_k^{-1} will be very dominant and the behaviour of the non-equilibrium intermediate resistance state can be theoretically described by (6) in conjunction with Ohm's law and an approximate relation given below for the specific resistivity $\varrho(T)$ in this region [10, 11] $(T_{c2} < T < T_{c2} + \delta T)$,

$$\varrho(T) \approx \frac{\varrho_0(T_{c2} + \delta T - T)}{\delta T}, \tag{8}$$

where δT is the finite width of the reentrant phase transition [10, 11] and ϱ_0 is the specific resistance in the ferromagnetic normal state below T_{c2}.

5. Discussion and Conclusions

If the sample is immersed in a helium bath, the relevant h_K for the solid–liquid helium interface [3, 5, 6] shall be used. In any case, the present system will serve as a better candidate for the study of non-equilibrium superconductivity, as well as thermal boundary conductance of solid–fluid interfaces at low temperatures, since uncertain thermal contacts with other media are not involved. While the recent observations in point contact junctions [12] are suggestive of the nonlinear behaviour [10, 11] of the I–U curve, experimental studies on cylindrical wires would be required to test the present theoretical ideas.

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References

[1] J. J. CHANG and D. J. SCALAPINO, J. low-Temp. Phys. **31**, 1 (1978).
[2] G. DHARMADURAI, Indian J. Cryogenics **5**, 53 (1980).
[3] K. E. GRAY, Nonequilibrium Superconductivity, Phonons, and Kapitza Boundaries, Plenum Press, New York 1981 and references therein.
[4] G. DHARMADURAI, phys. stat. sol. (a) **62**, 11 (1980).
[5] I. M. KHALATNIKOV, An Introduction to the Theory of Superfluidity, W. A. Benjamin, Inc., New York 1965 (p. 138).
[6] A. C. ANDERSON and W. L. JOHNSON, J. low-Temp. Phys. **7**, 1 (1972).
[7] G. DHARMADURAI, phys. stat. sol. (b) **116**, 307 (1983).
[8] G. DHARMADURAI, Phys. Letters A **78**, 481 (1980).
[9] T. R. ROBERTS, R. H. SHERMAN, S. S. SYDORIAK, and F. G. BRICKWEDDE, Progress in Low Temperature Physics, Ed. C. J. GORTER, North-Holland Publ. Co., Amsterdam 1964 (p. 506).
[10] J. P. REMEIKA, G. P. ESPINOSA, A. S. COOPER, H. BARZ, J. M. ROWELL, D. B. MCWHAN, J. M. VANDENBERG, D. E. MONCTON, Z. FISK, L. D. WOOLF, H. C. HAMAKER, M. B. MAPLE, G. SHIRANE, and W. THOMLINSON, Solid State Commun. **34**, 923 (1980).
[11] G. DHARMADURAI, Physica (Utrecht) B **108**, 1233 (1981); Nuovo Cimento B **64**, 503 (1981).
[12] Y. KUWASAWA, S. NAKANO, and L. RINDERER, Phys. Letters A **85**, 94 (1981).

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Effect of Supercurrent on Charge Imbalance Relaxation in a Phase Slip Centre

By

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The quasiparticle charge imbalance relaxation time in the nonequilibrium state of a phase slip centre in indium whiskers is calculated taking into account the depairing effect due to supercurrent and the finite size of the core. The normal-like length for the first phase slip centre for various temperatures below the transition temperature is calculated and compared with the experimental data. The analysis shows that the core of phase slip centre is about four times the Ginzburg-Landau coherence length. This result is new. Further the effect of pair breaking due to supercurrent on the charge imbalance relaxation time for various values of τ_E is calculated for temperature 1 to 50 mK away from the transition temperature.

Die Quasiteilchen-Ladungsungleichgewichts-Relaxationszeit im Nichtgleichgewichtszustand eines Phasengleitzentrums in Indiumwhiskern wird unter Berücksichtigung des Depaarungseffekts infolge von Superströmen und der endlichen Größe des Kerns berechnet. Die normalähnliche Länge für das erste Phasengleitzentrum wird für verschiedene Temperaturen unterhalb der Übergangstemperatur berechnet und mit experimentellen Werten verglichen. Die Analyse zeigt, daß der Kern des Phasengleitzentrums etwa das Vierfache der Ginzburg-Landau-Kohärenzlänge beträgt. Dieses Ergebnis ist neu. Weiterhin wird der Einfluß des Zerbrechens der Paare infolge des Superstroms auf die Ladungsungleichgewichts-Relaxationszeit für verschiedene τ_E-Werte für Temperaturen berechnet, die von der Übergangstemperatur 1 bis 50 mK entfernt sind.

1. Introduction

The nonequilibrium superconductivity has received great attention in the past few years and especially superconductors out of thermal equilibrium have been intensivly studied [1]. The breakdown of superconductivity in current carrying filaments (whiskers and microbridges) has been investigated by many workers [2]. In the one-dimensional superconductors with transverse dimensions small compared to coherence length $\xi(T)$ and penetration depth $\lambda(T)$, the transition from superconducting to normal state takes place through a series of regular steps in the I–U characteristics, when a current greater than critical current is passed through the filament. [2-5]

It is seen that each of these successive voltage jumps adds the same amount to the differential resistance. Further, each voltage step is due to the appearance of an additional, similar, localized resistance centre in series along the filament. The experimental studies conducted by Skocpol, Beasley, and Tinkham (SBT) [6] excluded the possibility of the successive voltage steps as corresponding to a succession of distributed modes involving the whole filament and thereby established the localized model (called SBT model) for an individual resistance centre. Since there is a net voltage drop across such a centre, it implies through the ac Josephson effect that the phase of the superconducting order parameter is increasing at different rates on the two sides of the centre. These local voltage carrying states are referred to as phase slip centre (PSC) or phase slip oscillators [7]. The SBT considers the differential

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resistance of the phase slip centre proportional to the quasiparticle relaxation length Λ_{Q^*}.

It has been suggested that the order parameter goes to zero in a region of the order of Ginzburg-Landau (GL) coherence length $\xi(T)$ called the beating heart of the PSC. Since the temperature dependences of ξ and Λ_{Q^*} are different, we propose that the size of the beating heart can be estimated by studying the details of the temperature dependence of the normal-like length L_n of the first PSC close to the transition temperature T_c. With this motivation we have reanalyzed the existing experimental data. Further, the correction to the relaxation time of the quasiparticles due to supercurrent is also temperature dependent and is taken into account. We find that the existing experimental data are consistent with the proposal that the size of the beating heart is nearly four times the GL coherence elngth.

2. Relaxation Time in Presence of Depairing

The charge imbalance relaxtion time τ_{Q^*} for nonequilibrium superconductors in presence of pair breaking effects due to magnetic impurities and supercurrent derived by using microscopic technique [1, 9], is given by

$$\tau_{Q^*} = \frac{4k_B T_{c0}}{\pi\Delta} \left\{ \frac{\tau_E}{2\Gamma} \left(1 + \frac{\hbar^2 \Gamma}{2\Delta^2 \tau_E} \right) \right\}^{1/2}, \tag{1}$$

where $\Delta(T)$ is temperature-dependent magnitude of the order parameter, and

$$\Gamma = \frac{1}{\tau_s} + \frac{1}{2\tau_E} + \frac{D}{2}\,(4m^2 v_s^2)\,. \tag{2}$$

Γ is the pair breaking parameter, τ_s the conduction electron spin-flip scattering time caused by paramagnetic impurities and the applied magnetic field, D the diffusion coefficient $D = \frac{1}{3} v_F l$, l being electron mean free paren and v_F the Fermi velocity. v_s the supercurrent velocity. In absence of any paramagnetic impurities or applied magnetic field, the spin-flip rate is zero.

When critical current is passed through a superconductor, the supercurrent velocity v_s has a maximum value called the critical supercurrent velocity v_c. The corresponding critical momentum is given by [10]

$$P_c = 2mv_c = \frac{\hbar}{\sqrt{3}\,\xi_{GL}(T)}, \tag{3}$$

where $\xi_{GL}(T)$ is the temperature-dependent Ginzburg-Landau coherence length. Substituting this in (2), the depairing parameter in presence of critical supercurrent becomes

$$\Gamma = \frac{1}{2\tau_E} + \frac{D}{6\xi_{GL}^2(T)}, \tag{4}$$

where [11]

$$\xi_{GL}(T) = 0.74 \left(1 + 0.754 \frac{\xi_0}{l} \right)^{-1/2} \left(\frac{T_{c0}}{T_{c0} - T} \right)^{1/2}, \tag{5}$$

ξ_0 being the BCS coherence length given by

$$\xi_0 = \frac{0.18\hbar v_F}{k_B T_{c0}} \tag{6}$$

and T_{c0} is the thermodynamic critical temperature in absence of current.

temperature T_{c0} the U–T curves do not show voltage steps and the transition to

3. Normal-Like Length of the Phase Slip Centre

When small currents are passed through indium whiskers [8] very close to the critical normal state is sharp but consistent with fluctuation effects. However, at temperature a few mK away from T_{c0} the corresponding critical currents are not small and the transition width of the U–T curves is considerable. In this regime for sufficiently large currents, voltage steps are observed in the U–I characteristics at fixed temperatures. The transition width from the first onset of the voltage to the normal state is considerably large.

At currents larger than the minimum critical current a finite electric field appears due to the appearance of the voltage step, and accelerates the supercurrent above the critical velocity resulting in a collapse of the order parameter. The whole current is then carried as a normal current. This in turn allows superconductivity to reappear and the cycle repeats at the Josephson frequency. Thus the phase of the order parameter slips by 2π (or possibly a multiple of 2π) each time when the magnitude of the order parameter goes to zero in the middle of the PSC. All this action with its associated strong ac supercourrent occurs in the "beating heart" of the PSC, a region which governs the spatial variation of the order parameter (energy gap) Δ.

Recently it has been proposed [12] that the oscillating core of the phase slip centre may extended over a few coherence lengths ξ matched to the outer regions where the quasiparticle charge imbalance and the associated dc elector-chemical potential μ decay exponentially over a distance Λ_{Q^*}. Therefore, the normal-like length L_n of the first phase slip centre can be written as

$$L_n = 2[\alpha\xi_{GL}(T) + \Lambda_{Q^*}(T)]\,, \tag{7}$$

where $\Lambda_{Q^*}(T)$ is the quasiparticle relaxation length given by

$$\Lambda_{Q^*} = (\tfrac{1}{3}\, v_F l \tau_{Q^*})^{1/2}\,. \tag{8}$$

We have reanalysed the experimental data on In whiskers by using the quasiparticle relaxation time in presence of pair breaking due to a critical supercurrent and have calculated the normal-like length L_n of the first phase slip centre and find that $\alpha = 4$ for the best fit. Thus nonequilibrium region L_n of the first phase slip centre which is thought of in terms of potentials μ_p and μ_Q for the paired electrons and quasiparticles. The pair potential μ_p varies spatially over a distance of the order of $4\xi_{GL}$ across PSC and μ_Q decays exponentially towards μ_p with characteristic length Λ_{Q^*} on either side of PSC. This model of the PSC is in conformity with the earlier experimental results of Dolan and Jackel [13]. Through normal and superconducting probes they measured the spatial variation of voltage across a PSC and have noted that the exponential decay of quasiparticles on the two sides of the PSC do not coincide at one space point when extrapolated (see Fig. 3 of [13], where $X_0 \neq X_0'$). This suggests that there is a non-equilibrium region across PSC in which charge imbalance remains almost constant. According to the present analysis this region is of the order of $2\xi_{GL}$ on either side of the PSC. Using (1), (4), (7), and (8) the values of L_n for In whiskers at different $\Delta T = T_{c0} - T$ have been plotted in Fig. 1, along with the experimental data of Tideeks and Slama [8]. $L_n \approx 100\,\mu m$ close to T_{c0} at $\Delta T = 1$ mK and falls to about $50\,\mu m$ at $\Delta T = 10$ mK. Except for the ξ-dominated initial fall, the normal-like length of PSC falls more slowly and remains almost constant for ΔT larger than 5 to 7 mK.

Fig. 1
Fig. 2

256

4. Effect of Depairing on Relaxation Time

The ratio of quasiparticle relaxation time τ_{Q^*} in presence of depairing supercurrent and the relaxation time $\tau_{Q^*}(0)$ in absence of supercurrent for various values of τ_E is plotted in Fig. 2 as a function of ΔT. It is observed that there is an appreciable decrease in $\tau_{Q^*}/\tau_{Q^*}(0)$ as ΔT increases. The fall in the ratio increases with the increase in the value of τ_E. For temperature close to transition temperature, i.e. $\Delta T < 5$ mK, $\xi_{GL}(T)$ is large. In this case one can take $\Gamma = (2\tau_E)^{-1}$ (ignoring the spin-flip scattering), so that (1) becomes

$$\tau_{Q^*} = \frac{4k_B T_{c0}}{\pi\Delta}\,\tau_E\left(1 + \frac{\hbar^2}{4\Delta^2\tau_E^2}\right)^{1/2}. \tag{9}$$

Looking at this expression, one expects the second term to give rise to a strong temperature dependence, because it is this factor which acts as the pair breaking one. We thus have

$$\frac{\tau_{Q^*}}{\tau_{Q^*}(0)} \approx \left(1 + \frac{\hbar^2}{4\Delta^2\tau_E^2}\right)^{1/2}. \tag{10}$$

This is much greater than one for $\Delta T \to 0$, and τ_E is fixed. Fig. 2 indicates this behaviour. For $\tau_E = 1.8 \times 10^{-10}$ s, as used here, $\tau_{Q^*}/\tau_{Q^*}(0)$ falls by about 25% as ΔT increases from 1 to 10 mK for indium whiskers.

5. Discussions and Conclusions

The early investigations on PSC in thin microbridges and microstrips were carried out at temperatures not too close to the critical temperature. In these investigations it appeared that the differential resistance of the PSC is independent of temperature. However, subsequent experiments [14] measured the differential resistance in presence of a magnetic field and concluded that the differential resistance depends on temperature and is in agreement with the temperature dependence implied by (1) in presence of a pair breaking magnetic field for ΔT larger than 10 mK.

The expression (1) for charge imbalance relaxation has been estimated by making several simplifying assumptions. The assumption that the superfluid velocity v_s is small needs closer examination. Very close to T_c the supercurrent and therefore the superfluid velocity are small and this assumption is valid. However, away from T_c the supercurrent is not small and its pair breaking effect is substantial as can be seen from Fig. 2.

The Boltzmann equation for the transverse mode which leads to (1) for charge relaxation is decoupled from the longitudinal mode only if v_s is zero. It is not clear whether (1) will continue to hold as a solution to the coupled equation. This is being investigated. Further the supercurrent depresses the energy gap and thereby increases L_n by about 10% for the Ginzburg-Landau critical current.

The In whiskers we have analysed have a short mean free path due to residual impurities or imperfections. The elastic nonparamagnetic impurity scattering processes do not relax the charge imbalance in absence of the supercurrent. However, the elastic impurity scattering processes contribute to charge relaxation in presence of superfluid velocity v_s by acting as pair breaker. The pair breaking parameter Γ in (2) depends on mean free path l through the diffusion coefficient D. Our analysis shows that the effect of nonparamagnetic impurities is adequately taken into account by (2) for small values of ΔT. However, away from T_c these elastic impurity scattering effects may be more serious, as appears from the interpretation of thermo-electric phenomena in superconductors [15].

References

[1] K. E. GRAY, Nonequilibrium Superconducting, Phonons, and Kapitza Boundaries, Plenum Press, New York 1981.

[2] J. MEYER and G. V. MINNIGERODE, Phys. Letters A 38, 529 (1972); Low-Temperature Physics, Plenum Press, New York 1974. → LT-13 (Proc. 13th Int. Conf. Low Temp. Phys. Boulder, 1972)

[3] J. D. MEYER, Appl. Phys. 2, 303 (1973).

[4] J. D. MEYER and R. TIDECKS, Solid State Commun. 18, 305 (1976).

[5] R. J. WARBURTON and W. W. WEBB, Bull. Amer. Phys. Soc. 13, 379 (1968); Phys. Rev. Letters 25, 1180 (1970).

[6] W. J. SKOCPOL, M. P. BEASLEY, and M. TINKHAM, J. low-Temp. Phys. 16, 145 (1974).

[7] R. J. WATTS-TOBIN, Y. KRAHENBUHL, and L. KRAMER, J. low-Temp. Phys. 42, 459 (1981).

[8] R. TIDECKS and G. SLAMA, Z. Phys. B 37, 103 (1980).

[9] A. SCHMID and G. SCHÖN, J. low. Temp. Phys. 20, 207 (1975).

[10] M. TINKHAM, Introduction to Superconductivity, McGraw-Hill Publ. Co., 1975.

[11] R. TIDECKS and J. D. MEYER, Z. Phys. B 32, 303 (1979).

[12] A. BARATAFF, Proc. Internat. Conf. Low-Temperature Physics, [illegible] 1981 (p. [illegible]).

[13] G. J. DOLAN and L. D. JACKEL, Phys. Rev. Letters 39, 1628 (1977).

[14] A. M. KADIN, W. J. SKOCPOL, and M. TINKHAM, J. low-Temp. Phys. 33, 481 (1978).

[15] G. SCHÖN, Festkörperprobleme 21, 341 (1981).

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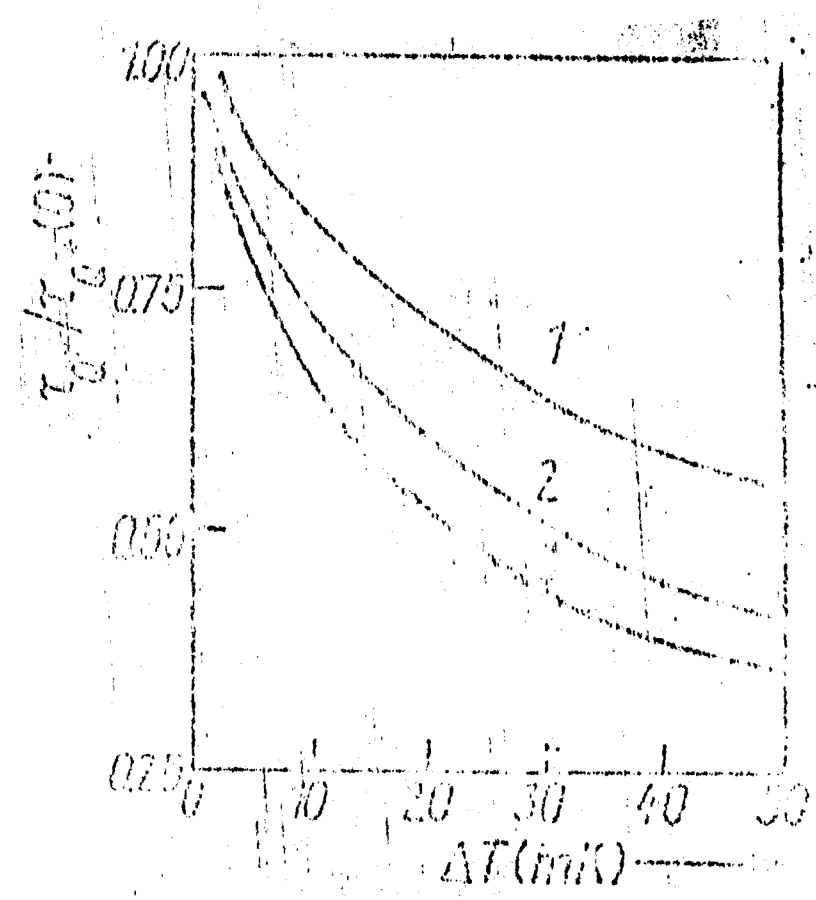
Fig. 1. The normal-like length L_n of the first phase slip centre in In whisker as a function of temperature. $T_{c0} = 3.350$ K, $T = T_{c0} - \Delta T$. o experimental results of [8]

Fig. 2. The ratio of quasiparticle relaxation time in presence of depairing due to supercurrent to the relaxation time in absence of supercurrent for In whisker as a function of temperature. (1) $\tau_E = 0.1$, (2) 0.2, (3) 0.3 ns



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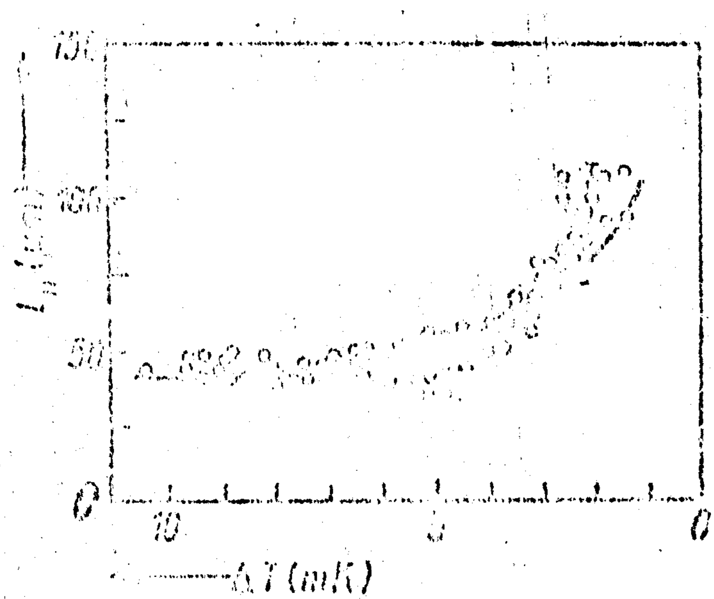

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