Measurement of Complex Dielectric Constant of Soil Particle from Bulk Measurement

INTRODUCTION:

Recently considerable interest has been devoted to evaluate the attenuation due to different dielectric bodies such as water drop, Solid precipitations, rain drop etc. For this purpose the concept of Mie scattering from different forms of the bodies has been utilized. Consequently, the expressions for the received power, and scattering cross-section have been obtained, which were further utilize to evaluate the attenuation of electromagnetic wave. The details of entire investigations are given in different sections of the chapter.
CALCULATION OF ATTENUATION BY WATER DROPS:

The fundamental calculation of the scattering and absorption of electromagnetic waves by dielectric sphere is due to Mie. Therefore using the concept of Mie scattering the total cross section is given as

\[
Q = -\frac{\lambda^2}{2\pi} \Re \sum_{n=1}^{\infty} (2n+1) \left( a_n^s + b_n^s \right)
\]  
(3.1)

Where \( \lambda \) is the free-space wavelength and \( a_n^s \) and \( b_n^s \) are given as

\[
a_n^s = -j \frac{2^n n!(n+1)!}{(2n)!(2n+1)} \frac{X^{2n+1}}{\mu_1 n + \mu_2 n + 1}
\]

\[
b_n^s = j \frac{2^n n!(n+1)!}{(2n)!(2n+1)} \frac{X^{2n+1}}{\mu_1 (n+1) + \mu_2 m^2 n}
\]

(3.2)
\[
\left\{ \mu - \mu m^2 - \frac{\mu [(n+1)+(n+3)m^2] - \mu m^2 [(n+3)+(n+1)m^2]}{(2n+2)(2n+3)} \right\}X^2 \tag{3.3}
\]

In the similar fashion the complex index of refraction of the water for the micro frequency can be given by the relation as

\[n_c = n \left( n - ix \right), \tag{3.4}\]

Solving Equation (3.5) and neglecting the higher powers of \(x\) one has

\[Q = \frac{\lambda^2}{2\pi} X^3 \left( c_1 + c_2 X^2 + c_3 X^3 \right), \quad p = X, \tag{3.5}\]

Where

\[c_1 = \frac{6 \varepsilon_2}{\left( \varepsilon_1 + 2 \right)^2 + \varepsilon_2^2}, \tag{3.6}\]

\[c_2 = \frac{\varepsilon_2}{15} \left[ \frac{3 \left( 7 \varepsilon_1^2 + 4 \varepsilon_1 - 20 + 7 \varepsilon_2^2 \right) + 25 \left( \varepsilon_1 + 3 \right)^2 (2 \varepsilon_1 + 3)^2 4 \varepsilon_2^2 + 1 \right] \tag{3.7}\]
\[ c_3 = \frac{4}{3} \left( \varepsilon_1 - 1 \right)^2 \left( \varepsilon_1 + 2 \right)^2 + \varepsilon_2^2 \left[ 2 (\varepsilon_1 - 1)(\varepsilon_1 + 2) - 9 \right] + \varepsilon_2^4 \left[ (\varepsilon_1 + 2)^2 + \varepsilon_2^2 \right]^2 \] (3.8)

It will be noticed that both \(C_1\) and \(C_2\) vanish if \(\varepsilon_2 = 0\) indicating that these terms are caused essentially by absorption. On the other hand if \(\varepsilon_2 = 0\), \(C_3\) reduces to the familiar Rayleigh scattering form,

\[ c_3 = \frac{4}{3} \left( \varepsilon_1 - 1 \right)^2 \] \(\text{ (3.9)}\)

Now for water \(\varepsilon_2\) is not zero but it is in the same order of the magnitude of \(\varepsilon_1\) Hence for sufficiently small \(x\) the absorption term \(C_1\) must predominate and the value of \(Q\) is modified as

\[ Q = \frac{4 \pi^2}{\lambda} c_1 a^3 \] \(\text{ (3.10)}\)
Empirically it is found that in the region from $\lambda = 0.5\text{cm}$ to $\lambda = 10\text{cm}$, $C_1$ varies as $1/\lambda$. Now, the corresponding attenuation $\gamma$ due to water drop can be obtained as

$$\gamma = \frac{0.433M}{\lambda^2} \text{db/km}$$

(3.11)

Where $M = 1\text{gm/m}^3$.

**Calculation of Attenuation by Precipitation in Solid Form**

The refractive index of ice has been measured by Dunsmuir and Lamb between wavelengths of 3 and 9 cm. Within these limits it appears that the real and imaginary parts of the index of refraction are independent of wavelength. It therefore seems reasonable to assume that the same values held for even shorter wavelengths. The most noteworthy difference between the
properties of water in liquid or solid form is the smallness of the absorption in the latter case. Besides greatly facilitating the calculations, the small value of refractive index $n_x$ means that the attenuation for very small particle size will be small compared with the attenuation from droplets of equal water content.

Now for this purpose ice and snow particles that are not spherical. Hence, it is difficult to use Mie formula to evaluate the scattering cross section for the solid precipitation. Now, for the simplicity of the problem, the calculations have been extended to ellipsoidal particle. Further, if the size of the dielectric body is very small compared with the wavelength it is found that the attenuation reduces to a term similar to the $C_1$ terms for spheres. For spherical dielectric body where $X^2$ terms may be neglected $C_1$ reduces to

$$C_1 = x \frac{12 n^2}{(n^2 + 2)^2}$$

(3.12)
Similarly, for long needles

\[ c_1 = x \frac{4n^2}{9} \left[ 1 + \frac{8}{(n^2 + 1)^2} \right] \]  

(3.13)

And for flat plates

\[ c_2 = x \frac{4n^2}{9} \left( 2 + \frac{1}{n^4} \right) \]  

(3.14)

With the values of n and x the attenuation \( \gamma \) can be easily calculated. The resultant formulas are summarized in grams per cubic meter. Combining Equation 3.10, 3.12, 3.13 and 3.14 the value of total cross section for spherical and non spherical dielectric bodies can be obtained as respectively.

For the spherical dielectric bodies

\[ Q = \frac{4 \pi^2}{\lambda} X \frac{12n^2}{\left( n^2 + 2 \right)^2} \]  

(3.15)
For long needles

\[
Q = \frac{4\pi^2}{\lambda} \times \frac{4n^2}{9} \left[ 1 + \frac{8}{(n^2 + 1)^2} \right]
\]

(3.16)

And for flat plates

\[
Q = \frac{4\pi^2}{\lambda} \times \frac{4n^2}{9} \left[ 2 + \frac{1}{n^4} \right]
\]

(3.17)

The corresponding attenuation \( \alpha \), can be found from relation given as

\[
\alpha = 433NQ dB / Km.
\]

Where \( N \) is the no. of dielectric bodies in the path of communication.
SCATTERING BY RAIN

In this case consider a single drop of water located at a point defined by the spherical coordinates $r_1, \phi, \theta$ as shown in Figure 01 if $P_t$ is the transmitted power then the incident power per unit area location of water droplet is given by

$$P_{inc} = \frac{P_t G (\theta, \phi)}{4 \pi r^2}$$

(3.18)

Where $G$ is the antenna gain. The backscattered power at the radar, location is

$$dP_{Bs} = P_{inc} = \frac{P}{dp} \frac{\sigma_{BS}}{4\pi r^2}$$

(3.19)

Where

$$\sigma_{BS} = 4\pi^2 (k_0 a)^4$$

(3.20)

And received power will be
\[ dP_r = \frac{\lambda_0^2}{4\pi} G(\theta, \phi) dp_{BS} \]  

(3.21)

Combining Equation 6.18, 6.19, 6.20 and 6.21 one has

\[ dP_r = \frac{\lambda_0^2}{(4\pi)^3 r^4} P_t G^2(\theta, \phi) 4\pi a^2 (K_0a)^4 \]  

(3.22)

Since the cross section depends on \((K_0a)^4\) the scattering is very weak at the longer wavelengths. The cross section given by Equation 3.20 is bases on the Rayleigh scattering theory which requires that \(a \ll \lambda_0\). At the shorter wavelengths when the drop radius becomes comparable to the wavelength in size, the backscatter cross section approaches the geometrical cross section \(\pi a^2\) and resultant backscattered power is much greater. In order to obtain an expression for total received power from an extended volume of rain, one has to assume that the scattered field from the drops is very weak compared with the incident field and thus
polarizing field acting on each drop is assumed to be the incident field only. In other words, multiple scattering is neglected. The second assumption is that the position of drops are random, so the phase of scattered electric from various drops can be considered to be a random variable uniformly distributed over the interval 0 to $2\pi$.

For a drop located at $r = r_i$, the phase of received voltage is determined by two way propagation factor $e^{-2jk_0r_j}$. The difference in phase angle is $2k_0(r_i - r_j)$ and when this is averaged over all drop locations the result is that there is no coherent in phase addition of scattered field at radar site. Consequently the average received power is simply the addition of that contributed by each drop. It is however necessary to take into account the drop size distribution since the cross section is dependent on drop radii.

We again let $N(a)$ da be number of drops per unit volume with radii in interval $a$ to $a+da$. The average backscatter cross section per unit volume is thus given by the relation as.

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\((\sigma BS) = \int_0^n 4\pi a^2 \left( k_0 a^4 \right) N(a) da \) \hspace{1cm} (3.23)

In Rayleigh region the total backscattered power is obtained by multiplying the average cross section by

\[
P_r = \frac{\lambda_0^2}{(4\pi)^3} P_t \int G^2(\theta, \phi) \frac{\left[ \sigma_{BS}(r, \theta, \phi) \right]}{r^2} \sin \theta d\theta d\phi dr \hspace{1cm} (3.24)
\]

Now the average received power may be given as

\[
P_r = \frac{\lambda_0}{(4\pi)^3} P_t \left( \frac{\sigma_{BS}}{r_0^2} \right) \frac{CT}{2} \int G^2(\theta, \phi) \sin \theta d\theta d\phi \hspace{1cm} (3.25)
\]

For a circular symmetric pattern with half power beam width \(2\theta_1/2\) we then obtain

\[
\int_0^{2\pi} \int_0^{\theta_1/2} G^2(0) \sin \theta d\theta d\phi = G^2(0) 2\pi \left(1 - \cos \theta_{1/2}\right) = G^2(0) \pi \theta_{1/2}^2 \hspace{1cm} (3.26)
\]

For a high gain antenna, the volume \(V\) of rain that is illuminated is given by
\[ V = \frac{CT}{2} \int_0^{\frac{\theta_1}{2}} \int_0^{r_0^2} r^2 \sin \theta d\theta d\phi \frac{CT}{2} r_0^2 \pi \theta_1^{\frac{1}{2}} \]  

(3.27)

Where \( V \) is defined earlier.

Then

\[ P_r = \frac{\lambda_0}{(4\pi)^3} P_l G^2(0) \left( \frac{\sigma_{BS}}{r_0^4} \right) V \]  

(3.28)

Hence the value \( \sigma_{BS} \) is obtained as

\[
\left( \sigma_{BS} \right) = \frac{2^6 \pi^5}{\lambda_0^4} N_0 \int_0^{\infty} a^6 e^{-\lambda_0 a} da
\]

\[ = 2^6 \pi^5 / \lambda_0^4 \]  

(3.29)

In this equation the numerical factor has been adjusted so that \( \lambda_0 \) is given in meters and \( R \) is the rain rate in millimeters per hour.
Bistatic Scattering from Rain

Consider a single drop of rain located at \( r_1, \theta_1, \phi_1 \) relative to antenna number. The incident field form antenna 01. Will be produced an equivalent polarization current \( j \omega P_0 \) in this drop given by

\[
j \omega P_0 = -k_0^2 Y_0 I_1 \frac{\cos \theta_1}{2\pi r_1} 4\pi a^3 f_1 \left( k x_1 \right) e^{-jk_0 r_1}
\]

Where the subscript 01 refers to antenna 01. In the above equation we have replaced \((k-1)/(k+2)\) by unit, since k is very large in the microwave region. The received open-circuit voltage at a second antenna, as shown in Figure 02, is given by the interaction of the field of this antenna, when it is radiating with unit input current, with the polarization current produced by antenna 01. There may, of course, also be direct coupling between the two antennas. However, we will assume that the antennas are positioned and
oriented such that there is no direct coupling. The received open-circuit voltage produced by scattering from one drop is thus given as

$$dV_0 = -\frac{jk_0 \cos \theta_2}{2\pi r_2} e^{-jk_0p_2} j\omega f_2(k_{x_2}k_{y_2})P_0$$

$$jk_0^3 Y_0 l_1 \frac{\cos \theta_1 \cos \theta_2}{\pi r_1 r_2} a^3 f_1 f_2 e^{-jk_0(r_1-r_2)}$$ (3.31)

Further the coordinates $r_2, \theta_2, f_2$ describe the location of the drop relative to antenna 02, as shown in Figure 02.

All of the drops in the common volume, which is defined by the volume over which the two antenna beams overlap, will contribute to the received open-circuit voltage. The resultant open-circuit voltage is obtained by summing Equation 3.31 over all drops in common volume. When the expression for received power is formulated and an ensemble average is carried out, we again find
that the average received power is the sum of that contributed by
each drop. When the sum is replaced by appropriate integrals the
final result is found as.

\[ P_r = \frac{k_0^6 Y_0^2 P_l}{4\pi^2 R_L^2} \int_0^\infty \int_0^\infty \frac{\cos^2 \theta_1 \cos^2 \theta_2}{r_1^2 r_2^2} \left| f_1, f_2 \right|^2 a^6 N(a) d\alpha d\omega \]  

(3.32)

In this formula the variables \( x, y \) and \( z \) describe the common
volume, and \( r_1, r_2, \theta_1, \) etc. Must be expressed in terms of \( x, y \) and \( z, \)
when the rain ell is far away from both antennas, \( r_1, \) and \( r_2 \) are
nearly constant through the common volume. Further, for high-gain
antennas the maximum values of \( \theta_1 \) and \( \theta_2 \) are small, so the cosine
factors can be replaced by unit. For this situation Equation 3.32 can
be replaced by the simplified formula as:

\[ P_r = \frac{k_0^6 Y_0^2 P_l}{4\pi^2 R_L^2 r_1^2 r_2^2} \int_0^\infty \int_0^\infty |f_1, f_2|^2 a^6 N(a) d\alpha d\omega \]  

(3.33)
Where \( r_1 \) and \( r_2 \) now the distances to the center of the common volume.

An important modification of Equation 3.32 and Equation 3.33 must be included when assumption of spherical drops is not valid. Now for irregular-shaped scatterers such as flattened drops, snowflakes, etc. The dipole moment \( P_0 \) is generally not in the same direction as the polarizing electric field. Then we have

\[
P_0 = \alpha e \varepsilon_0 E_0 a_z
\]

(3.34)

Where \( \alpha_e = 4\pi a^3(k-1)/(k+2) \) is called the polarizability of the particle. For irregular-shaped particles \( \alpha_e \) must be replaced by a dyadic \( \alpha_e \), and we then have

\[
P_0 = \varepsilon_0 \alpha e \cdot E_0 a_z
\]

(3.35)

The polarizability will be a function of a size parameter \( a \) corresponding to a characteristic dimension of the particle. A
formula such as Equation 35 is valid only for small particles which the Rayleigh scattering theory is applicable. With this generalization the factor $f_1 \cdot f_2 \cdot a^3$ in Equations 6.32 and 6.35 must be replaced by $f_2 \cdot \alpha \cdot f_1$. Non spherical particles will in general produce cross-polarized scattered fields, and these will cause co-channel interference between communication links operating with orthogonal polarized beams in close proximity.

**DISCUSSION OF RESULTS :-**

Assuming water drop as a dielectric bodies the phenomenon of scattering has been explained and expression for received power has been found. The power after scattering from the water drop significantly depends upon the frequency of operation and the size of rain and water drops which has been assumed as dielectric body. The value of received power sincerely depends on the presence of dielectric bodies in unit volume of layer. The received power
increases with increasing frequency of operation and particle size

Figure 03.

Fig. 01 Volume of rain cell illuminated by pulsed radar system
Fig. 02 Bistatic scattering from rain
Fig. 03 Received power increases with increasing frequency and particle size.