Chapter 6

A FUZZY PROGRAMMING TECHNIQUE TO SOLVE LINEAR PROGRAMMING RELATIONSHIP MULTICRITERIA DECISION MAKING PROBLEMS INVOLVING FUZZY PARAMETERS

6.1. Introduction

In this chapter we describe a fuzzy programming technique to solve linear programming relationship multicriteria decision making problems involving fuzzy parameters. The fuzzy parameters are represented by LR-triangular numbers. There exists limitation in the classical multicriteria decision making problems. This is because of the fact that in multicriteria decision making problems, the constraint conditions of real problems often occur in a fuzzy environment. If we solve it neglecting fuzziness, a more proper solution may sometimes be lost. Moreover, there is no unbridgeable gap between multiple criteria decision making and single criterion decision making and also between the criterion and the constraint.

Some problems are originally of the multicriteria type, however, in order to simplify the solution process, the value of some criteria is artificially restricted and the criteria are turned into constraints, hence multiple criteria becomes single criterion decision making problem. However, this artificial simplification may sometimes be inadequate. This is one of the reasons that interest is being shown in the problems on multicriteria decision making. The important issues in multicriteria decision making are how and in what form additional information which comes from decision maker can be obtained. By considering these observations, we propose a model of decision making in terms of fuzzy set theory.
In section 6.2, the concepts of fuzzy feasible set, fuzzy optimal point set and fuzzy programming are defined. These concepts are the extensions to the concepts developed by Feng [35]. In section 6.3 we develop a technique to solve linear programming relationship multicriteria decision making problems involving fuzzy parameters where the decision situations are considered with infinite set of alternatives. An example is illustrated in section 6.4.

6.2. Fuzzy value set, Fuzzy constraint, Fuzzy optimum point set and Fuzzy programming

In this section we develop the concept of fuzzy multicriteria decision making model from classical multicriteria decision making problem. To develop this concept we require some definitions.

Generally, the classical problems of multicriteria decision making problem under infinite set of alternatives is stated as

\[
\begin{align*}
\text{max} & \quad F(x) \\
\text{subject to} & \quad g(x) \geq b
\end{align*}
\]  

(6.2.1)

where \( F(x) = (f_1(x), f_2(x), \ldots, f_p(x))^T \)

\( g(x) = (g_1(x), g_2(x), \ldots, g_m(x))^T \)

\( b = (b_1, b_2, \ldots, b_m)^T \)

\( x \in \mathbb{R}^n \)

In order to solve the above-mentioned problem, we consider a more general model of decision making in terms of fuzzy subsets theory. The classical model of decision making is only a special case of this fuzzy model which contains more realistic cases that cannot be included in the classical model. A method using fuzzy subset theory to solve the multi-objective problem was given in Feng ([33],[34],[35]) and Feng and Wei [32] we extend those results.

**Definition 6.2.1.** Let \( f(x) \) be a real valued function defined on \( X \subseteq \mathbb{R}^n \). A fuzzy set \( \tilde{B} \) is called a fuzzy value set of \( f(x) \) if \( \tilde{B} \) is a fuzzy subset on the value set \( R \) of \( f(x) \) with membership function \( \mu_{\tilde{B}}(f(x)) \).
Definition 6.2.2. Let $\tilde{A}$ be a fuzzy subset on $X \subset \mathbb{R}^n$ and $\tilde{B}_i$ be a fuzzy value set of $g_i(x)$. Then $\tilde{A}$ is called a fuzzy constraint of $g_i(x)$ if $
abla \mu_{\tilde{A}} = \mu_{\tilde{B}_i}(g_i(x))$.

Now, if

$$
\mu_{\tilde{A}}(x) = \mu_{\tilde{B}_i}(g_i(x)) = \begin{cases} 0 & \text{for } g_i(x) < b_i \\
1 & \text{for } g_i(x) \geq b_i
\end{cases}
$$

then the fuzzy constraint with respect to $\tilde{B}_i$ is equivalent to the classical constraint $g_i(x) \geq b_i$. However, a fuzzy constraint can provide more information than a classical constraint (Feng, [35]).

Definition 6.2.3. Let $\tilde{C}$ be a fuzzy subset on $X \subset \mathbb{R}^n$ and $\tilde{A}$ be a fuzzy constraint of $g_i(x)$ with respect to $\tilde{B}_i, i = 1, 2, ..., m$. Then $\tilde{C}$ is called a fuzzy feasible set with respect to $\tilde{B}_i$ if

$$
\mu_C(x) = \bigwedge_{1 \leq i \leq m} \mu_{\tilde{A}_i}(x) = \min \mu_{\tilde{A}_i}(x)
$$

Definition 6.2.4. Let $\tilde{B}_i^*$ be a fuzzy value set of $f_i(x)$ on $(-\infty, M_i)$ where $M_i = \sup_{x \in X} f_i(x)$. $\tilde{B}_i^*$ is called a fuzzy optimum set of component $f_i(x)$ of $F(x)$ if

$$
\mu_{\tilde{B}_i^*}(y) = \text{strictly monotone increasing function on } [m_i, M_i] \text{ and } \mu_{\tilde{B}_i^*}(y) = 0 \text{ for } f_i(x) < m_i \text{ where } m_i \geq \inf_{x \in X} f_i(x).
$$

Definition 6.2.5. Let $\tilde{A}^*$ be a fuzzy subset on $X \subset \mathbb{R}^n$. $\tilde{A}^*$ is called a fuzzy optimum point set of $F(x)$ if

$$
\mu_{\tilde{A}^*}(x) = \bigwedge_{1 \leq i \leq p} \mu_{\tilde{A}_i}(x) = \min \mu_{\tilde{A}_i}(f_i(x))
$$

Definition 6.2.6. Let $\tilde{H}^*$ be a fuzzy subset on $X \subset \mathbb{R}^n$. $\tilde{H}^*$ is called a fuzzy optimum point set of $F(x)$ on a fuzzy feasible set $\tilde{C}$ if $\tilde{A}^*$ is a fuzzy optimum point set of $F(x)$ and

$$
\mu_{\tilde{H}^*}(x) = \mu_{\tilde{A}^*}(x) \mu_C(x) = \min(\mu_{\tilde{A}^*}(x), \mu_C(x))
$$
Then $\mu_{y^*}(x) \rightarrow \max_{x \in \mathcal{X}}$ (6.2.2)
is called a fuzzy programming problem with respect to $\tilde{H}^*$.

**Definition 6.2.7.** Point $\mathcal{X}$ is called an optimal solution of fuzzy programming problem, (6.2.2) if

$$\mu_{y^*}(\mathcal{X}) = \max_{x \in \mathcal{X}}(\mu_{y^*}(x)) > 0$$

From the definitions (6.2.3) and (6.2.5), it is seen that the relationship between a fuzzy feasible set $\tilde{C}$ and a fuzzy optimum point set $\tilde{A}^*$ is symmetric. The unique difference is that the membership functions $\mu_{y^*}(y)$ of the fuzzy optimum point set $\tilde{B}_i^*, i = 1, 2, ..., p$, whose intersection forms the fuzzy optimum point set $\tilde{A}^*$ of $F(x)$, are strictly monotone increasing functions on $[m_i, M_i]$. The fuzzy optimum set $\tilde{B}_i^*$ is only a special case of the fuzzy constraint. In this way the concepts of criterion and constraint are unified. Obviously, it is easier to represent the decision maker’s requirements by membership functions $\mu_{y^*}(y)$ of the fuzzy optimum set $\tilde{B}_i^*$. The decision maker can express different requirements for the individual criteria by using different membership functions $\mu_{y^*}(y)$.

In definition 6.2.4, it is realistic to require the membership function $\mu_{y^*}(y)$ of the fuzzy optimum set $\tilde{B}_i^*$ to be strictly monotone increasing on $[m_i, M_i]$. If this was not fulfilled, the corresponding optimal solution of fuzzy programming, Eqn (6.2.2) would not always be an efficient or weak-efficient solution. The concepts of efficient and weak-efficient solution of multicriteria decision making problem are defined as follows.

**Definition 6.2.8.** Let $x \in \mathcal{R}, \mathcal{X}$ is called an efficient solution of $F(x)$ on $\mathcal{R}$ if there is no $x \in \mathcal{R}$, such that $f_i(x) \geq f_i(x), i \in \{1, 2, ..., p\}$ and $f_i(x) > f_i(x), \text{ for at least one } i \in \{1, 2, ..., p\}$. 
Again, $\tau$ is called a weak-efficient solution of $F(x)$ on $R$ if there is no $x \in R$, such that $f_i(x) > f_i(\tau)$, $i = 1, 2, \ldots, p$.

Thus, if $\tau$ is an efficient solution on $R$, then $\tau$ is a weak-efficient solution on $R$. The converse is true only under some restrictions i.e., when $F(x)$ and $g(x)$ are concave vector functions.

The determination of an optimal solution to fuzzy programming, Eqn(6.2.2) is equivalent to finding a solution to a nondifferentiable optimization problem (Feng, [36]) as given below:

$$
\lambda \rightarrow \max \\
\text{subject to } \mu_{\lambda}(x) = \mu_{\xi}(f_i(x)) \geq \lambda \quad i = 1, 2, \ldots, p \\
\mu_{\lambda}(x) = \mu_{\eta}(g_j(x)) \geq \lambda \quad j = 1, 2, \ldots, m.
$$

(6.2.3)

This transformation leads both to a method for solving fuzzy programming problems and yields a “grade of membership”, $\lambda \in [0, 1]$. If $\lambda = 0$, then there exists no solution fulfilling the decision maker’s requirement. In the following we propose a fuzzy model for linear programming relationship multicriteria decision making problem involving fuzzy parameters.

6.3. Linear Programming Relationship Multicriteria Decision Making involving Fuzzy parameters

In many practical multicriteria decision making situations, it is not reasonable to require that the constraints or the criteria in multicriteria decision making problems be specified in precise, crisp terms. In such situations, it is desirable to use the concepts of fuzzy programming.

The general type of fuzzy linear programming relationship multicriteria decision making model is formulated as follows
\[
\max \left\{ \sum_{j=1}^{n} \tilde{C}_{ij} \tilde{x}_j, \sum_{j=1}^{n} \tilde{C}_{2j} \tilde{x}_j, \ldots, \sum_{j=1}^{n} \tilde{C}_{pj} \tilde{x}_j \right\}
\]

subject to \[\sum_{j=1}^{n} \tilde{A}_{ij} \tilde{x}_j \leq \tilde{B}_i, \quad i = 1, 2, \ldots, m \quad (6.3.1)\]

\[\tilde{x}_j \geq 0, \quad j = 1, 2, \ldots, n\]

where \(\tilde{C}_{ij}, (k = 1, 2, \ldots, p)\), \(\tilde{A}_{ij}, \tilde{B}_i\) are fuzzy numbers, and \(\tilde{x}_j\) are fuzzy variables whose states are fuzzy numbers, the operations of addition and multiplication are operations of fuzzy arithmetic, and \(\preceq\) denotes the ordering of fuzzy numbers.

Instead of discussing this general type, we consider the issues involved by a special case of fuzzy multicriteria decision making problem in which the coefficients \(\tilde{A}_{ij}\) of the constraint and the right-hand-side members \(\tilde{B}_i\) are triangular fuzzy numbers and \(\tilde{x}_j\) are real variables \(x_j\) and \(\tilde{C}_{ij}\) are real constants \(c_{ij}\). Let any triangular fuzzy number \(\tilde{A}\) be represented by three real numbers \(s, l, r\) whose meaning are defined in Fig 6.3.1. Let it be denoted by \(\tilde{A} = (s, l, r)\)

\[
\text{Fig.6.3.1 Triangular fuzzy number}
\]

Then problem (6.3.1) can be rewritten as

\[
\max \left\{ \sum_{j=1}^{n} c_{1j} x_j, \sum_{j=1}^{n} c_{2j} x_j, \ldots, \sum_{j=1}^{n} c_{pj} x_j \right\}
\]

subject to \[\sum_{j=1}^{n} <s_{ij}, l_{ij}, r_{ij}> x_j \preceq <t_{ij}, u_{ij}, v_{ij}> \quad i = 1, 2, \ldots, m \quad (6.3.2)\]

\[x_j \geq 0, \quad j = 1, 2, \ldots, n.\]
where \( \tilde{A}_i = <s_i,l_i,r_i> \) and \( \tilde{B}_i = <t_i,u_i,v_i> \) are triangular fuzzy numbers, and the partial order \( \leq \) is defined by \( \tilde{A} \leq \tilde{B} \) iff \( \max(\tilde{A}, \tilde{B}) = \tilde{B} \). Alternatively, for any two fuzzy numbers \( \tilde{A} = <s_1,l_1,r_1> \) and \( \tilde{B} = <s_2,l_2,r_2> \), \( \tilde{A} \leq \tilde{B} \) iff \( s_1 \leq s_2 \), \( l_1 \leq l_2 \), and \( s_1 + r_1 \leq s_2 + r_2 \). Also, \( <s_1,l_1,r_1> + <s_2,l_2,r_2> = <s_1 + s_2, l_1 + l_2, r_1 + r_2> \) and \( <s_1,l_1,r_1> x = <s_1,l_1,r_1,x> \) for \( x \geq 0 \). Then, the problem can be rewritten (Klir and Yuan [63]) as

\[
\max \{ \sum_{j=1}^{n} c_{ij}x_j, \sum_{j=1}^{n} c_{ij}x_j, \ldots, \sum_{j=1}^{n} c_{ij}x_j \} \\
\text{subject to} \\
\sum_{j=1}^{n} s_jx_j \leq t_i, \quad i = 1,2,\ldots,m \\
\sum_{j=1}^{n} (s_j - l_j)x_j \leq t_i - u_i, \quad i = 1,2,\ldots,m \\
\sum_{j=1}^{n} (s_j + r_j)x_j \leq t_i + v_i, \quad i = 1,2,\ldots,m \\
x_j \geq 0, (j = 1,2,\ldots,n)
\]

(6.3.3)

In order to determine the infimum and supremum of each objective function \( \sum_{j=1}^{n} c_{ij}x_j, \quad k = 1,2,\ldots,p, \) we consider two categories of LPP as follows:

**Category I.** In this category we develop \( p \) linear programming problems:

\[
\max \sum_{j=1}^{n} c_{ij}x_j \quad (k = 1,2,\ldots,p) \\
\text{s.t.} \sum_{j=1}^{n} (s_j - l_j)x_j \leq t_i - u_i, \quad i = 1,2,\ldots,m \\
x_j \geq 0, \quad (j = 1,2,\ldots,n)
\]

(6.3.4)

**Category II.** In this category we consider another \( p \) linear programming problems:
\[
\begin{align*}
\max & \sum_{j=1}^{n} c_{ij} x_j \quad (k = 1,2,\ldots,p) \\
\text{s.t.} & \sum_{j=1}^{n} (s_{ij} + r_{ij}) x_j \leq l_i + v_i \quad (i = 1,2,\ldots,m) \\
& x_j \geq 0 \quad (j = 1,2,\ldots,n).
\end{align*}
\tag{6.3.5}
\]

If \( m_1,m_2,\ldots,m_p \) are the values of objective functions of \( p \) LP problems of category I for each \( k = 1,2,\ldots,p \) respectively, and \( M_1,M_2,\ldots,M_p \) are the values of the objective functions of category II for each \( k = 1,2,\ldots,p \), then we construct the membership functions of each objective function as follows:

\[
\mu_{\bar{b}_k}(f_k(x)) = \begin{cases} 
0 & \text{for } f_k(x) < m_k \\
\frac{f_k(x) - m_k}{M_k - m_k} & \text{for } m_k \leq f_k(x) \leq M_k, \quad k = 1,2,\ldots,p.
\end{cases}
\tag{6.3.6}
\]

where \( f_k(x) = \sum_{j=1}^{n} c_{ij} x_j \). It is noted that the choice of \( m_k \) and \( M_k \) depends on the values of \( m_k \) and \( M_k \).

Now, each \( \mu_{\bar{b}_k}(f_k(x)) \) is strictly monotone increasing function on \([m_k,M_k]\).

We then construct a fuzzy constraint, \( \tilde{C} \), for the problem (6.3.2) as follows:

\[
\mu_C(x) = \begin{cases} 
0 & \text{for } \sum_{j=1}^{n} s_{ij} l_j + r_{ij} > x_i \geq <t_i,u_i,v_i> \quad i = 1,2,\ldots,m \\
1 & \text{elsewhere}
\end{cases}
\tag{6.3.7}
\]

Therefore, the fuzzy programming becomes

\[
\max \{ \min(\mu_{\bar{b}_k}(f_k(x)),\mu_C(x)) \} \quad k = 1,2,\ldots,p
\tag{6.3.8}
\]

As well known, it is equivalent to solving the following LP problem:
Using centre of maxima defuzzification method where defuzzification value of

\[ <s, l, r> = \frac{(s - l) + (s + l)}{2}, \] the problem (6.3.9) can be rewritten as

\[
\begin{align*}
\max & \quad \lambda \\
\text{subject to} & \quad \frac{f_k(x) - m_k}{M_k - m_k} \geq \lambda \quad (k=1,2,...,p) \\
\sum_{j=1}^{n} s_j x_j & \leq t_i, \quad i = 1,2,...,m \\
x_j & \geq 0, \quad (j = 1,2,...,n). \\
0 \leq \lambda & \leq 1.
\end{align*}
\]

Since all numbers involved in problem (6.3.10) are real numbers, this is a classical linear programming problem which can be solved by using simplex method. If \( \bar{x} \in R^n \) is the optimal solution of problem (6.3.9), then \( \bar{x} \) be the optimal solution of fuzzy programming (6.3.2).

As the fuzzy programming (6.3.2) is involved in multicriteria decision making problem (6.2.1), we require to show a relation between an optimal solution of fuzzy programming (6.3.2) and important concepts of multicriteria decision making problem (6.2.1), the efficient or weak-efficient solution (definition 6.2.8), so that the theoretical stability of that solution be established. For this purpose we consider the following theorem.

**Theorem 6.3.1.** Let \( \bar{x} \) be a weak-efficient solution of \( F(x) \) on \( C = \{x : g(x) \geq b\} \).

Then there is a fuzzy feasible set \( \tilde{C} \) and a fuzzy optimal point set \( \tilde{A} \) such that \( \bar{x} \) is an optimum solution of fuzzy programming (6.3.1).
Proof: We construct the fuzzy constraint by defining the membership function

\[
\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(g_i(x)) = \begin{cases} 
1 & \text{for } g_i(x) \geq b_i, i = 1, 2, \ldots, m \\
0 & \text{for } g_i(x) < 0.
\end{cases}
\]

We define the membership function of a fuzzy feasible set as

\[
\mu_{\tilde{C}}(x) = \Lambda_{i \in \mathcal{S}_m} \mu_{\tilde{A}_i}(x)
\]

Then, we construct a fuzzy optimum point set \( \tilde{A}^* \) as follows.

Let \( 0 < \theta < 1 \). If \( f_k(\bar{x}) \neq M_k \) and \( f_k(\bar{x}) \neq m_k, \ k \in \{1, 2, \ldots, p\} \), then we define the membership function of a fuzzy value set as

\[
\mu_{\tilde{B}_i}(y) = \begin{cases} 
\frac{y - m_k}{f_k(\bar{x}) - m_k} \theta & \text{for } m_k \leq y < f_k(\bar{x}) \\
\frac{y - f_k(\bar{x})}{M_k - f_k(\bar{x})} (1 - \theta) + \theta & \text{for } f_k(\bar{x}) \leq y \leq M_k
\end{cases}
\]

If there is some \( k_0, \ k_0 \in \{1, 2, \ldots, p\} \) such that \( f_{k_0}(\bar{x}) = M_{k_0} \) or \( f_{k_0}(\bar{x}) = m_{k_0} \), then we construct the membership function of the fuzzy value set as

\[
\mu_{\tilde{B}_i}(y) = \frac{y - m_{k_0}}{M_{k_0} - m_{k_0}} \theta \quad \text{for } m_{k_0} \leq y \leq f_{k_0}(\bar{x}) \text{ and}
\]

\[
\mu_{\tilde{B}_i}(y) = \frac{y - m_{k_0}}{M_{k_0} - m_{k_0}} (1 - \theta) + \theta \quad \text{for } f_{k_0}(\bar{x}) \leq y \leq M_{k_0}
\]

where \( m_{k_0} \) and \( M_{k_0} \) are the infimum and supremum of \( f_{k_0}(x) \), respectively. Now, \( \mu_{\tilde{B}_i}(y) \)'s are strictly monotone increasing on \([m_{k_0}, M_{k_0}]\).

Suppose that \( \mu_{\tilde{A}_i}(x) = \mu_{\tilde{B}_i}(f_k(x)), k = 1, 2, \ldots, p \) and \( \mu_{\tilde{A}}(x) = \Lambda_{i \in \mathcal{S}_p} \mu_{\tilde{A}_i}(x) \), then \( \tilde{A}^* \) is a fuzzy optimum point set and

\[
\mu_{\tilde{A}_i}(\bar{x}) = \mu_{\tilde{B}_i}(f_k(\bar{x})) = \theta, k = 1, 2, \ldots, p.
\]
Now, we are to prove that a weak-efficient \( \bar{x} \) is an optimum solution of fuzzy programming (6.3.1).

Clearly, \( \mu_{H_j}(\bar{x}) > 0 \). Suppose that \( \bar{x} \) is not an optimum solution of (6.2.2). Then there exists an \( \bar{x} \in R = \{ x : g(x) \geq b \} \), such that \( \mu_{H_j}(\bar{x}) > \mu_{H_j}(\bar{x}) \).

Since \( \mu_{H_j}(\bar{x}) = \mu_{A_j}(\bar{x}), \mu_{H_j}(\bar{x}) = \mu_{A_j}(\bar{x}) \)

We have \( \min_{\lambda \leq k \leq p} \mu_{A_k}(\bar{x}) > \min_{\lambda \leq k \leq p} \mu_{A_k}(\bar{x}) \).

Therefore,
\[
\mu_{A_k}(\bar{x}) \geq \min_{\lambda \leq k \leq p} \mu_{A_k}(\bar{x}) > \min_{\lambda \leq k \leq p} \mu_{A_k}(\bar{x}) = \mu_{A_k}(\bar{x}) = \theta
\]
\( k = 1, 2, \ldots, p \).

This follows that \( \mu_{B_j}(f_k(\bar{x})) > \mu_{B_j}(f_k(\bar{x})), k = 1, 2, \ldots, p \).

Because \( \mu_{B_j}(y) \) are strictly monotone increasing functions for each \( k \), we obtain
\[
f_k(\bar{x}) > f_k(\bar{x}), \quad k = 1, 2, \ldots, p,
\]
which contradicts that \( \bar{x} \) is a weak – efficient solution of \( F(x) \) on \( C \).

This completes the relationships between optimal solution of fuzzy programming and multicriteria programming.

6.4. Numerical Example

Based on the proposed method, we have developed an illustrative example which is designed to test the model. We consider the following linear programming relationship two- criteria decision making problems with fuzzy parameters involved in the constraints:

A company manufactures two products A and B under given capacities, Product A yields a profit of 2 units per piece and product B of 1 units per piece. Product A needs imported row material of 1 unit per piece. The two goals are an optimal balance of trade and a maximum profit while the capacity constraints are, e.g.
The fuzzy parameters $\tilde{a}_{ij}, \tilde{b}_{ij}, i = 1, 2, 3, 4; j = 1, 2.$ are triangular fuzzy numbers as explained in Table 6.4.1.
This problem can be modelled as follows:

$$\max f_1(x) = x_1 + 2x_2 \quad \text{balance of trade}$$

$$\max f_2(x) = 2x_1 + x_2 \quad \text{profit}$$

Subject to

the capacity constraints of (6.4.1).

Table 6.4.1. Triangular fuzzy numbers.

<table>
<thead>
<tr>
<th>Triangular fuzzy nos.</th>
<th>$s$</th>
<th>1</th>
<th>$r$</th>
<th>1</th>
<th>$t$</th>
<th>2</th>
<th>$u$</th>
<th>3</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{a}_{11}$</td>
<td>-2</td>
<td>0.5</td>
<td>0.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{a}_{22}$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{a}_{21}$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{a}_{31}$</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{a}_{32}$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{a}_{41}$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
Writing in the full form the problem can be rewritten as follows:

\[
\begin{align*}
\text{Max} & \quad f_1(x) = x_1 + 2x_2 \\
\text{Max} & \quad f_2(x) = 2x_1 + x_2 \\
\text{Subject to} & \quad \begin{cases}
-1, 0.5, 0.8 & < x_1 + < 3, 1, 2 > x_2 \leq 21, 5, 6 \\
1, 1, 2 & > x_1 + < 3, 1, 2 > x_2 \leq 27, 6, 7 \\
4, 2, 3 & < x_1 + < 3, 1, 2 > x_2 \leq 45, 10, 12 \\
3, 1, 2 & > x_1 + < 1, 1, 2 > x_2 \leq 30, 8, 9 \\
x_1, x_2 \geq 0
\end{cases}
\end{align*}
\]

Now we solve the following LP problems.

**Category (I):** \( \text{max } f_1(x) = x_1 + 2x_2 \)

s.t.

\[
\begin{align*}
-1.5x_1 + 2x_2 & \leq 16 \\
0x_1 + 2x_2 & \leq 21 \\
2x_1 + 2x_2 & \leq 35 \\
2x_1 + 0x_2 & \leq 22
\end{align*}
\]

The solution is found to be \( f(x_1) = 28, x_1 = 7, x_2 = 10.50 \).
s.t. 
\[ \begin{align*} 
-0.2x_1 + 5x_2 & \leq 27 \\
3x_1 + 5x_2 & \leq 34 \\
7x_1 + 5x_2 & \leq 57 \\
5x_1 + 3x_2 & \leq 39 
\end{align*} \]  
(6.4.4)

The solution problem (6.4.4) is obtained as \( f_i(x) = 13.16, x_1 = 2.19, x_2 = 5.49. \)

**Category (II):** \[ \begin{align*} 
\text{max } f_2(x) &= 2x_1 + x_2 \\
\text{s.t.} \\
-1.5x_1 + 2x_2 & \leq 16 \\
0x_1 + 2x_2 & \leq 21 \\
2x_1 + 2x_2 & \leq 35 \\
2x_1 + 0x_2 & \leq 22 
\end{align*} \]  
(6.4.5)

The solution is found to be \( f_2(x) = 28.50, x_1 = 11, x_2 = 6.50. \) \[ \begin{align*} 
\text{max } f_2(x) &= 2x_1 + x_2 \\
\text{s.t.} \\
-0.2x_1 + 5x_2 & \leq 27 \\
3x_1 + 5x_2 & \leq 34 \\
7x_1 + 5x_2 & \leq 57 \\
5x_1 + 3x_2 & \leq 39 
\end{align*} \]  
(6.4.6)

The solution is \( f_2(x) = 15.60, x_1 = 7.80, x_2 = 0. \)

Then, we assign \( M_1 = 28, M_2 = 28.50, m_1 = 13.16, m_2 = 15.60. \)

Therefore, we have

\[ \mu_{\beta'}(f_1(x)) = \begin{cases} 
0 & \text{for } x_1 + 2x_2 < 13.16 \\
\frac{x_1 + 2x_2 - 13.16}{28 - 13.16} & \text{for } 13.16 \leq x_1 + 2x_2 \leq 28 
\end{cases} \]

\[ \mu_{\beta'}(f_2(x)) = \begin{cases} 
0 & \text{for } 2x_1 + x_2 < 15.60 \\
\frac{2x_1 + x_2 - 15.60}{28.50 - 15.60} & \text{for } 15.60 \leq 2x_1 + x_2 \leq 28.50 
\end{cases} \]

By using (6.3.10) problem (6.4.2) becomes
\[ \text{max } \lambda \]
\[ \text{s.t.} \]
\[ x_1 + 2x_2 - 14.84 \lambda \geq 13.16 \]
\[ 2x_1 + x_2 - 12.9 \lambda \geq 15.60 \]
\[ -x_1 + 3x_2 \leq 21 \]
\[ x_1 + 3x_2 \leq 27 \]
\[ 4x_1 + 3x_2 \leq 45 \]
\[ 3x_1 + x_2 \leq 30 \]
\[ \lambda \geq 0 \]
\[ \lambda \leq 1 \]
\[ x_1, x_2 \geq 0 \]

After 5\textsuperscript{th} iterations, the optimal solution is obtained as 
\[ z = 0.33, x_1 = 7.20, x_2 = 5.40, \lambda = 0.33. \]

6.5. Conclusion

In the proposed method we introduced a new concept to determine the worst value \( m_k \) and best values \( M_k \) of each objective function \( f_k(x) \), \( k = 1, 2, ..., p \) by introducing modified constraints. The modified constraints are constructed from the original constraints by considering the fuzzy coefficients involved in the problem. Considering the worst and the best values as the lower and upper bounds respectively, of the range of values of the objective function, we construct fuzzy membership function of each objective function where the membership function is monotonically increasing in the closed interval \([m_k, M_k]\). Introducing the suitable operator, say minimum operator among the membership functions of the objective functions and fuzzy constraints of the problem, the original problem is transformed into a linear parametric programming problem where the fuzzy coefficients of the constraints are represented by the centre of maximum defuzzification values. The solution procedure suggests a cost-effective technique in solving linear programming based multicriteria decision making problems under fuzzy environment. The numerical example illustrates the efficiency of the proposed technique.