CHAPTER IV
IDENTITY AS THE LOGIC
OF SCIENTIFIC DISCOVERY

4.1 Identity as Functional Dependence

The analysis (in the preceding chapter) identified the empirical constraint on scientific classification in terms of the concept of a primitive logic of classification. Furthermore it was argued that the logic of two relations viz. Wittgenstein's theory of Family Resemblance, and Leibniz's Law of Identity, qualify as primitive in the requisite sense. In addition (to this constraint) scientific classifications (like all taxonomies) need to satisfy constraints in terms of (specific) goals and purposes. This appears to be one question on which philosophers of science voice near unanimity viz. that the aim of science is explanation and prediction. It is now proposed to analyse the form of law generated by classifications based on the law of identity;\(^1\) which at the same time, satisfies the constraint of predictive inference.

First we reiterate the intimate connection between systems of classification and forms of law. In this context Quine's ([1977] p. 168-170) discussion of the relevance of

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\(^1\) Although Wittgenstein's relation of family resemblance is primitive, it is not proposed to explore the form of law, if any, appropriate to this form of classification.
kinds for explicating the 'dim' notions of cause, of dispositional terms and of subjective conditionals, is illuminating.\footnote{Quine ([1977] p.168-170) as already noted, whilst condemning similarity as 'logically repugnant' nevertheless holds that similarity is the constitutive principle or at least correlative with kinds. This is in contrast to the position developed in this thesis i.e. of identity as the logic of kinds. This difference does not however affect the point at issue viz. the close relation between laws, law-terms, and predictive inference on the one hand, and systems of classification on the other.} Using Carnap's example of 'soluble' as a dispositional (law-like) term, Quine says: To say of some individual object that it is soluble in water is not to say merely that it always dissolves when in water, because this would be true by default of any object, however insoluble, if it merely happened to be destined never to get into water. It is to say rather that it would dissolve if it were in water; but this account brings small comfort, since the device of a subjunctive conditional involves all the perplexities of dispositional terms and more. Thus far I simply repeat Carnap. But now I want to point out what could be done in this connection with the notion of kind. Intuitively what qualifies a thing as soluble though it never gets into water is that it is of the same kind as the things that actually did or will dissolve ....'. The point that Quine is making is that dispositional terms i.e. law-like terms are natural kind terms; and that subjunctive conditional (predictive) inference goes through only
relative to (natural kind) classification.

Again Quine ([1977] p. 169) maintains: 'Another dim notion, which has intimate connections with dispositions and subjunctive conditionals is the notion of cause; and we shall see that it too turns on the notion of kinds. Hume explained cause as invariable succession; and this makes sense as long as the cause and effect are referred to by general terms. We can say that fire causes heat and we can mean thereby, as Hume would have it, that each event classifiable under the head of fire is followed by an event classifiable under the head of heat, or heating up. But this account, whatever its virtues for these general causal statements, leaves singular causal statements unexplained'. Quine clarifies this point in the following manner: If a singular event is succeeded by another event then the simple fact of succession does not explain the law-likeness of the connection. Furthermore, an arbitrary assignment of the preceding event to a class (set) and the succeeding event to yet another class does not yet solve the problem. (For sets could always be rigged up arbitrarily). Singular causal statements make (law-like) sense only when the events concerned are referred to natural kind sets. Therefore Quine remarks: 'What I wanted to bring out is just the relevance of the notion of kinds, as the needed link between singular and general causal statements'. What Quine is now maintaining is that inferences to universal generalizations
go through only relative to (a primitively accessible) classification of natural kinds. It follows that the principle of classification of natural kinds cannot be universal generalisation.

Hesse\(^3\) ([1974] p. 71-72) emphasizes much the same points as Quine. She says: "In the history of philosophy the problems of universals and of natural laws are closely connected. Aristotle's "stabilisation of the universal in the mind" as a result of reflection on experience, is his account both of how we come to predicate a new object correctly as "swan", and also of how we know "all swans are white", for "swanness" is a complex universal incorporating "whiteness" ...... The account of causality or law-like relations, which depends on regularities of co-presence, co-absence and covariance, may thus be seen as parallel to an account of qualities as classes defined by their similarities and differences. Directly experienced spatial and temporal relations between objects required for causality are then seen as parallel to directly experienced resemblances required for the definition of reference classes'. Hesse can be interpreted very simply as making the point that laws are framed in terms of properties

\(^3\) Hesse like Quine upholds the resemblance view of universals. As in the case of Quine, this does not affect the point at issue viz. the dependence of law's upon universals.
relevant to natural kind classes; and that it is this feature which distinguishes natural laws from accidental generalisations. This is a reiteration of Quine's contention that claims to universal regularities go through, relative to natural kind classification; and that predictive and counterfactual inference rests therefore, not on universality per se, but on the underlying logic of kind classification.

Again Frederick Suppe ([1977] p. 628-629) points out that Hinkikka [1976] and his Finnish colleagues also employ natural kind inference to justify probabilistic induction to universal generalizations. They attempt to modify Carnap's basic approach to inductive logic so as to obtain probability measures which assign non-zero probabilities to generalisations. Whereas Carnap's state descriptions are descriptive of individuals and their attributes, their approach is to construe state descriptions as being about kinds of individuals Suppe remarks that this shifts the problem of justifying probabilistic induction to the question of justifying the classification (of kinds) which underlies probabilistic inductive inferences to universal generalisations.

More recently, John MacNamara [1991] claims to offer a better understanding of induction; one that assimilates it to induction based on essential properties rather than to
statistical inference. This is made possible, according to MacNamara, by appealing to the logic of common nouns and applying it to the logic of natural kind-terms.

The extensive literature on natural-kind inference (exemplified above) indicates (i) the crucial relevance of classification to laws: on this view laws are explicit articulations of the underlying classificatory structure, and (ii) claims to or assumptions of universality go through only relative to natural kind classification. It follows that the constitutive principles of class organization cannot be that of universal generalisation.

This intuition seems to be supported by Popper ([1972] p. 422). Thus Popper in replying to the criticism of William Kneale (Popper [1972]) admits that there are structural theories in science (which include the atomic theory, Newton's laws of motion, and the law of universal gravitation) whose form is not really that of universal generalisation. Popper says that although these laws might be expressed as universal generalizations, yet the 'all' form is comparatively unimportant in the case of these laws. '... The difficulty with these structural theories is not so much to establish the universality of the law from repeated instances as to establish that it holds even for a single instance'. Yet, Popper does not offer any suggestion for the form of such laws, and maintains that William Kneale
does not succeed in making clear what the difference is between a universal statement and a 'principle of necessitation'.

The thesis is now put forth that in the case of classifications based on the principle of identity, the logic of identity (as exemplified by Leibniz's Law); in conjunction with the constraint of predictive inference (specified as the aim of scientific classification) indicates the appropriate form of law as that of functional dependence between properties of the same (not similar) object. This thesis reinterprets the controversial concept of nomic necessity both in terms of an intuitive notion of relational structure and more strongly, in the sense of a (functional) rule-bound correlation of properties.

This thesis can be defended in the following manner (i) Firstly, as has already been argued (in the preceding Chapter), the logic of the relation of identity is that of a primitive relation which in principle, can only be indexically indicated (extensionally exemplified) and not intensionally defined. From this it follows that properties and relations associated with a kind term (based on identity) do not constitute its intension; they only specify the structure of the kind. (ii) Furthermore since identity is a 'totally reflexive' relation. (Copi [1986] p.387) it holds only between the internal properties of a substance,
and (iii) finally since identity is a *relation*, it classifies kinds only on the basis of (internal) relations that hold (between properties); and not on the basis of the properties of kinds.

The implications of the foregoing analysis can be elaborated as follows: Point (iii) rules out the absolute theory of universals and the associated form of law viz. universal generalisation as appropriate for empirical systems. This can be understood in the following way: To reiterate the formulation of Hesse, [1976] according to the absolute theory *P* is predicated of an object *a* in virtue of its (absolute) possession of *P*-ness i.e. of a conjunction of properties. The emphasis of the absolute theory is on the properties (of objects/substances) and not on the relation of co-presence. This seems to be because mere co-presence (of properties) satisfies no intuition of necessary structure; nor does co-presence in individual cases permit predictive inference to future, counterfactual or subjunctive conditional cases. In brief, co-presence in individual cases is not a law-like relation - it exemplifies a Humean or radical empiricist conception of the Universe; wherein as the early Wittgenstein would put it, there is no metaphysical cement structuring properties into wholes.

Furthermore, neither the gratuitious assumption of universality (which amounts to Popper’s hypothetico-
deductive model) nor the (illegitimate) inductive inference to generality (implicit in Hempel's deductive nomological model) is sustained by the logic of natural kind classifications based on identity. To appreciate this we need only note that since identity is a primitive relation, properties associated with the kind-term do not constitute necessary or essential properties. Hence the assumption of, or inference to universality is unsupported by any intuition of necessity regarding (the conjunction of) properties associated with natural kind terms. Hence when universality is assumed or inferred, it legitimizes mere co-presence (in individual cases) to a (universal) law-like relation which permits predictive inference; but this transition is mediated not by logic, but by the stratagem of convention. This reflects a change in epistemic attitudes, wherein properties which are (observed to be) merely typical of the kind are converted into nominalist essences which define the kind. This has the effect of transforming an empirical classification based on identity into a conceptual framework peculiar to a language. Both Popper's hypothetico-deductive model and Hempel's deductive-nomological inferential structure exemplify this form of law (i.e. universal generalisation) whose rationale is convention; with all the attendant difficulties (analysed in Ch. II).

The empirical constraint on scientific classification, on the other hand, demands that (associated) laws specify
the structure and not define the kind. This leads directly to the conception of law as a relation between properties. Again, the constraint of prediction requires that the relation be necessary i.e. interdependent. It is suggested that both intuitions are satisfied by the relation of functional dependence between properties, the fundamental form of which is \( x \preceq y \) where \( \preceq \) signifies the relation of proportionality.

Before presenting Tarski’s formulation and discussion of relations of functional dependence; we might consider certain reflections of Kant’s which lend credence to the thesis that universal generalisations (from experience) do not exemplify causal necessity. Kant’s views on material or causal necessity as set forth by William Harper [1986] are:

i. Cosmological or structural theories (such as the laws of motion or of universal gravitation) constitute mixed items of knowledge, which according to Kant’s official characterization of necessity in the Postulates of Empirical Thought count as necessary.

ii. Experience never confers strict universality. Therefore universal generalisation through induction confers merely assumed and comparative universality which carries no necessity.

iii. Strict universality is derived not from experience but is valid absolutely a priori.
Characterization of Possibility and Necessity

4 (a) That which agrees with the formal conditions of experience is possible.

(b) That which is bound up with the material conditions of experience is actual.

(c) That which in its connection with the actual is determined in accordance with universal conditions of experience is necessary.

Thus there are two concepts of necessity (1) a judgement whose negation is not possible because it violates the formal conditions of experience and (2) material necessity which Kant identifies with causal necessity and whose form is:

If \( A^1 \) then, if \( A \) then \( B \).

where \( A^1 \) specifies some actuality, \( `A \) then \( B` \) specifies a causal necessity.

Obviously, universal generalisations (which are merely assumed and comparative) are not statements of causal necessity, on Kant's account. On William Harper's interpretation, Newton's inference to centripetal forces involves as actuality Kepler's law of areas and as the universal conditions of experience: Newton's laws of motion, the law of parallelogram of forces, Euclidean geometry, the calculus, the relativity of inertial motion. The final inference is to centrepetal forces.
The important point that emerges is: the 'actuality',
the background assumptions, as well as the final inference
to gravitational forces are laws of functional dependence
which exemplify the relation of proportionality.

Tarski [1965] gives the most general form of a relation
of functional dependence as \( x = R(y) \) or \( x = f(y) \) which is
read as: \( x \) is that value of the function \( f \) which corresponds
to (or is correlated with) the argument value \( y \). According
to Tarski a relation \( R \) is called a functional relation if to
everything \( y \), there corresponds at most one thing such that
\( xRy \), where the values of \( y \) are the argument values, and the
values of \( x \) are the function values. Tarski emphasizes that
functions are of particular significance as far as the
application of mathematics to the empirical sciences is
concerned. He says: 'Whenever we inquire into the
dependence between two kinds of quantities occurring in the
external world, we strive to give this dependence the form
of a mathematical formula, which would permit us to
determine exactly the quantity of the one kind by the
corresponding quantity of the other; such a formula always
represents some functional relation between the quantities
of two kinds.

An objection to the conception of law as functional
relationship in particular of the form \( x \leftrightarrow y \), might be that
it presupposes universality of precisely the form of
empirical generalisation. It says that for all values y the function (in particular of proportionality) assigns an unique value x. But a (tentatively offered) symbolic form of this might be \((y, x) \in (P \propto Qx)\) which may be read as: For all values \(y, x\), if the quantity (property) \(P\) takes the value \(y\), this implies (by the functional rule of proportionality) that the property \(Q\) takes the value \(x\). (This formulation is very tentatively offered and may be non-standard, but conveys the spirit of the conception being developed). In contrast the form of empirical generalisation is: \((x) \in (P \Rightarrow Qx)\) to be read as: For all objects \(x\), if \(x\) exhibits property \(P\) it exhibits property \(Q\). The former generalisation, and in general the notion of function, exemplifies a relational or structural view of the universe. From this perspective objects are not just bundles of properties, but are Knit together into systemic wholes, by possibly more than one functional relationship: The latter form of empirical generalisation on the other hand, is fundamentally a 'property' view of the universe, wherein atomic properties are only co-present without any necessary relationship between them.

To summarize the foregoing discussion: relations of functional dependence (whose most general form is that of proportionality) satisfy both the constraints i.e. (i) of specifying structure, and (ii) of permitting predictive
inference, imposed by classifications based on identity. At the same time, the logic of identity itself as a primitive relation, satisfies the empirical constraint on (scientific) systems of classification. In 4.2, examples of theoretical structure (from physics) are adduced, which illumine theoretical growth as a process of mathematical transformation, which employs as its fundamental principle of inference, Leibniz’s Principle of Identity of Indiscernibles (and the attendant laws of identity which follow from it).

4.2 Representation and Reduction: The Changing Faces of Realism

The issue of theoretical growth trifurcates into: (i) a preliminary clarification of the distinction between the logic of mathematical derivation (based on Leibniz’s Law) and the logic of propositional and quantification theory based deduction (which invokes rules of the propositional calculus and of quantification theory). This corresponds to Margaret Morrison’s [1990] distinction between theory as (mathematical) Representation and theory as (truth-functional) Reduction (ii) A critical exposition of Nagel’s [1979] development by reduction thesis in terms of Michael Redhead’s [1990] defence of it; which largely assimilates the current literature on this position, and (iii) a critical analysis of the shortcomings of the reduction
thesis by Margaret Morrison [1990] who advocates the representationalist view as better illuminating certain features of actual theoretical evolution in (physical) science. The terms of Morrison's discussion relate it quite naturally to the realism-anti-realism debate in (the philosophy of) science. These points can be elaborated in the following manner.

Before presenting Tarski's [1965] and Copi's [1986] distinction between the rules of inference based on the propositional and quantification calculi on the one hand; and the principles of inference for the relational calculus of Identity, based on Leibniz's Law on the other hand, it would be instructive to consider the traditional schema for the deduction of events/laws. This will help us to understand Tarski's and Copi's distinction.

The traditional schematism for the explanation (deduction) of individual events is exemplified by Hempel's [1965] D-N model. Redhead ([1990] p. 137) presents it thus: "In the deductive-nomological (D-N) model of Hempel the explanans cites one or more scientific laws. In the

4 Morrison's [1990] discussion focuses largely on the derivation of the Gas Laws from the Kinetic theory (of gases). But she also invokes other examples of theoretical growth to substantiate her points.

5 Redhead's [1990] notation is slightly modified in presentation.
usual schematic fashion adopted by philosophers of science, let us represent a typical scientific law in the universally quantified form \((x) (Px \rightarrow Qx)\) - succinctly all P's are Q's.

If a is a P i.e. Pa is true, then we seek to explain why a is a Q by deducing Qa from the premisses:

\[(x) (Px \rightarrow Qx)\]  
\[Pa\]  

Thus, from (1) by Universal Instantiation

\[Pa \rightarrow Qa\]  

whence, from (2) and (3) by *modus ponens*

Qa - This is the traditional schematism for the deduction of individual events. The schematism for the deduction of a law from other laws is presented by Nagel ([1979] p. 35) as:

'A schematic illustration .... is provided for an explanation of a law having the form "All A's are B's" when it is deduced from two laws having the forms, respectively "All A's are C's" and "All C's are B's".

Copi ([1986] p. 353-355) represents both schema succinctly thus:

6 Copi's [1986] notation is slightly altered in presentation.
Schematism I  
(Deduction of Individual events)  

1. (x) (Hx ---> Mx)  
2. Hs / .. Ms  
3. Hs --> Ms 1, U.I.  
4. Ms 3, 2, M.P.  

Schematism II  
(Deduction of laws)  

1. (x) (Hx ---> Mx)  
2. (x)(Gx-->Hx)/..(x)(Gx-->Mx)  
3. Hy --> My 1, U.I.  
5. Gy --> My 4,3, H.S.  
6. (x) (Gx--> Mx) 5, U.G.  

Notice that the above schema employ as rules of inference  
(1) rules from quantification logic viz. Universal  
Instantiation and Universal Generalisation, and (2) rules  
of the propositional calculus viz. Modus ponens and  
Hypotheticaal syllogism.  

If we now try to interpret laws of functional  
dependence in terms of these schema, the derivation gets  
blocked at the very outset. This can be made clear by a  
single example. Thus Nagel ([1979] p. 77) cites as a law of  
functional dependence, the Boyle-Charles Law for ideal  
gases, which he formulates as 'PV = aT where P is the  
pressure of the gas, V its volume, T its absolute  
temperature, and a, a constant that depends on the mass and  
the nature of the gas under consideration'. Now if we try  
to interpret this law in terms of the above schema, we  
obtain:

(x) (PV = aT)  

The very first step is clearly invalid. Hence next steps  
i.e. the dropping of the universal quantifier by invoking  
the rule of universal instantiation is blocked, because the
formula (for Boyle's Law) contains no individual variable which might be replaced by an individual constant. All the symbols of the formula represent either properties (of the same individual/substance); or else numerical constants (which are experimentally determined). Also the further application of the rules from propositional calculus viz. modus ponens and hypothetical syllogism are blocked as well. Instead the derivation proceeds by substitution either by measured values of variables (properties) to obtain the value of the (functionally) related variable; or else by the substitution of a variable by an identical variable (in Leibniz's sense of identical properties). This pattern of substitution is based on the Rule of Replacement or Rule of Substitution for Identity; and not on the Rule of Replacement for Logical Equivalence. This is the fundamental distinction made by both Tarski [1965] and by Copi [1986] to which we now turn.

Tarski ([1965] p. 47) mentions as rules of proof for the propositional calculus, the rule of detachment (or the modus ponens rule) and the Rule of Substitution (for logical Equivalence). The Rule of Substitution defines logical equivalence, and its content is formulated by Tarski as follows: 'If a sentence of a universal character, that has already been accepted as true, contains sentential variables, and if these variables are replaced by the sentential variables or by sentential functions or by
sentences - always substituting equal expressions for equal variables throughout then the sentence obtained in this way may also be recognized as true', Parallel to but distinct from, the rule of substitution for the propositional calculus is the rule of substitution (or replacement) for identity. Tarski [p. 56] formulates this as follows: 'As a consequence of Leibniz's Law we have the following rule which is of great practical importance: If in a certain context a formula having the form of an equation e.g. 

$$x = y$$

has been assumed or proved, then it is permissible to replace, in any formula or sentence occurring in this context the left side of the equation by its right side e.g. 

$$\sim x$$ by $$\sim y$$ and conversely. It is understood that should $$x$$ occur at several places in a formula, it may at some places be left unchanged and at others replaced by $$y$$; there is thus an essential difference between the rule discussed..... which does not permit such a partial replacement of one symbol by another'. Tarski thus emphasizes the difference in the rule of proof for logical equivalence and the rule of proof for logical identity.

Copi ([1986] p. 319) articulates the same distinction in greater detail. He formulates the Rule of Replacement for logical equivalence thus: 'In any truth-functional compound statement, if a component in it is replaced by another statement having the same truth-value, the truth-
value to the compound sentence will remain unchanged. But the only compound statements that concern us are truth-functional compound statements. We may accept, therefore as an (additional) principle of inference, the Rule of Replacement, which permits us to infer from any statement the result of replacing any component of that statement by any other statement \textit{logically equivalent} to the component replaced. 7 Thus logical equivalence is a truth functional concept. But the corresponding rule of inference for identity mentioned by Copi (p. 387) is based not on truth-functionality but on Leibniz’s definition of identity. Copi, formulatizes this principle thus: ‘x=y iff every attribute of x is an attribute of y, and conversely. This principle permits us to infer from the premises \( r = u \) and any formula containing an occurrence of \( r \), as conclusion any formula that results from replacing any number of occurrence of \( r \) in the second premiss by the symbol \( u \).

The foregoing formulations of inference rules make it clear that (1) the rules for logical equivalence are truth

7 Copi [1986] lists a number of forms for the Rule of Replacement (for logical equivalence) which include Commutation, Association, Distributivity, Double Negation, Transposition, Material Implication, Material Equivalence, Exportation, Tautology, De Morgan’s Theorem et al.

Logical identity on the other hand, invokes as rules of inference the Rule of substitutivity for identity, and the laws of identity viz. reflexivity, transitivity and symmetry.
functional whereas the rules for logical identity are relational (ii) these concepts i.e. of equivalence and of identity lead to distinctive inferential structures. In the light of this, the thesis is put-forth that (explanatory) theoretical structures (in physical science) which use laws of functional dependence (to specify kinds) exemplify the (mathematical) calculus of (the relation of) identity; and not the truth-functional calculus of logical equivalence. This leads directly to the view of scientific theories as (mathematical) representation rather than as (truth-functional) deduction.

The latter position which is implicit in Nagel’s [1979] thesis of (theoretical) development by reduction is defended by Michael Redhead [1990]. Redhead’s discussion is in the context of the criteria for (good) explanation in science. The necessary criterion is deducibility (from universal laws and initial conditions) in the sense of Hempel’s [1965] covering law model (exemplified in schematism I above). But this leads at once to what Redhead (p. 1137-1138) terms the ‘circularity objection’.

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8 It is important to guard against a misunderstanding here. It is not being maintained that laws of functional dependence are not statements or statement functions; what is being claimed is that the inferential structure generated by laws of functional dependence (using Leibniz’s Principle and the laws based on it, as rules of proof) exemplifies a relational structure, and not a (truth-functional) deductive structure.
The circularity objection is formulated by Redhead in the following way: \(^9\) 'In (1) the implication as we have written it is material implication. On a Humean (regularity) view of laws that is all there is to (1) it is true in virtue of all its instances being true. But if (1) depends for its truth on the truth of (3), and this, given the premiss Pa, must turn on the truth Qa. So is not the argument completely circular? The truth of Qa, given Pa, is grounded in the truth of a universal statement, whose truth is grounded in the truth of Qa, the very fact we are trying to explain. What this amounts to is that (1) is nothing more or less, on the Humean account, than a compendium of all the instances (3) (In the case of a finite variety of instances the universal law is indeed nothing else than the conjunction of its instances). On the Humean account the instances are "loose" (there is no cement!) so effectively the Hempelian model under this interpretation of law, amounts to the assertion that facts only explain themselves'.

In an attempt to circumvent the circularity objection Redhead points out that the whole argument hinges on the

\(\begin{align*}
(x) (Px \rightarrow Qx) & \quad \text{--- (1)} \\
Pa & \quad \text{--- (2)} \\
Pa \rightarrow Qa & \quad \text{--- (3)} \\
Qa.
\end{align*}\)

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\(^9\) Redhead's [1990] schematism is abbreviated as follows to facilitate reading of the text:
assumption that universal laws are only deductively supported by the evidence. If we could construe evidence as in some sense, inductively supporting universal laws, the charge of circularity might yet be deflected. But here, says Redhead (p. 138) 'we are backing ourselves straight into the problem of induction'. Hence (to avoid circularity) it must be acknowledged that the explanans (i.e. universal laws) is never known definitely to be true. This converts the D-N model into Popper's hypothetico-deductive model; and Redhead considers this as 'an obvious, but unavoidable defect in scientific explanations'. This is because our puzzlement over individual events (i.e. over the problem-situation which demands explanation) can hardly be mitigated by (deduction from) conjectural laws which are not merely not known to be true, but (according to Popper) more strongly, cannot be true'. Therefore Redhead says: 'So a Popperian expects, insofar as he allows himself any expectations, that an essential part of the explanans, in any scientific explanation, is definitely false (although not currently known to be false). What Redhead is emphasizing is that the hypothetico-deductive model, whatever its merits for the testing of laws, hardly seems to fulfil the (scientific) requirement for the explanation of events (or laws). Nevertheless Redhead admits that the logic of explanation as deduction thesis forces upon us the hypothetico-deductive model.
It ensues from the foregoing discussion that the problem of explanation in science reduces to that of the (non) confirmation of universal laws required for explanation (deduction). Redhead's (attempted) solution to this invokes the concept of unification which is intended to both provide increased confirmation for laws and to, circumvent the circularity objection. This can be clarified in the following way: Redhead (p. 140) following Nagel [1979] maintains that 'In practice good explanations ... arise at the intersection of several universal laws, all of which are necessary to deduce the explanation'. This formulation points to the crucial 'unification' aspect of explanation Redhead (p. 140) says: 'The world at the surface level of immediate experience appears very complicated, very rich in diverse phenomena with no apparent connection. But at a "deeper" theoretical level, can all this diversity get reduced to a few interlocking explanatory principles? This has always provided an ideal of theoretical progress in science, the ideal of unification'.

Redhead acknowledges that the logical structure for unification involves several problems. Most importantly, since unification is intended at both a logical systematization (of diverse domains) and at an increase in confirmation (of experimental laws), the reducing theory (which consists of a set of axioms) must itself be
empirically confirmed. This brings in the element of novel prediction which imposes constraints on the logical structure for unification. Thus, for example, the simple conjunction of theories is precluded because this approach yields no significant predictions.\textsuperscript{10} What is required by the idea of a unified explanation is a certain 'interlocking working together of axioms', which results in both novel prediction and 'depth'. Redhead (and reduction theorists generally) however, are unable to provide the logical schematism for this concept of 'an interlocked working together of axioms'. He admits that there are many complications associated with the idea.

Primary among these (complications) is that the unifying (reducing) theory might correct the reduced theories (or laws); and then says Redhead 'the idea of increased empirical content becomes formally problematic? This contretemps (exemplified by the correction of Kepler's Laws, and Galileo's law for free fall by Newton's theory of gravitation) certainly creates formal (logical) problems for the thesis of growth by reduction which Redhead is concerned

\textsuperscript{10} In this context Redhead ([1990] p. 140) says: 'Suppose we have two sorts of phenomena, P\textsubscript{1} and P\textsubscript{2} which stand for the sets of law-like regularity ... and suppose that P\textsubscript{1} and P\textsubscript{2} are explained by theories T\textsubscript{1} and T\textsubscript{2}. Then P\textsubscript{1} U P\textsubscript{2} is certainly explained by T\textsubscript{1}, T\textsubscript{2} ... In a trivial sense there are new predictions that can be deduced from T\textsubscript{1}, T\textsubscript{2} but not from either theory separately ... But there are no interesting novel predictions....'.
to defend. For logically speaking, a theory can hardly be allowed to correct (i.e. falsify) its own (deductive) consequences. This problem (and the associated example) are exploited by Popper [1972, 1983] to argue his own case for (theoretical) growth by conjecture and refutation. 11

Redhead considers, however, that the fundamental intuition underlying the concept of unification is not the (increased) confirmation of experimental laws; nor even the display (ing) of phenomena as (logically) interconnected, but rather an intuitive notion of simplicity. This latter

11 Popper [1969, 1983] rejects inter-theoretic reduction in the same domain whilst considering (Popper [1972b]) reduction across domains as an acceptable thesis. For the former case Popper [1969, 1983] cites counterexamples of new theories correcting i.e. falsifying previously held theoreis (in the domain) to both (1) reject induction as a method of discovering new theories and (2) to reject the thesis of growth by reduction. Instead Popper maintains that growth takes place by conjecture and refutation. The competing theories are related by (the sharing of) a common problem-solution; and are (comparatively) evaluated in terms of the falsifiability criterion (which assimilates the criteria of unity, simplicity, depth, verisimilitate etc.). The logical schematism, therefore for Popper's model of growth by conjecture and refutation remains that of his basic schematism for explaining individual events. (i.e. universal law + initial conditions ---> prediction). This schema, when taken conjointly with his thesis of theory-ladenness leads (as analysed in Ch. II) to methodological conventionalism and to meaning - incommensurability.

As for the case of inter domain reduction, (e.g. the reduction of laws of chemistry from those of physics) Popper accepts this, but does not provide any logical schematism for it. Furthermore where no reduction has been effected, Popper advances the thesis of emergence.
notion, he further interprets as a reduction in the total number of laws which we have to accept without explanation.

There are several points about Redhead's exegesis of the reductionist position which are problematic: The most important of these is that reduction bases itself on what Rom Harre [1986] terms the bivalence principle i.e. of truth-falsity of statements, including theoretical statements. This is because laws are conceived as universal statements and theories as (axiomatic) sets of statements, and also because reduction is interpreted in terms of (truth-functional) deduction. This makes the confirmation of (explanatory) laws and theories a crucial issue for reductionists. But inasmuch as confirmation is implicated in the unificationist thesis, the inability (of reduction theorists) to provide a logical schematism for unification leads to problems for confirmation (of laws and theories). The interpretation of unification in terms of a simplicity criterion merely shifts the problem. We must conclude that without an appropriate logical schematism for unification (which can account for actual theoretical growth in science) reductionism is a vacuous model for theoretical growth in science.

Margaret Morrison [1990] offers a more detailed and penetrating criticism of the reductionist programme. Focusing on Friedman's [1983] model she emphasizes that (i)
unification as conjunction (of theories) presupposes realism instead of justifying it. Hence the role of unification in the confirmation of theories is circular.

(ii) Theoretical evolution in actual (scientific) practice does not support the unification as conjunction thesis, nor does it support scientific realism in general and (iii) The reductionists' unification concept does not correspond to Whewell's consilience of inductions; and that neither concept provides adequate support for scientific realism. Finally, Morrison makes a case for theory as mathematical representation. These points can be elaborated in the following manner.

Morrison presents the Friedman [1983] model thus: We postulate a theoretical structure A (possessing certain mathematical properties) and an observational structure B. A explains or reduces the properties of B. Using the kinetic theory we can explain the observable properties of gases characterized by B by embedding them in A, where A is literally construed as the world of molecular theory. This enables us to account for the behaviour of gases by identifying them with large configurations of molecules that interact according to the laws of Newtonian mechanics. Due to the properties and relations provided by the theoretical structure we can derive laws that govern the behaviour of observable objects. Friedman sees the relation between A and B as that of model to submodel; which permits literal
identification of elements in A and B. On the representationalist account on the other hand, B is only "embedded" in A (p. 308).

Friedman prefers the literal construal because it yields greater unifying power and (hence) increased confirmation for both the unifying theory and the phenomenological laws reduced by it.\(^{12}\) Friedman claims two virtues for his reductivist programme: First there is a type of inference i.e. conjunctive inference that is valid on the hypothesis of a genuine reduction, but not in the case of a representation. Secondly there is the utility of conjunctive inference for confirmation.

But Morrison points to Putnam's [1975] conjunction objection to the effect that the conjunction of theories

\[^{12}\] Morrison ([1990] p. 308) exemplifies Friedman's point thus: She says that: "for example, we can conjoin molecular theory with atomic theory to explain chemical bonding, atomic energy and many other phenomena. Consequently, the molecular hypothesis will pick up confirmation in all the areas in which it is applied. The theoretical description then receives confirmation from indirect evidence (chemical, thermal and electrical phenomena) which it "transfers" to the phenomenological description. Without this transfer of confirmation the phenomenological description receives confirmation only from the behaviour of gases. So in cases where the confirmation of the theoretical description exceeds the prior probability of the phenomenological description the latter receives the appropriate boost in confirmation as well. Hence the phenomenological description is better confirmed in the context of a total theory that includes theoretical description than in the context of a theory that excludes such description."
presupposes belief (in the truth of) these theories; hence unification (by conjunction) presupposes a reductivist approach that construes theoretical structure as literally true. Therefore unification cannot be invoked for justifying conjunction. Apart from the logical issue, Morrison does not think that actual theoretical evolution in science supports conjunctive inference.

In fact, even apart from the special case of the evolution by conjunction thesis, Morrison does not consider reduction a viable approach because of (i) the idealized nature of theoretical assumptions involved. She cites the example of the reduction of thermodynamics by statistical mechanics and emphasizes that large parts of theoretical structure consists of mathematical representation which lacks physical significance. (ii) the problem of many models: Here Morrison invokes the example of the reduction of gas (the laws by the kinetic theory and points out that the Boyle-Charles law and the van der Waals law require different and incompatible models (of the kinetic theory). She concludes that the literal identification of observational with theoretical structure is precluded.

Furthermore Morrison (p. 326) thinks that the reductionists' notion of unification does not quite capture Whewell's concept of the consilience of inductions because it fails to explicate the 'conceptual reshuffling of the
phenomena' which takes place in a genuine consilience. Finally, Morrison emphasizes that neither unification nor consilience warrant realism, on account of the factor of contextuality and historical relativism involved.

The general conclusion that Morrison draws from her analysis of the reductionist position is that it does not constitute a viable approach to the problem of theoretical growth in science. This is so partly because reductionists are currently unable to offer a coherent logical schematism or mode of inference for unification (which is the cornerstone of reductionist strategy). But the thesis holds also because certain features of theory structure/evolution in science indicate that theoretical statements cannot be construed literally (as true-false) and that (truth-functional) deduction is not the appropriate form of inference in science. Morrison thinks that these features support the thesis of theory as (mathematical) representation. However she does not provide a detailed schematism or positive arguments in favour of her position.

13 Here Morrison cites as example the case of Newton's mechanics and Kepler's third law. The latter states that $a^2/T^2 = \text{constant}$. The Newtonian version of the law states that $a^2/T^2 = m + m'$ where $m$ is the mass of the sun and $m'$ is the mass of the planet in question. By ignoring $m'$ on the grounds that it is much smaller than $m$ we can assume that the two laws are roughly the same. In what sense is the Keplerian formulation true? Only by leaving out the fundamental qualitative aspects of Newton's theory. But if we consistently ignore $m$ it becomes impossible to apply Newton's theory because there is no gravitational force on a body with zero rest mass.
4.3 The Creativity of Identity

In this Section we extend the notion of theory as mathematical representation to encompass that of theory as mathematical transformation. This interpretation reveals the underlying logic of scientific discovery (in the mathematical sciences) to be that of identity as defined by Leibniz's Law. It is argued furthermore that this form of inference is characterized by (a) its intrinsic creativity (b) the hypothetical or conjectural character of its conclusions, and that (c) it sustains a position of referential realism. This analysis can be elaborated in the following way:

First, we note that laws of functional dependence specifically of the form of mathematical proportionality are expressed as statements of mathematical identity (i.e. as mathematical equations) by adducing the constant of proportionality. 14 What is also important to note is that when the constant of proportionality is not numerically specified, then the laws (of functional dependence) assume a purely symbolic form. This is emphasized by both Kuhn [1977] and Duhem [1976].

Kuhn ([1977] p. 464-467) says: 'In the Sciences,

14 As already noted in Ch. III, Tarski gives the general form of functional law as \( x = f(y) \). When the relation of mathematical proportionality i.e. \( x \propto y \) is expressed in Tarski's formulation it assumes the form \( x = ky \) where \( k \) is the constant of proportionality.
particularly in physics, generalizations are often found in symbolic form: \( f = ma \), \( I = V/R \), or ... others are ordinarily expressed in words: "action equals reaction", "chemical composition is in fixed proportions by weight..." Kuhn goes on to say "a shared commitment to a set of generalizations justifies logical and mathematical manipulation and induces commitment to the result. It need not however imply agreement about the manner in which the symbols, individually and collectively are to be correlated with the results of experiment and observation. To this extent the shared symbolic generalizations function as yet like expressions in a pure mathematical system, Kuhn however goes on to distinguish between a pure mathematical system and a scientific theory (consisting of mathematical equations). According to him, whilst the pure system is expressed as only one formulation (e.g. \( f = ma \)) scientific theories are more like schematic forms, which can express the same law variously. This introduces, he thinks, an empirical element into scientific theories even when they are expressed symbolically. However, Kuhn agrees that scientific theories are symbolically expressed, and are mathematically and logically operated by syntactic devices including the substitutivity of identities (p. 465). Moreover, Kuhn does not say that all the schematic forms of scientific theories have empirical significance, but only those that 'attach to nature'.
Duhem [1976] anticipates both the points that Kuhn makes viz. that in the context of theory, experimental laws are symbolically expressed, and that only some of these symbolic forms are physically interpreted; (i.e. attach to nature) whilst large parts of theoretical structure consists of pure mathematical representation/manipulation which lacks physical significance. Thus Duhem ([1976] p. 17) says: 'The facts of experience taken in their primitive rawness cannot serve mathematical reasoning; in order to feed this reasoning they have to be transformed and put into a symbolic form'. Again, he (p.20) maintains: 'In the first place, no experimental law can serve the theorist before it has undergone an interpretation transforming it into a symbolic law. Finally, Duhem says: 'The materials with which (this) theory is constructed are, on the one hand, the mathematical symbols serving to represent the various quantities and qualities of the physical world and on the other hand, the general postulates serving as symbols'. Duhem (p.28) goes on to stress that it is an error to insist that all the operations performed by the mathematician connecting postulates with conclusions should have a physical meaning. According to him such a requirement is legitimate only when it comes to the final formulas of the theory, but has no justification if applied to the intermediary formulas or to the logical-mathematical operations establishing the transition from postulates to
conclusion.

More recently, Peter Clark [1990] also emphasizes the pivotal role of mathematics in the articulation and testing of physical theory. He insists that no divide is possible between the purely mathematical context and the physical content of theory in mathematical physics; by which he means that the core notions and concepts of physics cannot be formulated without pre-supposing a very definite mathematical structure.

The larger point that all these philosophers can be interpreted as making is that all of physical theory consists of symbolic representation (only some of which need have physical significance).

The implications of these views for the thesis of identity as creative can be drawn as follows: First we make a preliminary clarification viz. that the philosophers whose views are exemplified above work from within very different and varied frameworks of philosophical assumptions regarding theoretical structure. 15 Yet they concur in emphasizing the symbolic form that all statements, including experimental laws assume in the context of theoretical structure/evolution. The significance of this for our thesis consists in this: We have already argued (in Section 4.1) that

15 Thus Kuhn operates with the concept of paradigm, Duhem with the holistic thesis, whilst Clark seems to accept Nagel's development by reduction thesis.
identity is not a concept of the propositional calculus, and that therefore, transformations effected in accordance with Leibniz’s law are not based on truth-functionality. This seems to indicate that the chain of inferences based on identity can operate only on symbolic formulations (algebraic expressions or sentential functions) which are not (true-false) propositions. It is therefore necessary that laws (in the context of theory) be symbolically expressed to facilitate logical manipulation in accordance with Leibniz’s law. This leads immediately to the ‘creative’ aspect of identity — since assumptions (premises) are only symbolically formulated, there is no constraint on the free creation of premises in terms of adherence to experimental facts. This enables a proliferation of theoretical assumptions which emphasizes the fecundity of identity.

The second aspect of the creativity of identity has to do specifically with the substitutivity of identity (i.e. the Rule of Replacement for Identity). An example of the ‘conceptual reshuffling of the phenomena’ (Morrison [1990]) achieved by this rule is presented from Redhead ([1990] p. 146). Redhead, in the context of discussing the notion of cause in modern physics says: ‘The surprising thing is that physicists long ago gave up the notion of cause as being of any particular interest. In physics the explanatory laws are laws of functional dependence, how one physical
magnitude is related in a regular (and law-like on the necessitarian account) fashion with another physical magnitude .... What we actually have in physics is a force law such as the inverse square law of gravitational attraction, which relates via Newton's second law, the acceleration of the body to the relative location of the bodies such as the earth. Instead of \( S = \frac{1}{2} gt^2 \) (Galileo's law), we have in idealized approximation \( S = \frac{1}{2} \left( \frac{GM_2}{R^2} \right) t^2 \), where \( M \) is the mass of the earth, \( R \) its radius and \( G \) is a new gravitational constant. So we are back with a regularity connecting \( S \) with \( t \), but also now with \( M \) and \( R \). But the force of gravity has been eliminated between the force law and Newton's second law ...'. This 'conceptual reshuffling of the phenomena' is achieved by inference according to the substitutivity of identity. This can be shown in the following way:

According to Copi if \( x = y \) and \( x = z \) then by Leibniz's Law \( y = z \). Thus if we have:

\[
F_g = \frac{GM_1 M_2}{R^2} \quad \text{(Law of gravitation)}
\]

and \( F = m, a \) (Newton's second law)

then by Leibniz's law we derive \( \frac{GM_1 M}{R^2} = m, a \) (assuming Mach's principle viz. gravitational mass \( M_1 = \text{inertial mass } m_1 \)).
Thereupon (by dividing $m_1$) we get \[ \frac{GM}{R^2} = a \]

Again since $a = g$

we get, by the transitivity of identity \[ \frac{GM_2}{R^2} = g \]

Finally, by the substitivity of identity,
we transform \( S = \frac{1}{2} gt^2 \) into \( S = \frac{1}{2} \left( \frac{GM_2}{R^2} \right) t^2 \)

Thus the relation between Newton's gravitational law and Galileo's law is not that of truth-functional entailment or conjunction (i.e., conjunction of Galileo's law and Kepler's law to obtain Newton's law); but that of mathematical transformation in accordance with the laws of identity. This process (of derivation) is transformational not only because it leads to the 'conceptual reshuffling' of the phenomena; but also because being non-truth-functional, it presupposes neither the truth of the old conceptual (classificatory structures) nor implies the truth of the new ones. Peter Clark ([1979] p.158) presents another example of conceptual reshuffling, this time invoking the 'transitivity of identity'.

Several other examples (e.g., the transformational derivation of Kepler's law from Newton's law; or of the gas laws from the Kinetic theory) could be presented from the corpus of physics. They point to the 'conceptual'
creativity of the relation of identity as a mode of inference which permits transformational derivations in accordance with its laws.

That scientific discovery is a creative process is also emphasized by Popper [1972]. From this Popper draws the conclusion that it is a process which is not amenable to logical analysis. But Popper construes logic only in the sense of truth-functional deduction (in accordance with the rules of the propositional and quantification - logic calculus). However in the calculus of identity, which is not a concept of the propositional calculus, we have a mode of inference which is valid, but nevertheless non-deductive (i.e. non truth-functional). It would therefore appear to be the appropriate form of reasoning for the logic of scientific discovery.

The fact that identity is not a concept of the sentential calculus and that therefore transformations in accordance with this principle are not truth-functional, also explicates the hypothetical character of (theoretical) formulations arrived at by this mode of reasoning. This thesis however needs careful interpretation. 16

16 It must be strongly emphasized that the analysis at this stage is concerned with areas in philosophical logic, which are currently very fluid and controversial. Hence conclusions are tentative, and an attempt is made to substantiate them with views/arguments from various sources/philosophers and with examples from science. However, the main argument is independently developed.
In this context we note that both Duhem and Popper emphasize that the truth of a theoretical formulation like Newton's law of gravitation is not entailed by the truth or falsity of Kepler's law (or vice-versa). Duhem's and Popper's emphasis (on the mutual inconsistency of theoretical formulation and experimental law) is in the context of a rejection of induction as the method of scientific discovery. But both interpret induction in the sense of (in)valid (truth-functional) deduction, and in this sense it is true that induction cannot account for the discovery of laws/theories. But transformations according to the principle of identity permit formulations which are only hypothetical relative to their inferential basis. This aspect of identity has to do with the peculiar character of the relation as formulated by Leibniz. Since $x$ and $y$ are identical properties iff they share all their properties in common, it follows that if $x = y$ they are properties of the same object/substance/system (or else $x$ and $y$ are the self-same entity). This is because only properties of the same system share all their other attributes in common. From this it follows that identity holds iff the properties are indeed properties of the same system. But it is just this assumption that cannot be substantiated in the case of statements of theoretical identity (e.g. Temperature = mean kinetic energy of molecules) which are mediated by a process of inference according to Leibniz's Law. This is because
the calculus of identity is not truth-functional.

The foregoing analysis has significant implications for the realism, anti-realism debate in the philosophy of science. These can be interpreted in the following way. We have emphasized that the concept of logical identity unlike that of logical equivalence, is not a concept of the propositional calculus. Creative 'transformational' inference based on the laws of identity, therefore, operates only on symbolic forms which are not propositions/statements. From this it follows that the conception of the growth (structure of a theory as a creative process in accordance with Leibniz's Law, cannot sustain a realism based on the bivalence principle (of truth/falsity). On the other hand, the relation of identity, as formulated by the principle of the Identity of Indiscernibles, purports to hold between entities or properties (of entities); (i.e. x is identical with y iff all properties of x are properties of y). Therefore theoretical structure, as a system of identities (mathematical equations), might be interpreted as supporting a position of referential realism (which presupposes existents). The concepts of 'bivalence realism' and referential realism can be clarified in the following way.

First we make a distinction (which is often conflated) between scientific realism based on the strict bivalence
principle (i.e. the truth falsity principle) and referential realism. Scientific realism in the first sense is largely defined in terms of the truth and falsity of statements. Newton-Smith [1981] calls this the minimal form of realism. But Rom Harre ([1986] p. 35) prefers to call this position one of 'maximal realism' since accepting it would commit one to an epistemological ideal that incorporates the strongest possible relationship between scientific discourse and the world, namely truth and falsity. Hence the principle of bivalence can be interpreted as maintaining that the theoretical statements of a science are true or false by virtue of the way the world is. Obviously, our position cannot be interpreted in terms of this form of realism.

Referential realism, by contrast is concerned with the existence/non-existence of theoretical entities. The classical statement of the referential position is due to Sellars (Harre [1986]) who expresses it thus: 'To have good reasons for holding a theory is ei ipso to have good reasons for holding that the entities postulated by the theory exist'. Thus the question of the truth or falsity of statements is replaced by that of the existence, non-existence of entities at the heart of a specification of scientific realism.

In interpreting our position in terms of this form of
scientific realism however, we should like to make a delicate distinction: Since our argument is based on identity, both as a principle of scientific classification (of natural kinds), and as a creative mode of transformational inference for theory structure/evolution; the conclusions of this evolution are also expressed as statements of theoretical identities (e.g. Temperature = mean kinetic energy of molecules or water is H₂O etc.). The question now arises: within the context of referential realism, how are these statements of theoretical identity to be interpreted. The answer lies in Leibniz's concept of the identity of indiscernibles. From this principle it follows that a statement of theoretical identity hold iff the entities/properties related by identity are properties of the same substance/system (or else if they are the self-same entity). Just in case this condition is satisfied, the statement of theoretical identity holds necessarily true. From this it follows that statements of theoretical identity are true/false by virtue of (i) the internal structure of objects/substances i.e. of existents; and (ii) the statements if true are tautologously true. Thus the truth/falsity of statements of theoretical identity presupposes existents. This conception of the truth/falsity of statements of theoretical identity obviously differs from the concept of truth/falsity in the correspondence theory (of truth). Whereas the former
presupposes existents, the latter presupposes facts. Our argument from identity permits us to make this delicate distinction, and so interprets the thesis of referential realism in its own terms. We can therefore agree with Van Frassen’s [1980] formulation of scientific realism according to which a realist holds (with respect to a theory) that sentences are true/false, and that what makes them true/false is something external. But this agreement is subject to the proviso that the statements are statements of theoretical identity, and provided ‘external’ is interpreted to imply existents.

The notions of ‘existents’ and of reference can be further clarified as follows: Reference according to Harre ([1986] p. 68) ‘consists in achieving a physical tie between embodied scientist and the being in question. Its existence is thus tied to that of the scientist’. On this view referring is a ‘material’ practice which encompasses both indexicality and manipulation of existents. It therefore presupposes ontological realism. As Harre (p.67) says: ‘Referential realism requires that some of the substantive terms in a discourse denote or purport to denote beings of various metaphysical categories such as substance, quality and relation, that exist independently of that discourse’. Hacking [1983] can also be interpreted as supporting referential realism.
In the light of these contrasting formulations of scientific realism, the argument based on identity, developed in this thesis can be interpreted as supporting a position of referential realism. This follows from: (i) The conception of theory as symbolic representation (of mathematical identities) large parts of which lack physical significance, and hence are not true/false. (ii) The concept of theoretical growth by mathematical transformations, according to Leibniz's Law of Identity, which is not truth-functional. (iii) The 'primitive' nature of identity as a principle of classification (for natural kinds) which can only be extensionally exemplified and not intensionally defined. This thesis is correlative with the thesis of indexicality and hence presupposes ontological realism.