Chapter 4

COTS Selection Based on Cohesion and Coupling under Multiple Applications Environment
The previous two chapters deal with the decision making situation in which a software developer selects COTS components for single application development task. However, in many decision making situations, a software developer may face a problem of COTS selection where multiple applications are handled concurrently. In this chapter, we consider the design of a modular software system consisting of multiple applications under the assumption that each module belongs to an application uniquely and there is no module that is common to different applications. In CBSD, it is assumed that COTS components within a set of alternative COTS components have similar functionality. Since COTS components are provided by different software component suppliers (or vendors), functions of COTS components could be different from each other. Therefore, functional contributions of COTS components towards the functional requirements of a component-based software system should also be considered in the COTS selection process. Furthermore, in CBSD, an important concern is cohesion and coupling. Coupling is the measure of interactions among software modules, while cohesion is the measure of interactions among COTS components that are within a software module. A good software system should possess software modules with high cohesion and low coupling. A highly cohesive module exhibits high reusability, and loosely coupled systems enable easy maintenance. The relationship between cohesion and coupling of modules can be measured by using intra-modular coupling density \((ICD)\). Additionally, while developing modular software systems consisting of multiple applications, reusability of components in different applications must also be considered. The main tasks considered in this chapter are how to select COTS components available in the COTS components’ market and deploy them into which application in order to maximize the functional requirements of the software system and minimize the total development cost of the software system which includes the cost needed for the procurement of COTS components and adaptation of components to various modules in all the undertaken applications.

This chapter is divided into two sections. To formulate the mathematical models in both the sections the following notation are used:

\(N\): the number of applications handled concurrently,

\(M\): the number of modules in the software system,
$L$: the number of alternative COTS components,

$T$: the number of sets of alternative COTS components,

$sc_k$: the $k$-th COTS component, $k = 1, 2, \ldots, L$,

$s_t$: the set of alternative COTS components for the $t$-th functional requirement of the software system, $t = 1, 2, \ldots, T$,

$c^p_k$: the procurement cost of the $k$-th COTS component, $k = 1, 2, \ldots, L$,

$c^a_{jk}$: the adaptation cost if the $k$-th COTS component is adapted into the $j$-th module, $j = 1, 2, \ldots, M$, $k = 1, 2, \ldots, L$,

$r_{kk'}$: the number of interactions between the $k$-th and the $k'$-th COTS components, $k, k' = 1, 2, \ldots, L$,

$f_{jk}$: the functional rating of the $k$-th COTS component for the $j$-th module, $f_{jk} \in [0, 1]$, $j = 1, 2, \ldots, M$, $k = 1, 2, \ldots, L$,

$s_{ij}$: the binary parameter,

\[
s_{ij} = \begin{cases} 
1, & \text{if the } j\text{-th module belongs to the } i\text{-th application}, \\
0, & \text{otherwise, } i = 1, 2, \ldots, N, j = 1, 2, \ldots, M,
\end{cases}
\]

$b_{jk}$: the binary parameter,

\[
b_{jk} = \begin{cases} 
1, & \text{if the } k\text{-th COTS component can be reused to achieve the } j\text{-th module}, \\
0, & \text{otherwise, } j = 1, 2, \ldots, M, k = 1, 2, \ldots, L,
\end{cases}
\]

$x_{jk}$: the binary variable,

\[
x_{jk} = \begin{cases} 
1, & \text{if the } k\text{-th COTS component is selected to implement the } j\text{-th module}, \\
0, & \text{otherwise, } j = 1, 2, \ldots, M, k = 1, 2, \ldots, L,
\end{cases}
\]
\( y_k \): the binary variable,

\[
y_k = \begin{cases} 
1, & \text{indicates that the } k\text{-th COTS component is selected,} \\
0, & \text{otherwise, } k = 1, 2, \ldots, L, 
\end{cases}
\]

\( H_i \): a threshold value of ICD of the \( i \)-th application, \( i = 1, 2, \ldots, N \).

### 4.1 Biobjective optimization model for COTS selection based on cohesion and coupling under multiple applications environment

Here, we propose a biobjective optimization model that maximizes the functional requirements and minimizes the total development cost of the modular software system. The total development cost includes the procurement and adaptation costs of COTS components. The model is constrained by many realistic constraints including a minimum threshold on the ICD for each application, reusability of COTS components, selection of only one COTS component from a set of alternative components for each functional requirement per application, and the selection of more than one component per module if required. The proposed model can be considered as a generalization and extension of the optimization models proposed in [69, 103] in terms of providing a systematic framework for COTS selection that facilitates software development process of a modular software system under multiple applications development task. In the case of MOOP, the multiple objectives usually conflict with each other and any improvement in one objective can be obtained only at the expense of another. Consequently, the aim in solving MOOP is to derive a compromise solution of the DM that is also pareto optimal. In this section, we use the weighted sum method to solve the biobjective optimization model. The model sensitivity has been shown with respect to changes in the minimum threshold value of the ICD for each application and also by varying the weight parameters of the two objective functions reflecting the preferences of the software development team (here the DM). A real-world scenario of developing two financial applications for two small-scale industries is included to demonstrate the efficiency of the model.
The rest of the section is organized as follows. Section 4.1.1 describes the criteria used for COTS selection. In Section 4.1.2, mathematical formulation of the optimization model is presented. Section 4.1.3 discusses solution methodology. Section 4.1.4 presents numerical illustrations of a real-world scenario inspired from CBSD to demonstrate the applicability of the proposed model. Finally, in Section 4.1.5, we give some concluding remarks.

4.1.1 Criteria for COTS selection under multiple applications environment

To select COTS components for modular software systems under multiple applications environment, the following criteria may be used.

- **Functional performance**

  The functional capabilities of COTS components are different for different components. Functionality of the COTS component is nothing but the ability of the component to perform according to the specific needs of the organization. We use functional ratings of COTS components to the software modules as coefficients in the objective function that maximizes the functional performance of the modular software system. We assume that these ratings are provided by the DM.

- **Cost**

  The cost criterion is used to assess cost related characteristics of COTS components. We consider cost based on procurement and adaptation costs of COTS components. The procurement cost contains licensing arrangement cost, product and technology cost, and consulting cost.

- **ICD**

  For a modular software system the COTS components should be selected such that the interactions of COTS components within a software module are maximized, and interactions of COTS components among software modules are minimized. Since, coupling is the measure of interactions among software modules and cohesion is the measure of interactions among the software components which are within a software module; for a good software system modules with high cohesion and low coupling are desired. Thus, in CBSD, a quantitative
way of minimizing coupling and maximizing the cohesion of modules must be addressed properly. Abreu and Goulao [2] have proposed quantitative measures of cohesion and coupling. The relationship between cohesion and coupling of modules in the development of a modular software system can be measured by using $ICD$ defined as

$$ICD = \frac{CI_{IN}}{CI_{IN} + CI_{OUT}} = \frac{CI_{IN}}{CA} \quad (4.1.1)$$

where $CI_{IN}$ is the number of class interactions within modules, $CI_{OUT}$ is the number of interactions between classes of distinct modules and $CA = CI_{IN} + CI_{OUT}$ corresponds to the total number of existing class interactions. Thus, $ICD$ presents the ratio between class interactions within modules and total number of class interactions. It is well known that loose coupling and tight cohesion can achieve high maintainability of a software system. Thus, the values of $ICD$ for each of the application would have great influence on the maintainability of the modular software system. Figure 4.1.1 (replicated from [69]) shows the diagrammatic depiction of cohesion and coupling of software modules in the development of a modular software system.

![Diagram showing cohesion and coupling of modules in CBSD](image.png)

**Figure 4.1.1:** Cohesion and coupling of modules in CBSD
• Reusability

Reusability of a component defines the extent to which parts of a software can be reused in other applications. Software reuse means to abstract the general logic from different applications, implement the logic, and be used into more applications with slight or no modification. Thus, reusability of a COTS component means that a component can be reused into different applications rather than one application.

4.1.2 Biobjective optimization model

Here, we assume that the DM concurrently undertakes \( N \) applications which consists of \( M \) modules as shown in Figure 4.1.2. Suppose the \( i \)-th application requires \( m_i \) modules, then \( M = \sum_{i=1}^{N} m_i \). Each module must contain at least one COTS component. The COTS components’ market contains \( L \) COTS components which are distributed among \( T \) different sets of alternative COTS components. The COTS components within a set fulfills the same functional requirement of the system and only one component from a given set is selected in each application.

Figure 4.1.2: Three layers of hierarchy of a modular software system with multiple applications
In the proposed optimization model for COTS selection, we consider the following objectives and constraints.

**Objectives**

- **Functional performance**
  The objective function corresponding to the functional performance of the software system is expressed as
  \[
  \max F(x) = \sum_{j=1}^{M} \sum_{k=1}^{L} f_{jk} x_{j,k}. \]

- **Cost**
  The objective function corresponding to the total cost of the software system is expressed as
  \[
  \min C(x) = \left( \sum_{k=1}^{L} c_{pk} y_k + \sum_{j=1}^{M} \sum_{k=1}^{L} c_{jk} x_{j,k} \right).
  \]

In the above objective function the first term represents the total procurement cost and the second term represents the total adaptation cost of all selected COTS components. Hence, the above objective minimizes the total development cost incurred for accomplishing all the undertaken applications.

**Constraints**

- **ICD constraints**
  Let the cohesion within the \( j \)-th module, \((CI_{IN})_j\), be given by
  \[
  (CI_{IN})_j = \sum_{k=1}^{L-1} \sum_{k'=k+1}^{L} r_{kk'} x_{j,k} x_{j,k'}.
  \]
  Then, the sum of cohesions within all modules of the \( i \)-th application, \((CI_{IN})^i\), can be expressed as
  \[
  (CI_{IN})^i = \sum_{j=1}^{M} s_{ij} (CI_{IN})_j = \sum_{j=1}^{M} s_{ij} \left( \sum_{k=1}^{L-1} \sum_{k'=k+1}^{L} r_{kk'} x_{j,k} x_{j,k'} \right). \]
Furthermore, let all interactions including cohesion and coupling associated with the \( j \)-th module, \((CA)_j\), be expressed as

\[
(CA)_j = (CI_{IN})_j + (CI_{OUT})_j = \sum_{k=1}^{L-1} \sum_{k' = k+1}^{L} r_{kk'} x_{j,k} \left( \sum_{j=1}^{M} s_{ij} x_{j,k'} \right).
\]

Then, all interactions including cohesion and coupling of the \( i \)-th application, \((CA)^i\), can be expressed as

\[
(CA)^i = \sum_{j=1}^{M} s_{ij} (CA)_j = \sum_{k=1}^{L-1} \sum_{k' = k+1}^{L} r_{kk'} \left( \sum_{j=1}^{M} s_{ij} x_{j,k} \right) \left( \sum_{j=1}^{M} s_{ij} x_{j,k'} \right). \tag{4.1.3}
\]

Now, using equations (4.1.1), (4.1.2), and (4.1.3), \(ICD\) for the \( i \)-th application is given by

\[
(ICD)_i = \frac{\sum_{j=1}^{M} s_{ij} \left( \sum_{k=1}^{L-1} \sum_{k' = k+1}^{L} r_{kk'} x_{j,k} x_{j,k'} \right)}{\sum_{k=1}^{L-1} \sum_{k' = k+1}^{L} r_{kk'} \left( \sum_{j=1}^{M} s_{ij} x_{j,k} \right) \left( \sum_{j=1}^{M} s_{ij} x_{j,k'} \right)}. \]

**Reusability constraints**

The reusability constraints of the \( k \)-th COTS component is given by

\[
\sum_{j=1}^{M} s_{ij} x_{j,k} \leq 1, \quad i = 1, 2, \ldots, N, \quad k = 1, 2, \ldots, L.
\]

The above constraints implies that each COTS component can be used into an application system at most once. If \( \sum_{j=1}^{M} s_{ij} x_{j,k} = 0 \), it means the \( k \)-th COTS component is not selected into the \( i \)-th application; otherwise, the \( k \)-th component is selected.

The logical relation between \( y_k \) and \( x_{j,k} \) can be represented by the constraint
\[
\sum_{j=1}^{M} x_{j,k} \leq y_k \cdot N, \; k = 1, 2, \ldots, L.
\]

It expresses that, if a COTS component is selected to implement a module into an application, then it can be reused at most \( N \) times in all the applications for implementing different modules; that is, \( \sum_{j=1}^{M} x_{j,k} \leq N \) while \( y_k = 1 \); and for \( j = 1, 2, \ldots, M, \; x_{j,k} = 0 \) when \( y_k = 0 \).

Since, the binary parameter \( b_{jk} = 1 \) denotes that the \( k \)-th COTS component can be reused to implement the \( j \)-th module, the ‘can be adapted to’ relation between COTS components and modules is represented using the constraint

\[
x_{j,k} \leq b_{jk}, \; j = 1, 2, \ldots, M, \; k = 1, 2, \ldots, L.
\]

- **COTS component selection constraints**

The constraint representing that from a set of alternative COTS components only one COTS component can be selected for a particular application is given by

\[
\sum_{k \in \mathcal{S}_i} \sum_{j=1}^{M} s_{ij} x_{j,k} = 1, \; i = 1, 2, \ldots, N, \; t = 1, 2, \ldots, T.
\]

Also, the constraint representing that each software module must contain at least one COTS component is given by

\[
\sum_{k=1}^{L} x_{j,k} \geq 1, \; j = 1, 2, \ldots, M.
\]

- **Selection or rejection of a COTS component**

\[
x_{j,k} \in \{0, 1\}, \; j = 1, 2, \ldots, M, \; k = 1, 2, \ldots, L.
\]
The decision model

The biobjective optimization model for COTS selection is formulated as follows:

\[
P(4.1.1) \quad \begin{align*}
\text{max } F(x) &= \sum_{j=1}^{M} \sum_{k=1}^{L} f_{jk}x_{j,k} \\
\text{min } C(x) &= \left( \sum_{k=1}^{L} c_{pk}y_{k} + \sum_{j=1}^{M} \sum_{k=1}^{L} c_{jk}x_{j,k} \right) \\
\text{subject to } & \\
\sum_{j=1}^{M} s_{ij} \left( \sum_{k=1}^{L-1} \sum_{k'=k+1}^{L} r_{kk'}x_{j,k,x_{j,k'}} \right) & \geq H_{i}, \quad i = 1,2, \ldots, N, \quad (4.1.4) \\
\sum_{j=1}^{M} s_{ij}x_{j,k} & \leq 1, \quad i = 1,2, \ldots, N, \quad k = 1,2, \ldots, L, \quad (4.1.5) \\
\sum_{j=1}^{M} x_{j,k} & \leq y_{k} \cdot N, \quad k = 1,2, \ldots, L, \quad (4.1.6) \\
x_{j,k} & \leq b_{jk}, \quad j = 1,2, \ldots, M, \quad k = 1,2, \ldots, L, \quad (4.1.7) \\
\sum_{k \in s_{t}} \sum_{j=1}^{M} s_{ij}x_{j,k} & = 1, \quad i = 1,2, \ldots, N, \quad t = 1,2, \ldots, T(4.1.8) \\
\sum_{k=1}^{L} x_{j,k} & \geq 1, \quad j = 1,2, \ldots, M, \quad (4.1.9) \\
x_{j,k}, y_{k} & \in \{0,1\}, \quad j = 1,2, \ldots, M, \quad k = 1,2, \ldots, L. \quad (4.1.10)
\end{align*}
\]

4.1.3 Solution approach

The proposed biobjective optimization model contain many optimal solutions which are called pareto optimal solutions or efficient solutions. Here, the weighted sum method is used to solve the biobjective optimization model P(4.1.1). Let \( F_{\text{max}} \) and \( F_{\text{min}} \) be the upper and lower bounds of the first objective function, respectively. Let \( C_{\text{max}} \) and \( C_{\text{min}} \) denote the upper and lower bounds of the second objective function, respectively. The value of \( F_{\text{max}} \) can be determined by solving model P(4.1.2) defined as follows:
\[ P(4.1.2) \quad F_{\text{max}} = \max \sum_{j=1}^{M} \sum_{k=1}^{L} f_{jk} x_{j,k} \]

subject to

Constraints (4.1.4)–(4.1.10).

Similarly, the value of \( F_{\text{min}} \) can be determined by solving model \( P(4.1.3) \) defined as follows:

\[ P(4.1.3) \quad F_{\text{min}} = \min \sum_{j=1}^{M} \sum_{k=1}^{L} f_{jk} x_{j,k} \]

subject to

Constraints (4.1.4)–(4.1.10).

The model \( P(4.1.4) \) determines the value of \( C_{\text{max}} \) as follows:

\[ P(4.1.4) \quad C_{\text{max}} = \max \left( \sum_{k=1}^{L} c_{k}^{p} y_{k} + \sum_{j=1}^{M} \sum_{k=1}^{L} c_{jk}^{a} x_{j,k} \right) \]

subject to

Constraints (4.1.4)–(4.1.10).

The model \( P(4.1.5) \) finds the value of \( C_{\text{min}} \) as follows:

\[ P(4.1.5) \quad C_{\text{min}} = \min \left( \sum_{k=1}^{L} c_{k}^{p} y_{k} + \sum_{j=1}^{M} \sum_{k=1}^{L} c_{jk}^{a} x_{j,k} \right) \]

subject to

Constraints (4.1.4)–(4.1.10).

After calculating the values of upper and lower bounds of the two objective functions, the proposed biobjective optimization model can be reformulated into a weighted sum optimization model as follows:
4.1 Biobjective optimization model for COTS selection based on cohesion and coupling under multiple applications environment

\[ \text{P(4.1.6)} \quad \max \left( w_f \cdot F'(x) + w_c \cdot C'(x) \right) \]

subject to

Constraints (4.1.4)–(4.1.10),

\[ w_f + w_c = 1 \]

\[ w_f \geq 0, \quad w_c \geq 0 \]

where \( F'(x) = \frac{F(x) - F_{\text{min}}}{F_{\text{max}} - F_{\text{min}}} \) and \( C'(x) = \frac{C_{\text{max}} - C(x)}{C_{\text{max}} - C_{\text{min}}} \). Also, \( w_f \) and \( w_c \) are the weights of the first and the second objective function, respectively.

**Theorem 4.1** ([30]). The optimal solution of the weighted sum optimization model \( \text{P(4.1.6)} \) is pareto optimal solution of the biobjective optimization model \( \text{P(4.1.1)} \) if the weighting coefficients \( w_f \) and \( w_c \) are positive.

The optimization models \( \text{P(4.1.2)}, \text{P(4.1.3)}, \text{P(4.1.4)}, \text{P(4.1.5)}, \) and \( \text{P(4.1.6)} \) are nonlinear optimization models. We use LINGO to solve them.

### 4.1.4 An illustrative example

To illustrate the proposed methodology of optimizing the selection of best-fit COTS components for modular software systems, a hypothetical small-scale scenario of software development discussed in [69, 103] is presented in this section. Let us consider that a software developer undertakes two financial applications for two different small-size industries, namely, Garment Industry Financial System (App1) and Pharmaceutical Industry Financial System (App2). App1 consists of three modules: Garment Business-Related module \((M_1)\), Garment Security module \((M_2)\), and Assistance for Garment Industry \((M_3)\). Similarly, App2 includes: Pharmaceutical Business-Related module \((M_4)\), Pharmaceutical Security module \((M_5)\), and Assistance for Pharmaceutical Industry \((M_6)\).

The software system should fulfill six basic functional requirements, namely, Facsimile/Fax \((R_1)\), Encryption \((R_2)\), Credit Card Authorization \((R_3)\), Automatic Updates \((R_4)\), e-Commerce \((R_5)\), and Financial Reporting \((R_6)\). E-commerce and Financial Reporting functions are provided by Business-Related module, the functions of Encryption and Authorization are provided by the
Security module while functions like Fax and Software Automatic Updating are mainly provided by the Assistance module. Let us consider that 12 COTS components are available in COTS components’ market which are denoted by $sc_1, sc_2, \ldots, sc_{12}$. Note that, corresponding to functional requirement $R_1$ the set $s_1$ of alternative COTS components include $sc_1, sc_2, sc_3, sc_4$; for $R_2$ the set $s_2$ includes $sc_5$; for $R_3$ the set $s_3$ includes $sc_6$; for $R_4$ the set $s_4$ include $sc_7, sc_8, sc_9$; for $R_5$ the set $s_5$ include $sc_{10}, sc_{11}$; and for $R_6$ the set $s_6$ includes $sc_{12}$. The membership matrix $S = [s_{ij}]_{N \times M}$ representing ‘belong to’ relation between applications and modules is presented in Table 4.1.1. The reuse matrix $S' = [b_{jk}]_{M \times L}$, representing ‘can be adapted to’ relation between modules and COTS components is presented in Table 4.1.2. The Individual functional requirements and their corresponding alternative COTS components, functional ratings of COTS components corresponding to the software modules, as well as the procurement cost of COTS components, and the adaptation costs of each component to various modules are given in Table 4.1.3. Also, ‘−’ in Table 4.1.3 denotes that the $k$-th COTS component cannot be reused to implement the $j$-th module.

**Table 4.1.1: Membership matrix $S$**

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<th>$M_1$</th>
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<th>$M_4$</th>
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**Table 4.1.2: Reuse matrix $S'$**

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Table 4.1.3: Initial parameters for COTS components

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<td>M3</td>
<td>18 12 13 11</td>
<td>-</td>
<td>21</td>
<td>17 19 16</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>0.51 0.63 0.72 0.57</td>
<td>0</td>
<td>0.45</td>
<td>0.94 0.86</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>M5</td>
<td>17 18 14 15</td>
<td>11</td>
<td>13</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>M6</td>
<td>0.35 0.24 0.13 0.21</td>
<td>0.85</td>
<td>0.70</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td></td>
<td>- - -</td>
<td>13</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>App2</td>
<td>0.45 0.65 0.49 0.54</td>
<td>0</td>
<td>0.40</td>
<td>0.90 0.65</td>
<td>0.97</td>
</tr>
</tbody>
</table>
The functional ratings describe the degrees of functional contributions of COTS components towards the software modules. The functional ratings which are fixed based on group decision making of the DM ranges from 0 to 1 where 1 refers to a very high degree of contribution while 0 indicates zero degree of contribution. Table 4.1.4 shows the degrees of interaction among the COTS components which are provided by the DM’s judgment and varies between 0-10, where the degree ‘10’ refers to a very high degree of interaction. It may be noted that cohesion and coupling are undirected relations; hence, Table 4.1.4 is symmetric.

### Table 4.1.4: Interactions among COTS components

<table>
<thead>
<tr>
<th></th>
<th>sc1</th>
<th>sc2</th>
<th>sc3</th>
<th>sc4</th>
<th>sc5</th>
<th>sc6</th>
<th>sc7</th>
<th>sc8</th>
<th>sc9</th>
<th>sc10</th>
<th>sc11</th>
<th>sc12</th>
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<tbody>
<tr>
<td>sc1</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>sc2</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>sc3</td>
<td>-</td>
<td>0</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>7</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>sc4</td>
<td>-</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>sc5</td>
<td>-</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sc6</td>
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<td>5</td>
<td>8</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>sc7</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td></td>
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</tr>
<tr>
<td>sc8</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sc9</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sc10</td>
<td>-</td>
<td>0</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sc11</td>
<td>-</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sc12</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Suppose the DM initially give equal importance to both the objective functions, that is, \( w_f = 0.5 \) and \( w_c = 0.5 \). Let \( H_1 = 0.3 \), \( H_2 = 0.3 \) and assume that each COTS component can be used twice in all the applications. Then, by solving models P(4.1.2), P(4.1.3), P(4.1.4), and P(4.1.5) the values of \( F_{max} \), \( F_{min} \), \( C_{max} \), and \( C_{min} \) are obtained as 9.65, 6.21, 569, and 509, respectively. Using these values and the data given in Tables 4.1.1–4.1.4, we obtain the following weighted sum model for COTS selection:
4.1 Biobjective optimization model for COTS selection based on cohesion and coupling under multiple applications environment

max = 0.5((0.35x_{1,5} + 0.98x_{1,10} + 0.89x_{1,11} + 0.75x_{1,12} + 0.32x_{2,1} + 0.22x_{2,2} + 0.15x_{2,3} + 0.23x_{2,4} + 0.94x_{2,5} + 0.68x_{2,6} + 0.51x_{3,1} + 0.63x_{3,2} + 0.72x_{3,3} + 0.57x_{3,4} + 0.45x_{3,6} + 0.94x_{3,7} + 0.86x_{3,8} + x_{3,9} + 0.15x_{4,5} + 0.90x_{4,10} + 0.83x_{4,11} + 0.60x_{4,12} + 0.35x_{5,1} + 0.24x_{5,2} + 0.13x_{5,3} + 0.21x_{5,4} + 0.85x_{5,5} + 0.70x_{5,6} + 0.45x_{6,1} + 0.65x_{6,2} + 0.49x_{6,3} + 0.54x_{6,4} + 0.40x_{6,6} + 0.90x_{6,7} + 0.65x_{6,8} + 0.97x_{6,9}) - 6.21)}/(3.44) + 0.5((569 - (49y_1 + 66y_2 + 54y_3 + 55y_4 + 62y_5 + 61y_6 + 74y_7 + 74y_8 + 79y_9 + 47y_{10} + 40y_{11} + 49y_{12} + 22x_{1,15} + 12x_{1,10} + 14x_{1,11} + 16x_{1,12} + 20x_{2,1} + 19x_{2,2} + 18x_{2,3} + 17x_{2,4} + 14x_{2,5} + 13x_{2,6} + 18x_{3,1} + 12x_{3,2} + 13x_{3,3} + 11x_{3,4} + 21x_{3,6} + 17x_{3,7} + 19x_{3,8} + 16x_{3,9} + 20x_{4,5} + 12x_{4,10} + 16x_{4,11} + 15x_{4,12} + 17x_{5,1} + 18x_{5,2} + 14x_{5,3} + 15x_{5,4} + 11x_{5,5} + 13x_{6,1} + 9x_{6,2} + 11x_{6,3} + 13x_{6,4} + 20x_{6,6} + 18x_{6,7} + 14x_{6,8} + 13x_{6,9})/60)

subject to

\[
\sum_{j=1}^{3} x_{j,1}(x_{j,6} + 6x_{j,7} + 8x_{j,8} + 7x_{j,9}) + \sum_{j=1}^{3} x_{j,2}(7x_{j,5} + 6x_{j,6} + 8x_{j,7} + 9x_{j,8} + 7x_{j,9}) + \sum_{j=1}^{3} x_{j,3}(8x_{j,5} + 7x_{j,6} + 9x_{j,7} + 7x_{j,8} + 6x_{j,9}) + \sum_{j=1}^{3} x_{j,4} (4x_{j,5} + 3x_{j,6} + 5x_{j,7} + 6x_{j,8} + 8x_{j,9}) + \sum_{j=1}^{3} x_{j,5}(8x_{j,6} + 7x_{j,10} + 7x_{j,11} + 8x_{j,12}) + \sum_{j=1}^{3} x_{j,6}(5x_{j,7} + 8x_{j,8} + 7x_{j,9}) + \sum_{j=1}^{3} x_{j,10}(8x_{j,12}) + \sum_{j=1}^{3} x_{j,11}(9x_{j,12})
\]

\[
\left(\sum_{j=1}^{3} x_{j,1} + \sum_{j=1}^{3} x_{j,6} + \sum_{j=1}^{3} x_{j,7} + \sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,9}\right)
\]

\[
+ \sum_{j=1}^{3} x_{j,2} \left(\sum_{j=1}^{3} x_{j,5} + \sum_{j=1}^{3} x_{j,6} + \sum_{j=1}^{3} x_{j,7} + \sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,9}\right) + \sum_{j=1}^{3} x_{j,3} \left(\sum_{j=1}^{3} x_{j,5} + \sum_{j=1}^{3} x_{j,6} + \sum_{j=1}^{3} x_{j,7} + \sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,9}\right) + \sum_{j=1}^{3} x_{j,4} \left(\sum_{j=1}^{3} x_{j,5} + \sum_{j=1}^{3} x_{j,6} + \sum_{j=1}^{3} x_{j,7} + \sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,9}\right) + \sum_{j=1}^{3} x_{j,5} \left(\sum_{j=1}^{3} x_{j,6} + \sum_{j=1}^{3} x_{j,7} + \sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,9}\right) + \sum_{j=1}^{3} x_{j,6} \left(\sum_{j=1}^{3} x_{j,7} + \sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,9}\right) + \sum_{j=1}^{3} x_{j,7} \left(\sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,9}\right) + \sum_{j=1}^{3} x_{j,8} \left(\sum_{j=1}^{3} x_{j,9}\right)
\]
\[
\begin{align*}
&\sum_{j=1}^{3} x_{j,4} \left( 4 \sum_{j=1}^{3} x_{j,5} + 3 \sum_{j=1}^{3} x_{j,6} + 5 \sum_{j=1}^{3} x_{j,7} + 6 \sum_{j=1}^{3} x_{j,8} + 8 \sum_{j=1}^{3} x_{j,9} \right) + \\
&\sum_{j=1}^{3} x_{j,5} \left( 8 \sum_{j=1}^{3} x_{j,6} + 7 \sum_{j=1}^{3} x_{j,10} + 7 \sum_{j=1}^{3} x_{j,11} + 8 \sum_{j=1}^{3} x_{j,12} \right) + \sum_{j=1}^{3} x_{j,6} \\
&\left( 5 \sum_{j=1}^{3} x_{j,7} + 8 \sum_{j=1}^{3} x_{j,8} + 7 \sum_{j=1}^{3} x_{j,9} \right) + \sum_{j=1}^{3} x_{j,10} \left( 8 \sum_{j=1}^{3} x_{j,12} \right) + \sum_{j=1}^{3} x_{j,11} \\
&\left( 9 \sum_{j=1}^{3} x_{j,12} \right) \geq 0.3,
\end{align*}
\]
4.1 Biobjective optimization model for COTS selection based on cohesion and coupling under multiple applications environment

\[
\sum_{j=1}^{3} x_{j,k} \leq 1, \ k = 1, 2, \ldots, 12, \quad \sum_{j=4}^{6} x_{j,k} \leq 1, \ k = 1, 2, \ldots, 12,
\]

\[
\sum_{j=1}^{6} x_{j,k} \leq 2y_k, \ k = 1, 2, \ldots, 12,
\]

\[
x_{1,1} \leq 0, \ x_{1,2} \leq 0, \ x_{1,3} \leq 0, \ x_{1,4} \leq 0, \ x_{1,5} \leq 0, \ x_{1,6} \leq 0, \ x_{1,7} \leq 0, \ x_{1,8} \leq 0, \ x_{1,9} \leq 0, \ x_{1,10} \leq 1, \ x_{1,11} \leq 1, \ x_{1,12} \leq 1, \ x_{2,1} \leq 1, \ x_{2,2} \leq 1, \ x_{2,3} \leq 1, \ x_{2,4} \leq 1, \ x_{2,5} \leq 1, \ x_{2,6} \leq 1, \ x_{2,7} \leq 0, \ x_{2,8} \leq 0, \ x_{2,9} \leq 0, \ x_{2,10} \leq 0, \ x_{2,11} \leq 0, \ x_{2,12} \leq 0, \ x_{3,1} \leq 1, \ x_{3,2} \leq 1, \ x_{3,3} \leq 1, \ x_{3,4} \leq 1, \ x_{3,5} \leq 0, \ x_{3,6} \leq 1, \ x_{3,7} \leq 1, \ x_{3,8} \leq 1, \ x_{3,9} \leq 1, \ x_{3,10} \leq 0, \ x_{3,11} \leq 0, \ x_{3,12} \leq 0, \ x_{4,1} \leq 0, \ x_{4,2} \leq 0, \ x_{4,3} \leq 0, \ x_{4,4} \leq 0, \ x_{4,5} \leq 1, \ x_{4,6} \leq 0, \ x_{4,7} \leq 0, \ x_{4,8} \leq 0, \ x_{4,9} \leq 0, \ x_{4,10} \leq 1, \ x_{4,11} \leq 1, \ x_{4,12} \leq 1, \ x_{5,1} \leq 1, \ x_{5,2} \leq 1, \ x_{5,3} \leq 1, \ x_{5,4} \leq 1, \ x_{5,5} \leq 1, \ x_{5,6} \leq 1, \ x_{5,7} \leq 0, \ x_{5,8} \leq 0, \ x_{5,9} \leq 0, \ x_{5,10} \leq 0, \ x_{5,11} \leq 0, \ x_{5,12} \leq 0, \ x_{6,1} \leq 1, \ x_{6,2} \leq 1, \ x_{6,3} \leq 1, \ x_{6,4} \leq 1, \ x_{6,5} \leq 0, \ x_{6,6} \leq 1, \ x_{6,7} \leq 1, \ x_{6,8} \leq 1, \ x_{6,9} \leq 1, \ x_{6,10} \leq 0, \ x_{6,11} \leq 0, \ x_{6,12} \leq 0,
\]

\[
\sum_{j=1}^{3} (x_{j,1} + x_{j,2} + x_{j,3} + x_{j,4}) = 1, \quad \sum_{j=1}^{3} x_{j,5} = 1, \quad \sum_{j=1}^{3} x_{j,6} = 1,
\]

\[
\sum_{j=1}^{3} (x_{j,7} + x_{j,8} + x_{j,9}) = 1, \quad \sum_{j=1}^{3} (x_{j,10} + x_{j,11}) = 1, \quad \sum_{j=1}^{3} x_{j,12} = 1,
\]

\[
\sum_{j=4}^{6} (x_{j,1} + x_{j,2} + x_{j,3} + x_{j,4}) = 1, \quad \sum_{j=4}^{6} x_{j,5} = 1, \quad \sum_{j=4}^{6} x_{j,6} = 1,
\]

\[
\sum_{j=4}^{6} (x_{j,7} + x_{j,8} + x_{j,9}) = 1, \quad \sum_{j=4}^{6} (x_{j,10} + x_{j,11}) = 1, \quad \sum_{j=4}^{6} x_{j,12} = 1,
\]

\[
\sum_{k=1}^{12} x_{j,k} \geq 1, \ j = 1, 2, \ldots, 6,
\]

\[
y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12}, x_{1,1}, x_{1,2}, x_{1,3}, x_{1,4}, x_{1,5}, x_{1,6}, x_{1,7}, x_{1,8}, x_{1,9}, x_{1,10}, x_{1,11}, x_{1,12}, x_{2,1}, x_{2,2}, x_{2,3}, x_{2,4}, x_{2,5}, x_{2,6}, x_{2,7}, x_{2,8}, x_{2,9}, x_{2,10}, x_{2,11}, x_{2,12}, x_{3,1}, x_{3,2}, x_{3,3}, x_{3,4}, x_{3,5}, x_{3,6}, x_{3,7}, x_{3,8}, x_{3,9}, x_{3,10}, x_{3,11}, x_{3,12}, x_{4,1}, x_{4,2}, x_{4,3}, x_{4,4}, x_{4,5}, x_{4,6}, x_{4,7}, x_{4,8}, x_{4,9}, x_{4,10}, x_{4,11}, x_{4,12}, x_{5,1}, x_{5,2}, x_{5,3}, x_{5,4}, x_{5,5}, x_{5,6}, x_{5,7}, x_{5,8}, x_{5,9}, x_{5,10}, x_{5,11}, x_{5,12}, x_{6,1}, x_{6,2}, x_{6,3}, x_{6,4}, x_{6,5}, x_{6,6}, x_{6,7}, x_{6,8}, x_{6,9}, x_{6,10}, x_{6,11}, x_{6,12} \in \{0, 1\}.
\]
By solving the above optimization model using LINGO, we obtain the optimal values of \( F(x) \) and \( C(x) \) as 9.58 and 511, respectively. The COTS components \( sc_{10} \) and \( sc_{12} \) selected for modules \( M1 \) and \( M4 \) provides the functional requirements of e-Commerce and Financial Reporting as desired. Modules \( M2 \) and \( M5 \) gets COTS components \( sc_{5} \) and \( sc_{6} \) which fulfills the functional requirements of Encryption and Credit Card Authorization. Modules \( M3 \) and \( M6 \) gets COTS components \( sc_{3} \) and \( sc_{9} \) which contributes towards the functional requirements of Fax and Automatic updates.

**Sensitivity analysis with respect to changes in the minimum threshold value of ICD**

In order to increase maintainability of the software system, we perform sensitivity analysis with respect to changes in the minimum threshold value of ICD for each application. For different values of \( H_i \), the obtained results are listed in Table 4.1.5. From Table 4.1.5, it is clear that if we increase the minimum threshold value of \( H_i \), that is, increase the maintainability of the software system, it has adverse effect on at least one of the two objective functions. Also, if the DM desires that the ICD level should be at least 0.35, then second and third solutions can be considered for implementation. Besides, considering the objective values, the DM can also select any one of the obtained solutions based on various criteria, such as their own preferences or customers’ expectations. For example, if a customer has a limited budget of 512 then only first two solutions can be considered for implementation.

**Table 4.1.5:** COTS selection corresponding to \( w_f = 0.5, w_c = 0.5 \)

<table>
<thead>
<tr>
<th>( H_1 )</th>
<th>( H_2 )</th>
<th>( F(x) )</th>
<th>( C(x) )</th>
<th>( M1 )</th>
<th>( M2 )</th>
<th>( M3 )</th>
<th>( M4 )</th>
<th>( M5 )</th>
<th>( M6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>9.58</td>
<td>511</td>
<td>( sc_{10}, sc_{12} )</td>
<td>( sc_{3}, sc_{6} )</td>
<td>( sc_{3}, sc_{9} )</td>
<td>( sc_{10}, sc_{12} )</td>
<td>( sc_{5}, sc_{6} )</td>
<td>( sc_{3}, sc_{9} )</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>9.48</td>
<td>512</td>
<td>( sc_{10}, sc_{12} )</td>
<td>( sc_{3}, sc_{6} )</td>
<td>( sc_{4}, sc_{9} )</td>
<td>( sc_{10}, sc_{12} )</td>
<td>( sc_{5}, sc_{6} )</td>
<td>( sc_{4}, sc_{9} )</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>9.33</td>
<td>513</td>
<td>( sc_{10}, sc_{12} )</td>
<td>( sc_{3}, sc_{6} )</td>
<td>( sc_{1}, sc_{9} )</td>
<td>( sc_{10}, sc_{12} )</td>
<td>( sc_{5}, sc_{6} )</td>
<td>( sc_{1}, sc_{9} )</td>
</tr>
</tbody>
</table>

**Sensitivity analysis with respect to changes in the weights of the two objectives**

By varying the weights of the two objectives different solutions can be obtained for a fixed value of ICD. For example, suppose the DM gives more importance
4.1 Biobjective optimization model for COTS selection based on cohesion and coupling under multiple applications environment

4.1.5 Concluding remarks

In this section, we have introduced a biobjective optimization model which maximizes the functional requirements and minimizes the total cost of the modular software system. Compared with the previous studies, the proposed model...
additionally considered components’ reusability and cohesion and coupling of software modules simultaneously under multiple applications environment. The model is used to assist the DM in selecting best-fit COTS components when multiple applications are undertaken concurrently. The effectiveness of the model is demonstrated through numerical examples constructed corresponding to a real-world scenario of multiple applications environment. The model sensitivity have been shown with respect to changes in the minimum threshold value of the ICD for each application and also by varying the weight parameters of the two objective functions reflecting the preferences of the DM. The proposed methodology involves some subjective judgments from the DM such as the determination of the scores of interaction and the functional ratings which may be considered as limitations of the study.

4.2 Fuzzy COTS selection based on cohesion and coupling under multiple applications environment

To implement the COTS selection optimization model P(4.1.1), we require exact estimates of the functional ratings and cost of each COTS component. However, in real life, these estimates cannot be determined exactly because DM’s assessment about these estimates may be based on incomplete knowledge of COTS components itself and other aspects including vendor’s credentials, which may affect the COTS selection. The selection of best-fit COTS components using a crisp optimization model thus may not be the right decision. Under such circumstances, the issue of selecting COTS components becomes the one of a choice from a ‘fuzzy’ set of subjective/intuitive interpretations, the term fuzzy being suggestive of the diversity of both the DM’s objective functions as well as that of the constraints.

The fuzzy biobjective optimization model proposed in this section is an extension of the optimization model P(4.1.1) presented in the Section 4.1.2. To provide flexibility in the exact estimation of functional ratings and the cost of COTS components, we use fuzzy mathematical programming based on vague aspiration levels of the DM characterized by nonlinear S-shape membership
functions. We use Bellman-Zadeh’s maximization principle (the max-min approach) to formulate the fuzzy biobjective optimization model under the assumption that both the fuzzy objectives are treated equivalently. Furthermore, to reflect the relative importance of the DM for the two objectives, a weighted additive model is solved by assigning different weights to the two objectives. The efficiency of the obtained solutions is verified by using a two-phase approach. The applicability of the model in real-world situations is demonstrated through a case study. The main advantage of the proposed models is that if the DM is not satisfied with any of the COTS selection obtained, more COTS selections can be generated by varying values of shape parameters of nonlinear S-shape membership functions.

The rest of the section is organized as follows. Section 4.2.1 presents the fuzzy mathematical programming models of COTS selection using nonlinear S-shape membership functions. Section 4.2.2 deals with numerical illustrations of the fuzzy biobjective optimization models for COTS selection of a modular software system. Finally, some concluding remarks are given in Section 4.2.3.

4.2.1 COTS selection models based on fuzzy set theory

Here, we use nonlinear S-shape membership functions to express vague aspiration levels of the DM. The membership function of the goal for the functional requirement is given by

$$\mu_F(x) = \frac{1}{1 + \exp\left(-\alpha_F \left( \sum_{j=1}^{M} \sum_{k=1}^{L} f_{jk} x_{j,k} - F_m \right) \right)};$$

where $F_m$ is the mid-point (middle aspiration level) at which the membership function value is 0.5 and $\alpha_F$ is given by the DM based on his own degree of satisfaction for the functional requirement (see Figure 4.2.1).
Similarly, the membership function of the goal for the cost is given by

$$\mu_C(x) = \frac{1}{1 + \exp\left(\alpha_C \left(\sum_{k=1}^{L} c_{yk}^k + \sum_{j=1}^{M} \sum_{k=1}^{L} c_{jk}^a x_{j,k} - C_m\right)\right)},$$

where $C_m$ is the mid-point (middle aspiration level for the cost) at which the membership function value is 0.5 and $\alpha_C$ is given by the DM based on his own degree of satisfaction for the cost (see Figure 4.2.2).

The shape parameters $\alpha_F$ and $\alpha_C$ and the mid points $F_m$ and $C_m$ are defined and interpreted on the same lines as given in Remarks (3.1.1)–(3.1.2).

Using, Bellman-Zadeh’s maximization principle along with the above defined fuzzy membership functions, the fuzzy biobjective optimization model for COTS selection is formulated as follows:
$$\textbf{P}(4.2.1) \quad \max \lambda$$

subject to

$$\lambda \leq \mu_F(x),$$
$$\lambda \leq \mu_C(x),$$
$$0 \leq \lambda \leq 1,$$

and Constraints (4.1.4)–(4.1.10).

The model P(4.2.1) is an NLPP. Now, proceeding on the same lines as in Section 3.1.2, the above model can be transformed into the following LPP using the transformation $\theta = \log \frac{\lambda}{1 - \lambda}$.

$$\textbf{P}(4.2.2) \quad \max \theta$$

subject to

$$\theta \leq \alpha_F \left( \sum_{j=1}^{M} \sum_{k=1}^{L} \tilde{f}_{jk} x_{j,k} - F_m \right),$$
$$\theta \leq \alpha_C \left( C_m - \sum_{k=1}^{L} c_k^p y_k - \sum_{j=1}^{M} \sum_{k=1}^{L} c_{jk}^a x_{j,k} \right),$$
$$\theta \geq 0,$$

and Constraints (4.1.4)–(4.1.10).

To incorporate relative importance of the fuzzy objectives in COTS selection problem, the weighted additive model for COTS selection with unequal importance between the two objectives is written as follows:

$$\textbf{P}(4.2.3) \quad \max \sum_{h=1}^{2} \omega_h \lambda_h$$

subject to

$$\lambda_1 \leq \mu_F(x),$$
$$\lambda_2 \leq \mu_C(x),$$
$$0 \leq \lambda_h \leq 1, \ h = 1, 2,$$

and Constraints (4.1.4)–(4.1.10),
where $\omega_h$ is the relative weight of the $h$-th objective function to be specified by the DM such that $\omega_h > 0$ and $\sum_{h=1}^{2} \omega_h = 1$. Furthermore, to ensure the efficiency of the obtained solution, we solve models P(4.2.4) and P(4.2.5) corresponding to models P(4.2.2) and P(4.2.3), respectively, in the second phase of the two-phase approach.

\[
\mathbf{P(4.2.4)} \quad \max \sum_{h=1}^{2} \omega_h \theta_h \\
\text{subject to} \\
\log \frac{\mu_F(x^*)}{1 - \mu_F(x^*)} \leq \theta_1 \leq \alpha_F \left( \sum_{j=1}^{M} \sum_{k=1}^{L} f_{jk} x_{j,k} - F_m \right), \\
\log \frac{\mu_C(x^*)}{1 - \mu_C(x^*)} \leq \theta_2 \leq \alpha_C \left( C_m - \sum_{k=1}^{L} c_p^k y_k - \sum_{j=1}^{M} \sum_{k=1}^{L} c_a^j x_{j,k} \right), \\
\theta_h \geq 0, \ h = 1, 2, \\
\text{and Constraints (4.1.4)–(4.1.10)},
\]

where $x^*$ is an optimal solution of model P(4.2.2), $\omega_1 = \omega_2 > 0$ and $\sum_{h=1}^{2} \omega_h = 1$.

\[
\mathbf{P(4.2.5)} \quad \max \sum_{h=1}^{2} \omega_h \lambda_h \\
\text{subject to} \\
\mu_F(x^{**}) \leq \lambda_1 \leq \mu_F(x), \\
\mu_C(x^{**}) \leq \lambda_2 \leq \mu_C(x), \\
0 \leq \lambda_h \leq 1, \ h = 1, 2, \\
\text{and Constraints (4.1.4)–(4.1.10)},
\]

where $x^{**}$ is an optimal solution of model P(4.2.3).

### 4.2.2 An illustrative example

Here, we consider the same hypothetical real-world scenario of developing two financial applications for two small-scale industries which was discussed in Section 4.1.4. We consider that 15 COTS components are available in COTS
components’ market denoted by \( sc_1, sc_2, \ldots, sc_{15} \). Note that, corresponding to functional requirement \( R_1 \) the set \( s_1 \) of alternative COTS components include \( sc_1, sc_2, sc_3 \); for \( R_2 \) the set \( s_2 \) include \( sc_4, sc_5 \); for \( R_3 \) the set \( s_3 \) include \( sc_6, sc_7, sc_8 \); for \( R_4 \) the set \( s_4 \) include \( sc_9, sc_{10} \); for \( R_5 \) the set \( s_5 \) include \( sc_{11}, sc_{12} \); and for \( R_6 \) the set \( s_6 \) include \( s_{13}, s_{14}, s_{15} \). The membership matrix \( S = [s_{ij}]_{N \times M} \) representing ‘belong to’ relation between applications and modules is presented in Table 4.2.1. The reuse matrix \( S' = [b_{jk}]_{M \times L} \), representing ‘can be adapted to’ relation between modules and COTS components is presented in Table 4.2.2. Individual functional requirements and their corresponding alternative COTS components, functional ratings of COTS components corresponding to the software modules, as well as the procurement cost of COTS components and the adaptation costs of each component to various modules are shown in Table 4.2.3. Also, ‘−’ in Table 4.2.3 denotes that the \( k \)-th COTS component cannot be reused to implement the \( j \)-th module. The functional ratings and the degree of interactions among the COTS components are defined on the same lines as discussed in Section 4.1.4. The degrees of interaction among the COTS components is given in Table 4.2.4.

**Table 4.2.1: Membership matrix \( S \)**

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**Table 4.2.2: Reuse matrix \( S' \)**

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### Table 4.2.3: Initial parameters for COTS components

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Table 4.2.4: Interactions among COTS components

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<td>$sc_9$</td>
<td>-</td>
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<td>$sc_{14}$</td>
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<td>$sc_{15}$</td>
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</table>
COTS selection using optimization model P(4.2.2)

We set $H_1 = 0.3, H_2 = 0.3, \alpha_F = 2, \alpha_C = 60, F_m = 8.5, \text{ and } C_m = 535$. Also, let each COTS component can be used twice in all the applications. Using this data and the data given in Tables 4.2.1, 4.2.2, 4.2.3, and 4.2.4, we formulate the fuzzy optimization model P(4.2.2) for COTS selection as follows:

$$\text{max } \theta$$

subject to

$$2((0.31x_{1,1} + 0.28x_{1,5} + 0.98x_{1,11} + 0.89x_{1,12} + 0.62x_{1,13} + 0.75x_{1,14} + 0.80x_{1,15} + 0.58x_{2,1} + 0.34x_{2,2} + 0.26x_{2,3} + 0.94x_{2,4} + 0.86x_{2,5} + 0.68x_{2,6} + 0.72x_{2,7} + 0.65x_{2,8} + 0.75x_{3,1} + 0.84x_{3,2} + 0.91x_{3,3} + 0.50x_{3,6} + 0.35x_{3,7} + 0.42x_{3,8} + 0.94x_{3,9} + 0.81x_{3,10} + 0.25x_{4,4} + 0.48x_{4,5} + 0.90x_{4,11} + 0.83x_{4,12} + 0.59x_{4,13} + 0.76x_{4,14} + 0.93x_{4,15} + 0.61x_{5,1} + 0.58x_{5,2} + 0.49x_{5,3} + 0.83x_{5,4} + 0.95x_{5,5} + 0.76x_{5,6} + 0.64x_{5,7} + 0.81x_{5,8} + 0.82x_{6,1} + 0.95x_{6,2} + 0.76x_{6,3} + 0.54x_{6,6} + 0.35x_{6,7} + 0.20x_{6,8} + 0.62x_{6,9} + 0.71x_{6,10}) - 8.5) \geq \theta,$$

$$60(535 - (48y_{1} + 66y_{2} + 54y_{3} + 55y_{4} + 62y_{5} + 61y_{6} + 74y_{7} + 74y_{8} + 65y_{9} + 68y_{10} + 47y_{11} + 42y_{12} + 49y_{13} + 55y_{14} + 52y_{15} + 17x_{1,4} + 20x_{1,5} + 14x_{1,11} + 14x_{1,12} + 16x_{1,13} + 18x_{1,14} + 15x_{1,15} + 20x_{2,1} + 20x_{2,2} + 19x_{2,3} + 15x_{2,4} + 18x_{2,5} + 19x_{2,6} + 12x_{2,7} + 11x_{2,8} + 18x_{3,1} + 17x_{3,2} + 15x_{3,3} + 21x_{3,4} + 20x_{3,7} + 21x_{3,8} + 20x_{3,9} + 19x_{3,10} + 16x_{4,4} + 16x_{4,5} + 11x_{4,11} + 13x_{4,12} + 14x_{4,13} + 15x_{4,14} + 18x_{4,15} + 15x_{5,1} + 16x_{5,2} + 15x_{5,3} + 14x_{5,4} + 12x_{5,5} + 11x_{5,6} + 9x_{5,7} + 8x_{5,8} + 10x_{6,1} + 13x_{6,2} + 15x_{6,3} + 10x_{6,6} + 9x_{6,7} + 7x_{6,8} + 12x_{6,9} + 14x_{6,10}) \geq \theta,$$

$$\theta \geq 0,$$

$$3 \sum_{j=1} x_{j,1}(2x_{j,4} + x_{j,5} + x_{j,6} + 3x_{j,7} + 5x_{j,8} + 8x_{j,9} + 9x_{j,10}) + 3 \sum_{j=1} x_{j,2}$$
\[
(3x_{j,5} + 6x_{j,6} + 4x_{j,7} + 2x_{j,8} + 8x_{j,10}) + \sum_{j=1}^{3} x_{j,3}(x_{j,4} + 2x_{j,5} + 3x_{j,6} + 4x_{j,7} + x_{j,8} + 6x_{j,10}) + \sum_{j=1}^{3} x_{j,4}(6x_{j,6} + 10x_{j,7} + 4x_{j,11} + 2x_{j,13} + 3x_{j,14} + x_{j,15}) + \sum_{j=1}^{3} x_{j,5}(8x_{j,6} + 7x_{j,7} + 8x_{j,8} + x_{j,11} + 3x_{j,12} + 2x_{j,14} + 4x_{j,15}) + \sum_{j=1}^{3} x_{j,6} (4x_{j,9} + 2x_{j,10}) + \sum_{j=1}^{3} x_{j,7}(x_{j,9} + x_{j,10}) + \sum_{j=1}^{3} x_{j,8}(2x_{j,9} + x_{j,10}) + \sum_{j=1}^{3} x_{j,11} (7x_{j,13} + 9x_{j,14}) + \sum_{j=1}^{3} x_{j,12}(5x_{j,13} + 6x_{j,14} + 8x_{j,15}) \bigg/ \left( \sum_{j=1}^{3} x_{j,1} \left(2 \sum_{j=1}^{3} x_{j,4} + \sum_{j=1}^{3} x_{j,5} + \sum_{j=1}^{3} x_{j,6} + 3 \sum_{j=1}^{3} x_{j,7} + 5 \sum_{j=1}^{3} x_{j,8} + 8 \sum_{j=1}^{3} x_{j,9} + 9x_{j,10} \right) \right) + \sum_{j=1}^{3} x_{j,2} \left(3 \sum_{j=1}^{3} x_{j,5} + 6 \sum_{j=1}^{3} x_{j,6} + 4 \sum_{j=1}^{3} x_{j,7} + 2 \sum_{j=1}^{3} x_{j,8} + 8 \sum_{j=1}^{3} x_{j,10} \right) + \sum_{j=1}^{3} x_{j,3} \left(3 \sum_{j=1}^{3} x_{j,4} + \sum_{j=1}^{3} x_{j,5} + 3 \sum_{j=1}^{3} x_{j,6} + 4 \sum_{j=1}^{3} x_{j,7} + x_{j,8} + 6 \sum_{j=1}^{3} x_{j,10} \right) + \sum_{j=1}^{3} x_{j,4} \left(6 \sum_{j=1}^{3} x_{j,6} + 10 \sum_{j=1}^{3} x_{j,7} + 8 \sum_{j=1}^{3} x_{j,11} + 3 \sum_{j=1}^{3} x_{j,13} + 3 \sum_{j=1}^{3} x_{j,14} + 3 \sum_{j=1}^{3} x_{j,15} \right) + \sum_{j=1}^{3} x_{j,5} \left(8 \sum_{j=1}^{3} x_{j,6} + 7 \sum_{j=1}^{3} x_{j,7} + 8 \sum_{j=1}^{3} x_{j,8} + \sum_{j=1}^{3} x_{j,11} + 3 \sum_{j=1}^{3} x_{j,12} + 2 \sum_{j=1}^{3} x_{j,14} + 4 \sum_{j=1}^{3} x_{j,15} \right) + \sum_{j=1}^{3} x_{j,6} \left(4 \sum_{j=1}^{3} x_{j,9} + 2 \sum_{j=1}^{3} x_{j,10} \right) + \sum_{j=1}^{3} x_{j,7} \left(3 \sum_{j=1}^{3} x_{j,9} + 3 \sum_{j=1}^{3} x_{j,10} \right) + \sum_{j=1}^{3} x_{j,8} \left(2 \sum_{j=1}^{3} x_{j,9} + 3 \sum_{j=1}^{3} x_{j,10} \right) + \sum_{j=1}^{3} x_{j,10} \left(3 \sum_{j=1}^{3} x_{j,11} + 7 \sum_{j=1}^{3} x_{j,13} + 9 \sum_{j=1}^{3} x_{j,14} \right) + \sum_{j=1}^{3} x_{j,11} \left(5 \sum_{j=1}^{3} x_{j,13} + 6 \sum_{j=1}^{3} x_{j,14} \right) + \sum_{j=1}^{3} x_{j,12} \left(3 \sum_{j=1}^{3} x_{j,13} + 6 \sum_{j=1}^{3} x_{j,14} \right) + \sum_{j=1}^{3} x_{j,15} \geq 0.3
\]
(4.2.1)
\[
\left( \sum_{j=1}^{3} x_{j,1}(2x_{j,4} + x_{j,5} + x_{j,6} + 3x_{j,7} + 5x_{j,8} + 8x_{j,9} + 9x_{j,10}) + \sum_{j=4}^{6} x_{j,2}(3x_{j,5} + \\
6x_{j,6} + 4x_{j,7} + 2x_{j,8} + 8x_{j,10}) + \sum_{j=4}^{6} x_{j,3}(x_{j,4} + 2x_{j,5} + 3x_{j,6} + 4x_{j,7} + x_{j,8} + \\
6x_{j,10}) + \sum_{j=4}^{6} x_{j,4}(6x_{j,6} + 10x_{j,7} + 4x_{j,11} + 2x_{j,13} + 3x_{j,14} + x_{j,15}) + \sum_{j=4}^{6} x_{j,5}
\right)
\]

\[
(8x_{j,6} + 7x_{j,7} + 8x_{j,8} + x_{j,11} + 3x_{j,12} + 2x_{j,14} + 4x_{j,15}) + \sum_{j=4}^{6} x_{j,6}(4x_{j,9} + \\
2x_{j,10}) + \sum_{j=4}^{6} x_{j,7}(x_{j,9} + x_{j,10}) + \sum_{j=4}^{6} x_{j,8}(2x_{j,9} + x_{j,10}) + \sum_{j=4}^{6} x_{j,11}(7x_{j,13} + 9x_{j,14})
\]

\[
+ \sum_{j=4}^{6} x_{j,12}(5x_{j,13} + 6x_{j,14} + 8x_{j,15}) \right) / \left( \sum_{j=4}^{6} x_{j,1}(2\sum_{j=4}^{6} x_{j,4} + \sum_{j=4}^{6} x_{j,5} + \sum_{j=4}^{6} x_{j,6}
\right)
\]

\[
+ 3\sum_{j=4}^{6} x_{j,7} + 5\sum_{j=4}^{6} x_{j,8} + 8\sum_{j=4}^{6} x_{j,9} + 9x_{j,10}) + \sum_{j=4}^{6} x_{j,2}(3\sum_{j=4}^{6} x_{j,5} + 6\sum_{j=4}^{6} x_{j,6}
\right)
\]

\[
+ 4\sum_{j=4}^{6} x_{j,7} + 2\sum_{j=4}^{6} x_{j,8} + 8\sum_{j=4}^{6} x_{j,9} + 9x_{j,10}) + \sum_{j=4}^{6} x_{j,3}(6\sum_{j=4}^{6} x_{j,4} + 2\sum_{j=4}^{6} x_{j,5} + 3\sum_{j=4}^{6} x_{j,6}
\right)
\]

\[
+ 4\sum_{j=4}^{6} x_{j,7} + 6\sum_{j=4}^{6} x_{j,10}) + \sum_{j=4}^{6} x_{j,4}(6\sum_{j=4}^{6} x_{j,6} + 10\sum_{j=4}^{6} x_{j,7} + 4\sum_{j=4}^{6} x_{j,11}
\right)
\]

\[
+ 2\sum_{j=4}^{6} x_{j,13} + 3\sum_{j=4}^{6} x_{j,14} + \sum_{j=4}^{6} x_{j,15}) + \sum_{j=4}^{6} x_{j,5}(8\sum_{j=4}^{6} x_{j,6} + 7\sum_{j=4}^{6} x_{j,7} + \\
8\sum_{j=4}^{6} x_{j,8} + \sum_{j=4}^{6} x_{j,9} + 3\sum_{j=4}^{6} x_{j,11} + 3\sum_{j=4}^{6} x_{j,12} + 2\sum_{j=4}^{6} x_{j,14} + 4\sum_{j=4}^{6} x_{j,15}) + \sum_{j=4}^{6} x_{j,6}(4\sum_{j=4}^{6} x_{j,9}
\right)
\]

\[
+ 2\sum_{j=4}^{6} x_{j,10}) + \sum_{j=4}^{6} x_{j,7}(\sum_{j=4}^{6} x_{j,9} + \sum_{j=4}^{6} x_{j,10}) + \sum_{j=4}^{6} x_{j,8}(2\sum_{j=4}^{6} x_{j,9} + \sum_{j=4}^{6} x_{j,10}
\right)
\]

\[
+ \sum_{j=4}^{6} x_{j,11}(7\sum_{j=4}^{6} x_{j,13} + 9\sum_{j=4}^{6} x_{j,14}) + \sum_{j=4}^{6} x_{j,12}(5\sum_{j=4}^{6} x_{j,13} + 6\sum_{j=4}^{6} x_{j,14} + \\
8\sum_{j=4}^{6} x_{j,15}) \geq 0.3,
\]

\[(4.2.2)\]
\[ \sum_{j=1}^{3} x_{j,k} \leq 1, \ k = 1, 2, \ldots, 15, \quad \sum_{j=4}^{6} x_{j,k} \leq 1, \ k = 1, 2, \ldots, 15, \]  
\[ \sum_{j=1}^{6} x_{j,k} \leq 2y_k, \ k = 1, 2, \ldots, 15, \]  
\[ x_{1,1} \leq 0, \ x_{1,2} \leq 0, \ x_{1,3} \leq 0, \ x_{1,4} \leq 1, \ x_{1,5} \leq 1, \ x_{1,6} \leq 0, \ x_{1,7} \leq 0, \ x_{1,8} \leq 0, \]  
\[ x_{1,9} \leq 0, \ x_{1,10} \leq 0, \ x_{1,11} \leq 1, \ x_{1,12} \leq 1, \ x_{1,13} \leq 1, \ x_{1,14} \leq 1, \ x_{1,15} \leq 1, \]  
\[ x_{2,1} \leq 1, \ x_{2,2} \leq 1, \ x_{2,3} \leq 1, \ x_{2,4} \leq 1, \ x_{2,5} \leq 1, \ x_{2,6} \leq 1, \ x_{2,7} \leq 1, \ x_{2,8} \leq 1, \]  
\[ x_{2,9} \leq 0, \ x_{2,10} \leq 0, \ x_{2,11} \leq 0, \ x_{2,12} \leq 0, \ x_{2,13} \leq 0, \ x_{2,14} \leq 0, \ x_{2,15} \leq 0, \ x_{3,1} \leq 1, \]  
\[ x_{3,2} \leq 1, \ x_{3,3} \leq 1, \ x_{3,4} \leq 0, \ x_{3,5} \leq 0, \ x_{3,6} \leq 1, \ x_{3,7} \leq 1, \ x_{3,8} \leq 1, \ x_{3,9} \leq 1, \ x_{3,10} \leq 1, \ x_{3,11} \leq 0, \ x_{3,12} \leq 0, \ x_{3,13} \leq 0, \ x_{3,14} \leq 0, \ x_{3,15} \leq 0, \ x_{4,1} \leq 0, \ x_{4,2} \leq 0, \]  
\[ x_{4,3} \leq 0, \ x_{4,4} \leq 1, \ x_{4,5} \leq 1, \ x_{4,6} \leq 0, \ x_{4,7} \leq 0, \ x_{4,8} \leq 0, \ x_{4,9} \leq 0, \ x_{4,10} \leq 0, \]  
\[ x_{4,11} \leq 1, \ x_{4,12} \leq 1, \ x_{4,13} \leq 1, \ x_{4,14} \leq 1, \ x_{4,15} \leq 1, \ x_{5,1} \leq 1, \ x_{5,2} \leq 1, \ x_{5,3} \leq 1, \]  
\[ x_{5,4} \leq 1, \ x_{5,5} \leq 1, \ x_{5,6} \leq 1, \ x_{5,7} \leq 1, \ x_{5,8} \leq 1, \ x_{5,9} \leq 0, \ x_{5,10} \leq 0, \ x_{5,11} \leq 0, \ x_{5,12} \leq 0, \ x_{5,13} \leq 0, \ x_{5,14} \leq 0, \ x_{5,15} \leq 0, \ x_{6,1} \leq 1, \ x_{6,2} \leq 1, \ x_{6,3} \leq 1, \ x_{6,4} \leq 0, \]  
\[ x_{6,5} \leq 0, \ x_{6,6} \leq 1, \ x_{6,7} \leq 1, \ x_{6,8} \leq 1, \ x_{6,9} \leq 1, \ x_{6,10} \leq 1, \ x_{6,11} \leq 0, \ x_{6,12} \leq 0, \ x_{6,13} \leq 0, \ x_{6,14} \leq 0, \ x_{6,15} \leq 0, \]  
\[ \sum_{j=1}^{3} (x_{j,1} + x_{j,2} + x_{j,3}) = 1, \quad \sum_{j=1}^{3} (x_{j,4} + x_{j,5}) = 1, \quad \sum_{j=1}^{3} (x_{j,6} + x_{j,7} + x_{j,8}) = 1, \]  
\[ \sum_{j=1}^{3} (x_{j,9} + x_{j,10}) = 1, \quad \sum_{j=1}^{3} (x_{j,11} + x_{j,12}) = 1, \quad \sum_{j=1}^{3} (x_{j,13} + x_{j,14} + x_{j,15}) = 1, \]  
\[ \sum_{j=4}^{6} (x_{j,1} + x_{j,2} + x_{j,3}) = 1, \quad \sum_{j=4}^{6} (x_{j,4} + x_{j,5}) = 1, \quad \sum_{j=4}^{6} (x_{j,6} + x_{j,7} + x_{j,8}) = 1, \]  
\[ \sum_{j=4}^{6} (x_{j,9} + x_{j,10}) = 1, \quad \sum_{j=4}^{6} (x_{j,11} + x_{j,12}) = 1, \quad \sum_{j=4}^{6} (x_{j,13} + x_{j,14} + x_{j,15}) = 1, \]  
\[ \sum_{k=1}^{15} x_{j,k} \geq 1, \ j = 1, 2, \ldots, 6, \]  
\[ y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12}, y_{13}, y_{14}, y_{15}, x_{1,1}, x_{1,2}, x_{1,3}, \]  
\[ x_{1,4}, x_{1,5}, x_{1,6}, x_{1,7}, x_{1,8}, x_{1,9}, x_{1,10}, x_{1,11}, x_{1,12}, x_{1,13}, x_{1,14}, x_{1,15}, x_{2,1}, x_{2,2}, \]  
\[ x_{2,3}, x_{2,4}, x_{2,5}, x_{2,6}, x_{2,7}, x_{2,8}, x_{2,9}, x_{2,10}, x_{2,11}, x_{2,12}, x_{2,13}, x_{2,14}, x_{2,15}, x_{3,1}, \]
By solving the above optimization model using LINGO, we obtain the optimal values of $F(x)$ and $C(x)$ as 10.21 and 533, respectively. The COTS components $sc_{11}$ and $sc_{15}$ which are selected for modules $M1$ and $M4$ provides the functional requirements of e-Commerce and Financial Reporting as desired. Modules $M2$ and $M5$ selects COTS components $sc_5$ and $sc_6$ which fulfils the functional requirement of Encryption and Authorization. Modules $M3$ and $M6$ selects COTS components $sc_2$ and $sc_9$ which contribute towards the functional requirements of Fax and Software Automatic Updating. Suppose the DM is not satisfied with the COTS selection then more COTS selection strategies can be generated by varying the minimum threshold value of $ICD$ for each application and also by varying values of shape parameters $\alpha_F$ and $\alpha_C$ in model P(4.2.2) (see Table 4.2.5). Furthermore, in order to check the efficiency of the obtained solutions, we apply the two-phase approach and solve the corresponding model P(4.2.4). We find that the solutions obtained are efficient; that is, their criteria vector are non-dominated. Thus, if the DM desires a software system with high functional performance then second and fourth solutions are preferred and if a software system with minimum cost is desired then first and third solutions are preferred.

**COTS selection using weighted optimization model P(4.2.3)**

By varying the weights of the two objectives functions, we can also obtain different solutions as per the preferences of the DM. Here, we consider two cases.

**Case 1.** The DM gives more importance to cost as compared to functional performance of the software system by considering $\omega_1 = 0.1$ and $\omega_2 = 0.9$. We set $H_1 = 0.3$, $H_2 = 0.3$, $\alpha_F = 2$, $\alpha_C = 60$, $F_m = 8.3$, $C_m = 529$ and formulate the fuzzy optimization model for COTS selection as follows:
Table 4.2.5: COTS selection corresponding to model P(4.2.2)

<table>
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<th>$\alpha_F$</th>
<th>$\alpha_C$</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$F_m$</th>
<th>$C_m$</th>
<th>$F(x)$</th>
<th>$C(x)$</th>
<th>COTS components selected</th>
</tr>
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<td>0.0</td>
<td>0.0</td>
<td>0.977687</td>
<td>1</td>
<td>8.3</td>
<td>529</td>
<td>10.19</td>
<td>527</td>
<td>$sc_{11}, sc_{15}$  $sc_4, sc_8$ $sc_2, sc_9$ $sc_{11}, sc_{15}$ $sc_4, sc_8$ $sc_2, sc_9$</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>0.3</td>
<td>0.3</td>
<td>0.968324</td>
<td>1</td>
<td>8.5</td>
<td>535</td>
<td>10.21</td>
<td>533</td>
<td>$sc_{11}, sc_{15}$  $sc_5, sc_6$ $sc_2, sc_9$ $sc_{11}, sc_{15}$ $sc_5, sc_6$ $sc_2, sc_9$</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>1</td>
<td>1</td>
<td>8.3</td>
<td>529</td>
<td>10.07</td>
<td>515</td>
<td>$sc_{11}, sc_{15}$  $sc_4, sc_8$ $sc_3, sc_9$ $sc_{11}, sc_{15}$ $sc_4, sc_8$ $sc_3, sc_9$</td>
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<td>0.3</td>
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<td>1</td>
<td>8.5</td>
<td>535</td>
<td>10.11</td>
<td>523</td>
<td>$sc_{11}, sc_{15}$  $sc_5, sc_6$ $sc_3, sc_9$ $sc_{11}, sc_{15}$ $sc_5, sc_6$ $sc_3, sc_9$</td>
</tr>
</tbody>
</table>
\[
\text{max } \left( 0.1\lambda_1 + 0.9\lambda_2 \right) \\
\text{subject to} \\
\frac{1}{(1 + \exp(-2((0.31x_{1,4} + 0.28x_{1,5} + 0.98x_{1,11} + 0.89x_{1,12} + 0.62x_{1,13} + 0.75x_{1,14} + 0.80x_{1,15} + 0.58x_{2,1} + 0.34x_{2,2} + 0.26x_{2,3} + 0.94x_{2,4} + 0.86x_{2,5} + 0.68x_{2,6} + 0.72x_{2,7} + 0.65x_{2,8} + 0.75x_{3,1} + 0.84x_{3,2} + 0.91x_{3,3} + 0.50x_{3,6} + 0.35x_{4,7} + 0.42x_{3,8} + 0.94x_{3,9} + 0.81x_{3,10} + 0.25x_{4,4} + 0.48x_{4,5} + 0.90x_{4,11} + 0.83x_{4,12} + 0.59x_{4,13} + 0.76x_{4,14} + 0.93x_{4,15} + 0.61x_{5,1} + 0.58x_{5,2} + 0.49x_{5,3} + 0.83x_{5,4} + 0.95x_{5,5} + 0.76x_{5,6} + 0.64x_{5,7} + 0.81x_{5,8} + 0.82x_{6,1} + 0.95x_{6,2} + 0.76x_{6,3} + 0.54x_{6,6} + 0.35x_{6,7} + 0.20x_{6,8} + 0.62x_{6,9} + 0.71x_{6,10} - 8.3)))} \geq \lambda_1,
\]
\[
\frac{1}{(1 + \exp(60((48y_1 + 66y_2 + 54y_3 + 55y_4 + 62y_5 + 61y_6 + 74y_7 + 74y_8 + 65y_9 + 68y_{10} + 47y_{11} + 42y_{12} + 49y_{13} + 55y_{14} + 52y_{15} + 17x_{1,4} + 20x_{1,5} + 14x_{1,11} + 14x_{1,12} + 16x_{1,13} + 18x_{1,14} + 15x_{1,15} + 20x_{2,1} + 20x_{2,2} + 19x_{2,3} + 15x_{2,4} + 18x_{2,5} + 19x_{2,6} + 12x_{2,7} + 11x_{2,8} + 18x_{3,1} + 17x_{3,2} + 15x_{3,3} + 21x_{3,6} + 20x_{3,7} + 21x_{3,8} + 20x_{3,9} + 19x_{3,10} + 16x_{4,4} + 16x_{4,5} + 11x_{4,11} + 13x_{4,12} + 14x_{4,13} + 15x_{4,14} + 18x_{4,15} + 15x_{5,1} + 16x_{5,2} + 15x_{5,3} + 14x_{5,4} + 12x_{5,5} + 11x_{5,6} + 9x_{5,7} + 8x_{5,8} + 10x_{6,1} + 13x_{6,2} + 15x_{6,3} + 10x_{6,6} + 9x_{6,7} + 7x_{6,8} + 12x_{6,9} + 14x_{6,10}) - 529)))} \geq \lambda_2,
\]
\[
0 \leq \lambda_1 \leq 1, \\
0 \leq \lambda_2 \leq 1,
\]
and Constraints (4.2.1)–(4.2.8).

By solving the above optimization model using LINGO, the optimal values of \( F(x) \) and \( C(x) \) are obtained as 9.71 and 529, respectively. The COTS components \( sc_{11} \) and \( sc_{13} \) which are selected for modules \( M1 \) and \( M4 \) provide the functional requirements of e-Commerce and Financial Reporting as desired. COTS components \( sc_{5} \) and \( sc_{8} \) are selected for modules \( M2 \) and \( M5 \) which
fulfil the functional requirements of Encryption and Authorization. Modules $M_3$ and $M_6$ get COTS components $sc_2$ and $sc_9$ which contribute toward the functional requirements of Fax and Software Automatic Updating. The efficiency of the obtained solution is verified by solving model $P(4.2.5)$ in the second phase. The achievement levels of the two membership functions are: $\lambda_1 = 0.918339$, $\lambda_2 = 1$. Note that these achievement levels are consistent with the DM’s preferences $\omega_1 = 0.1$ and $\omega_2 = 0.9$. In other words, $(\lambda_1 < \lambda_2)$ agrees with $(\omega_1 < \omega_2)$. The corresponding computational results are summarized in Table 4.2.6.

**Case 2:** The DM gives more importance to functional performance as compared to cost of the system by considering $\omega_1 = 0.9$ and $\omega_2 = 0.1$. We set $H_1 = 0.3$, $H_2 = 0.3$, $\alpha_F = 2$, $\alpha_C = 60$, $F_m = 8.3$, $C_m = 529$ and solve the optimization model $P(4.2.3)$ using LINGO. The corresponding computational results are summarized in Table 4.2.6. The optimal values of $F(x)$ and $C(x)$ are obtained as 10.15 and 531, respectively. The COTS components $sc_{11}$ and $sc_{15}$ which are selected for modules $M_1$ and $M_4$ provide the functional requirements of e-Commerce and Financial Reporting as desired. Modules $M_2$ and $M_5$ get COTS components $sc_4$ and $sc_8$ which fulfil the functional requirements of Encryption and Authorization. Modules $M_3$ and $M_6$ get COTS components $sc_2$ and $sc_{10}$ which contribute toward the functional requirements of Fax and Software Automatic Updating. The efficiency of the solution is verified by solving model $P(4.2.5)$ in the second phase. The achievement levels of the membership functions are consistent with the DM’s preferences, that is, $(\lambda_1 > \lambda_2)$ agrees with $(\omega_1 > \omega_2)$. It is important to point out that for some choices of the membership functions there may be no solution to the COTS selection model $P(4.2.2)/P(4.2.3)$. In such instances, we need to modify the aspiration levels for the various scenarios to find a satisfactory solution.
<table>
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<tr>
<th>$\omega_1$</th>
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<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$F(x)$</th>
<th>$C(x)$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$M_4$</th>
<th>$M_5$</th>
<th>$M_6$</th>
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<td>SC5, SC8</td>
<td>SC7, SC9</td>
<td>SC15, SC16, SC17, SC19, SC20</td>
<td>SC4, SC6, SC7, SC8, SC9, SC10, SC12, SC14, SC18</td>
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<td>0.1</td>
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<td>531</td>
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<td>SC5, SC8</td>
<td>SC11, SC13</td>
<td>SC5, SC8</td>
<td>SC2, SC9</td>
<td>SC14, SC6, SC7, SC8, SC9, SC10</td>
</tr>
</tbody>
</table>

Table 4.2.6: COTS selection corresponding to $F_m = 8.3, C_m = 529, H_1 = 0.3, H_2 = 0.3, \alpha_F = 3, \alpha_C = 60$. 

COTS components selected: $M_1 \rightarrow \{SC2, SC3, SC5, SC8\}, M_2 \rightarrow \{SC7, SC9\}, M_3 \rightarrow \{SC15, SC16, SC17, SC19, SC20\}, M_4 \rightarrow \{SC4, SC6, SC7, SC8, SC9\}, M_5 \rightarrow \{SC1, SC3\}, M_6 \rightarrow \{SC11, SC13\}$. 


4.2.3 Concluding remarks

We developed fuzzy biobjective optimization models which maximize functional requirements and minimize cost of the modular software system. The proposed methodology involves subjective judgments from the DM, such as the determination of the scores of interaction and the functional ratings. Fuzzy set theory is used here as a tool to deal with the fuzziness caused by subjective judgments. The main advantage of the proposed models is that if the DM is not satisfied with any of the COTS selection obtained, more COTS selections can be generated by varying values of shape parameters of nonlinear S-shape membership functions. The effectiveness of the models is demonstrated through numerical examples constructed corresponding to a real-world scenario of multiple applications environment. The models’ sensitivity have been shown with respect to changes in the minimum threshold value of the \( ICD \) of each application and also by varying the weight parameters of the two objective functions, reflecting relative importance between them.