CHAPTER - 4
OTRA BASED FILTERS

The content and results of the following papers have been reported in this chapter.


4.1 INTRODUCTION

Electronic filter is an essential building block of communication and instrumentation systems. It is a linear two port frequency selective network which allows signals in a specified frequency range to be passed (the filter passband), while rejecting frequencies outside this range (the filter stop band). Ideally in pass band the magnitude of signal transmission is unity whereas in stop band it is zero. Filters are classified according to the function they perform in terms of frequency range as pass band and stop band. A filter for which passband extends from zero to a definite frequency, known as cutoff frequency, is a low pass filter. A high pass filter stops the frequency range from zero to cutoff frequency and allows rest to pass. In band pass filter a continuum of signal frequencies is allowed to pass while all other frequencies are stopped. The band reject filter is complement of the band pass filter where a continuum frequency band is rejected allowing rest of the frequencies to pass. These filter structures can be classified as voltage input voltage output (voltage mode-VM), current input current output (Current mode-CM), voltage input current output (Transadmittance mode-TAM), and current input voltage output (Transimpedance mode-TIM) type as per the choice between voltage and current as two possible input and output signals. A further classification may be based upon the filter order; number of inputs and outputs available, cascadability and number of active building blocks used. This classification can be used as an assessment criterion to select a particular structure for a specific application.

This chapter concerns with the realization of OTRA based biquadratic and higher order filters. The work has been classified as single and multiamplifier filters. For the applications having power consumption as an important design constraint, single amplifier based biquadratic (SAB) filter is a useful choice. However SABs are less versatile and more sensitive to parameter changes when compared to multiamplifier filters. Multiamplifier filter is a preferred option for designing high quality factor ($Q_0$) filters. This leads to the development of OTRA based multiamplifier configurations which are further categorised as second order and higher-order filters in the work presented.
In the following section a brief record of earlier work dealing with the filter design using OTRA is presented.

4.2 REVIEW OF EXISTING LITERATURE

In synchronization with the theme of this chapter the filter structures reported in the literature are categorised in terms of single and multi-amplifier filters in voltage/current/transimpedance mode.

VM filters presented in [44], [47], [51] - [53] are single amplifier filters and are further classified as first order [51] - [53] or biquadratic [44], [47], [52], [53] filters. Few first order all pass topologies are presented in [51] - [53] whereas in [52], [53] second order notch and all pass filter structures are also presented. A biquad single input single output (SISO) configuration is presented in [44] which can synthesize LP, HP, BP, BR and AP responses by appropriate component selection. The SAB presented in [47] is a multiple input single output (MISO) structure which provides all five standard filter responses; namely LP, HP, BP, BR and AP, by lifting the nodes for input excitation.

The structures proposed in [36], [42], [45], [46], [48] - [50] are VM multi-amplifier biquadratic filters. The filters presented in [36], [46] are single input multiple output (SIMO) structures. OTRA based Tow Thomas and KHN biquad structures are presented in [36] whereas [46] presents a different approach of realizing three standard filter functions available in KHN biquad. In [42] two different MISO universal filter configurations are presented. The biquad of [45] is a SISO structure which provides all five standard filter responses and is based on Fleischer-Tow scheme whereas in [48] a VM multiple input multiple output (MIMO) structure providing LP, HP, BP, and BR responses is reported.

The filter structures presented in [42], [49], [50] are VM higher order multi-amplifier filters. A third order Chebyshev LPF is presented in [42] and the design approach for higher order filters using linear transformation method is outlined in [49], [50].

Current mode single amplifier based first order all pass structure is reported in [55]. The structures of [54], [56] are multi-amplifier filters. Two different universal biquad
configurations are available in [54] which are MISO structures whereas in [56] linear transformation based CM higher order filter design method is presented.

TIM filters have not gained much attention as revealed by the reported work. TIM type single amplifier based first order all pass filter is available in [57] whereas [58] presents multiamplifier MISO universal biquad providing all five standard responses.

4.3 SINGLE OTRA BASED BIQUADRATIC FILTERS

Several OTRA based VM SAB filters [44], [47], [52], [53] have been proposed in literature. A detailed study of these structures shows that

- Component selection is required for obtaining various filter responses in [44], [47], [52], [53]
- Component matching and/or condition on components need to be satisfied for filter realization in [44], [47], [52], [53]
- Independent adjustment of angular frequency and quality factor is not possible in [44], [47], [52]
- Large component spread is required for obtaining high $Q_0$ value in [44], [47], [52], [53]
- Filter responses are obtained by lifting of nodes for input excitation in [47]

Thus all SAB filters not only require component selection for realization of various responses but also need to fulfill the component matching constraint and/or condition on components. Further these structures do not provide high $Q_0$ value with moderate component spread. Therefore in this work, two SAB filter configurations are presented which impose neither any condition on components nor require component matching constraints.

4.3.1 SAB Topology-I

The proposed SAB topology-I is shown in Fig. 4.1 and is based on Sallen Key approach. The transfer function of the circuit of Fig. 4.1 can be expressed as
\[ \frac{v_o}{v_{in}} = \frac{KY_3Y_4}{D} \]  \hspace{1cm} (4.1)

Where \( D \) and \( K \) are respectively given as

\[ D = Y_3(Y_2 + Y_4 + Y_6) + Y_4(Y_2 + Y_5 + Y_6) + Y_2Y_5 + Y_5Y_6 - KY_4Y_5 \]  \hspace{1cm} (4.2)

\[ K = \frac{Y_2}{Y_1} \]  \hspace{1cm} (4.3)

It realizes the LP, HP and BP filter functions by appropriate admittance selection. The appropriate admittance choices, which result in three required filter responses, are listed in Table 4.1. It may be noted that admittances \( Y_1 \) and \( Y_2 \) remain \( G_1 \) and \( G_2 \) respectively for all the responses so the value of \( K \) will be given as ratio of \( G_2 \) and \( G_1 \). Using the admittance choices given in Table 4.1 the LP response can be expressed as

\[ \frac{V_o}{V_{in,LP}} = \frac{KG_3 G_4/C_1 C_2}{s^2 + s \left( \frac{C_2(G_3+G_4) + C_1 G_2 + C_1 G_4(1-K)}{C_1 C_2} \right) + \left( \frac{G_3 G_4 + G_2 G_4 + G_2 G_3}{C_1 C_2} \right)} \]  \hspace{1cm} (4.4)

![Proposed SAB topology-I.](image)

Fig. 4.1 Proposed SAB topology-I.

From (4.4) the resonant angular frequency \( (\omega_0) \), the quality factor \( (Q_0) \) and filter Gain \( (H_0) \) for LP response can be obtained as

\[ \omega_0|_{LP} = \sqrt{\frac{G_3 G_4 + G_2 G_4 + G_2 G_3}{C_1 C_2}} \]  \hspace{1cm} (4.5)
\[ Q_0 \big|_{LP} = \frac{\sqrt{(G_3G_4+G_2G_4+G_2G_3)C_1C_2}}{C_2(G_3+G_4)+C_1G_2+C_1G_4(1-K)} \]  

(4.6)

\[ H_0 \big|_{LP} = \frac{KG_3G_4}{(G_3G_4+G_2G_4+G_2G_3)} \]  

(4.7)

Table 4.1: Admittance Selection for SAB topology-I.

<table>
<thead>
<tr>
<th>Filter Function</th>
<th>(Y_1)</th>
<th>(Y_2)</th>
<th>(Y_3)</th>
<th>(Y_4)</th>
<th>(Y_5)</th>
<th>(Y_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LowPass</td>
<td>(G_1)</td>
<td>(G_2)</td>
<td>(G_3)</td>
<td>(G_4)</td>
<td>(sC_1)</td>
<td>(sC_2)</td>
</tr>
<tr>
<td>HighPass</td>
<td>(G_1)</td>
<td>(G_2)</td>
<td>(sC_1)</td>
<td>(sC_2)</td>
<td>(G_3)</td>
<td>(G_4)</td>
</tr>
<tr>
<td>Band Pass</td>
<td>(G_1)</td>
<td>(G_2)</td>
<td>(G_3)</td>
<td>(sC_1)</td>
<td>(G_4)</td>
<td>(sC_2)</td>
</tr>
</tbody>
</table>

For appropriate admittance choices the HP response can be obtained as below

\[ \frac{v_0}{v_{in}} \big|_{HP} = \frac{Ks^2}{s^2 + s\left(\frac{C_1(G_2+G_4)+C_2(G_2+G_4)+G_3C_2(1-K)}{C_1C_2}\right) + \left(\frac{G_2G_3+G_3G_4}{C_1C_2}\right)} \]  

(4.8)

The filter parameters \(\omega_0\), \(Q_0\) and \(H_0\) for HP response can be expressed as

\[ \omega_0 \big|_{HP} = \sqrt{\frac{G_3(G_2+G_4)}{C_1C_2}} \]  

(4.9)

\[ Q_0 \big|_{HP} = \frac{\sqrt{(G_2G_3+G_3G_4)C_1C_2}}{C_1(G_2+G_4)+C_2(G_2+G_4)+G_3C_2(1-K)} \]  

(4.10)

\[ H_0 \big|_{HP} = K \]  

(4.11)

The BP response with proper admittance choices, enlisted in Table 4.1, is given by
\[
\frac{V_{\omega}}{V_{in}}_{BP} = \frac{K G_2}{C_2} \frac{G_3}{C_1} (4.12)
\]

From (4.12) the values of \(\omega_0\), \(Q_0\) and \(H_0\) for the BP response can be computed as

\[
\omega_0 \bigg|_{BP} = \sqrt{\frac{G_2 (G_3 + G_4)}{C_1 C_2}} \quad (4.13)
\]

\[
Q_0 \bigg|_{BP} = \sqrt{\frac{C_1 C_2 G_2 (G_3 + G_4)}{C_1 (G_2 + G_3) + C_2 (G_3 + G_4) + C_1 G_4 (1 - K)}} \quad (4.14)
\]

\[
H_0 \bigg|_{BP} = \frac{K C_1 G_3}{C_1 (G_2 + G_3) + C_2 (G_3 + G_4) + C_1 G_4 (1 - K)} \quad (4.15)
\]

Table 4.2 lists the \(H_0\), \(\omega_0\) and \(Q_0\) for all the three responses when all the conductances except \(G_1\), the \(K\) determining component, are set equal to \(G\) and all capacitors are set equal to \(C\). This suggests that the \(H_0\) and \(Q_0\), for all the responses, can be controlled by varying \(K\) without affecting the \(\omega_0\) and by adjusting the capacitance value \(\omega_0\) can be tuned independently. With MOS transistor implementation of \(G_1\) as outlined in chapter 2, the filter parameters \(Q_0\) and \(H_0\) can be electronically tuned. Filter circuit of Fig. 4.1 can be redrawn as Fig. 4.2 wherein \(G_1\) is implemented using MOS transistors \(M_a\) and \(M_b\).

### 4.3.1.1 Sensitivity Analysis

The passive sensitivities of \(\omega_0\) and \(Q_0\) for the LPF configuration can be expressed as

\[
S_{G_1}^{\omega_0} = S_{G_2}^{\omega_0} = -\frac{1}{2} , \quad S_{G_1}^{\omega_0} = 0 , \quad S_{G_2}^{\omega_0} = \frac{1}{2} - \frac{G_3 G_4}{2 (G_3 G_4 + G_2 G_4 + G_2 G_3)},
\]

\[
S_{G_3}^{\omega_0} = \frac{1}{2} - \frac{G_2 G_4}{2 (G_3 G_4 + G_2 G_4 + G_2 G_3)}, \quad S_{G_4}^{\omega_0} = \frac{1}{2} - \frac{G_2 G_3}{2 (G_3 G_4 + G_2 G_4 + G_2 G_3)} \quad (4.16)
\]
\[ S_{C_1}^{\omega_0} = \frac{1}{2} \frac{C_1G_2+C_1G_4(1-K)}{C_2(G_3+G_4)+C_1G_2+C_1G_4(1-K)} \quad S_{C_2}^{\omega_0} = \frac{1}{2} \frac{C_2(G_3+G_4)}{C_2(G_3+G_4)+C_1G_2+C_1G_4(1-K)} \] (4.17)

Table 4.2: Filter parameters for equal component design.

<table>
<thead>
<tr>
<th>Filter Function</th>
<th>Filter Gain ((H_0))</th>
<th>Angular Frequency ((\omega_0))</th>
<th>Quality Factor ((Q_0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Pass</td>
<td>(K/3)</td>
<td>(\sqrt{3}G/C)</td>
<td>(\sqrt{3}/(4-K))</td>
</tr>
<tr>
<td>High Pass</td>
<td>(K)</td>
<td>(\sqrt{2}G/C)</td>
<td>(\sqrt{2}/(5-K))</td>
</tr>
<tr>
<td>Band Pass</td>
<td>(K/(5-K))</td>
<td>(\sqrt{2}G/C)</td>
<td>(\sqrt{2}/(5-K))</td>
</tr>
</tbody>
</table>

Fig. 4.2 Electronically tunable SAB Topology-I.

The passive sensitivities of \(\omega_0\) and \(Q_0\) for the HPF configuration can be computed as

\[ S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}, \quad S_{G_2}^{\omega_0} = \frac{1}{2} - \frac{G_4}{2(G_4+G_2)}, \quad S_{G_1}^{\omega_0} = 0, \quad S_{G_3}^{\omega_0} = \frac{1}{2}, \quad S_{G_4}^{\omega_0} = \frac{1}{2} - \frac{G_2}{2(G_4+G_2)} \] (4.18)
The passive sensitivities of \( \omega_0 \) and \( Q_0 \) for the BPF configuration can be calculated as

\[
S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}, \quad S_{G_2}^{\omega_0} = 0, \quad S_{G_3}^{\omega_0} = \frac{1}{2}, \quad S_{G_4}^{\omega_0} = \frac{1}{2} - \frac{G_3}{2(G_3+G_4)}, \quad S_{G_4}^{Q_0} = \frac{1}{2} - \frac{G_3}{2(G_3+G_4)} \quad (4.20)
\]

\[
S_{C_1}^{Q_0} = \frac{1}{2} - \frac{C_1(G_2+G_3)+C_1G_4(1-K)}{C_1(G_2+G_4)+C_2(G_2+G_4)+C_2G_3(1-K)},
\]

(4.21)

It is clearly observed from (4.16) and (4.21) that the passive sensitivities for all the filter responses are lower than 1/2 in magnitude so the filter configurations may be termed as insensitive.

### 4.3.1.2 Nonideality Analysis

The response of the filter may deviate due to nonideality of OTRA in practice. Taking the nonideality effect as discussed in section 2.3, into account (4.4), (4.8) and (4.12) modify to (4.22), (4.23) and (4.24) respectively

\[
\left. \frac{V_\theta}{V_{in}} \right|_{LP,n} = \frac{K}{1+sC_p/G_1} \frac{G_3G_4}{C_1C_2} S^2 + S \left( \frac{C_2(G_3+G_4)+C_1G_2+C_2G_3}{C_1C_2} \frac{1-\frac{K}{1+sC_p/G_1}}{C_1C_2} \right) + \left( \frac{G_2G_4+G_2G_4+G_2G_3}{C_1C_2} \right)
\]

(4.22)

\[
\left. \frac{V_\theta}{V_{in}} \right|_{HP,n} = \frac{K}{1+sC_p/G_1} \frac{G_2G_4}{C_1C_2} S^2 + S \left( \frac{C_1(G_2+G_4)+C_2G_2+C_2G_3}{C_1C_2} \frac{1-\frac{K}{1+sC_p/G_1}}{C_1C_2} \right) + \left( \frac{G_2G_3+G_2G_4}{C_1C_2} \right)
\]

(4.23)
\[
\frac{V_\theta}{V_{in}}_{BP,n} = \left. \frac{KsG_1}{(1+sC_p/G_1)C_2} \right\} \frac{C_1(G_2+G_3)+C_2(G_3+G_4)+C_1G_4\left(1-\frac{K}{(1+sC_p/G_1)}\right)}{C_1C_2} + \frac{G_2(G_3+G_4)}{C_1C_2}
\]

(4.24)

The \(sC_p\) term appearing in parallel to \(G_1\) will result in introduction of parasitic pole having radian frequency as \(\omega = G_1/C_p\). The pole frequency for the biquad is typically expressed as \(G_i/C_i\). The effect of the parasitic pole can be ignored by selecting \(C_i >> C_p\), so that the pole frequency of the biquad is much lower than the parasitic pole frequency.

### 4.3.1.3 Simulation Results

The workability of the proposed SAB is verified through SPICE simulation using OTRA CMOS schematic of Fig. 2.9. The LPF and HPF have been designed for \(f_0 = 500\) KHz for which capacitance values are chosen as \(C_1 = C_2 = 100\) pF. Various conductances to obtain the LP and HP responses are computed as

LPF: \(G_1 = 0.05\) mS, \(G_2 = 0.1\) mS, \(G_3 = G_4 = 0.2\) mS

HPF: \(G_1 = G_2 = 0.1\) mS, \(G_3 = 0.2\) mS, \(G_4 = 0.1\) mS.

The BP function is demonstrated through a design having \(f_0 = 750\) KHz and \(Q_0 = 0.56\). The capacitors are chosen as \(C_1 = C_2 = 30\) pF and conductance values are computed to be \(G_1 = 0.04\) mS, \(G_2 = G_3 = G_4 = 0.1\) mS. The ideal and simulated LP, HP and BP responses are shown in Fig. 4.3(a), (b) and (c) respectively. It can be observed that there is a close agreement between the theoretical and simulated responses except that low pass transmission deviates from ideal response at frequencies well above cutoff and starts increasing at a rate of 20dB/dec. This behaviour can be explained using high frequency model of LPF shown in Fig. 4.4 which is based on the assumption that \(C_1\) and \(C_2\) become effectively short at high frequencies as compared to \(G_3\) and \(G_4\). The filter transfer function can now be expressed as

\[
\frac{v_0}{v_{in}} = \frac{1}{G_4 + \frac{1}{G_3G_0+1}}
\]

(4.25)
Fig. 4.3 Frequency response of proposed SAB topology - (a) LP, (b) HP, (c) BP.
Assuming \( Z_0 \ll \frac{1}{G_3} \), (4.25) reduces to

\[
\frac{v_0}{v_{in}} \approx Z_0 G_3
\]

where \( Z_0 \) is the closed-loop output impedance and can be expressed as

\[
Z_0(s) = \frac{R_{out}}{1 + \frac{R_m(s)}{K}}
\]  

(4.27)

The \( R_{out} \) represents the output resistance of OTRA. The feedback factor, \( K \), is constant however the open loop gain, \( R_m(s) \), is frequency dependant. Representing \( R_m(s) \) with a single pole model as discussed in section 2.3, (4.27) modifies to

\[
Z_0(s) = R_{out}sKC_p
\]

(4.28)

Assuming \( R_{out} \) is mainly resistive, \( Z_0 \) becomes inductive and increases at a rate of 20 dB/dec at high frequencies and the transfer function expressed by (4.26) appears to be a first order HP response.

\[ \text{Fig. 4.4 High frequency model of LPF.} \]

To show the orthogonal adjustment of \( f_0 \) and \( Q_0 \) the BP response for \( f_0 = 375 \text{ KHz}, 750 \text{ KHz} \) and 1.5 MHz has been plotted in Fig. 4.5(a) while keeping \( Q_0 \) fixed at 0.56. The simulation results for \( Q_0 = 14, 1 \) and 0.47, when \( f_0 \) remains constant at 750 KHz, is shown in Fig. 4.5(b). The \( Q_0 \) tuning can be accomplished through \( G_1 \) and \( f_0 \) can be adjusted by simultaneous change in capacitance values. It is evident from Fig. 4.5(a) and (b) that \( f_0 \) and \( Q_0 \) are orthogonally tunable. For electronic tuning of the filter parameters the gate voltages of the transistors used to implement \( G_1 \) need to be controlled. For the transistors \( M_a \) and \( M_b \) used for resistance realization as shown in Fig.4.2, the aspect ratio is taken as \( W/L = 10 \mu m/2 \mu m \). The
values of gate bias voltages used for tuning of $G_1$ and $Q_0$ as obtained are listed in Table 4.3.

![Graph a](image1)

(a)

![Graph b](image2)

(b)

Fig. 4.5 Orthogonal tunability of BP response (a) $\omega_0$ adjustment (b) $Q_0$ adjustment.

Table 4.3: Component values used for orthogonal tunability of $Q_0$.

<table>
<thead>
<tr>
<th>$V_a$ (V)</th>
<th>$V_b$ (V)</th>
<th>$G_1$ (mS)</th>
<th>$K$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>0.556</td>
<td>0.5</td>
<td>2</td>
<td>0.47</td>
</tr>
<tr>
<td>1.0</td>
<td>0.635</td>
<td>0.28</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>1.0</td>
<td>0.73</td>
<td>0.2</td>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>
4.3.2 SAB Topology-II

The second filter topology proposed is shown in Fig. 4.6 and using routine analysis the transfer function of this circuit can be expressed as

\[
\frac{v_o}{v_{in}} = \frac{-Y_1 Y_3}{D}
\]  

(4.29)

where \( D = (Y_1 + Y_2 + Y_3 + Y_4)Y_5 + Y_3 Y_4 \)  

(4.30)

![Proposed SAB topology-II](image)

Fig. 4.6 Proposed SAB topology-II.

The appropriate admittance choices, which result in three required filter responses, are listed in Table 4.4.

<table>
<thead>
<tr>
<th>Filter Function</th>
<th>Y_1</th>
<th>Y_2</th>
<th>Y_3</th>
<th>Y_4</th>
<th>Y_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Pass</td>
<td>G_1</td>
<td>sC_1</td>
<td>G_2</td>
<td>G_3</td>
<td>sC_2</td>
</tr>
<tr>
<td>High Pass</td>
<td>sC_1</td>
<td>G_1</td>
<td>sC_2</td>
<td>sC_3</td>
<td>G_2</td>
</tr>
<tr>
<td>Band Pass</td>
<td>G_1</td>
<td>G_2</td>
<td>sC_1</td>
<td>sC_2</td>
<td>G_3</td>
</tr>
</tbody>
</table>
Using the admittance choices given in Table 4.4 the LP response can be expressed as

\[
\frac{V_o}{V_{in}}_{LP} = \left. \frac{-G_1 G_2 / C_1 C_2}{s^2 + s\left(\frac{(G_1+G_2+G_3)}{C_1}\right) + \frac{(G_2 G_3)}{C_1 C_2}} \right|_{LP}
\]

(4.31)

The LPF can be characterized by filter parameters given by (4.32), (4.33) and (4.34) respectively.

\[
\omega_0 \left|_{LP} = \sqrt{\frac{G_2 G_3}{C_1 C_2}} \right.
\]

(4.32)

\[
Q_o \left|_{LP} = \frac{\sqrt{G_2 G_3 C_1}}{\sqrt{C_2 (G_1+G_2+G_3)}} \right.
\]

(4.33)

\[
H_o \left|_{LP} = \frac{-G_1}{G_3} \right.
\]

(4.34)

Using appropriate admittance choices as given in Table 4.4 the HP response can be obtained as below

\[
\frac{V_o}{V_{in}}_{HP} = \left. \frac{-s^2 C_1 / C_3}{s^2 + s\left(G_2 (C_1 + C_2 + C_3) \right) + \frac{(G_1 G_2)}{(C_2 C_3)}} \right|_{HP}
\]

(4.35)

and can be characterized by \(\omega_o\), \(Q_0\) and \(H_0\) as given by (4.36), (4.37) and (4.38) respectively.

\[
\omega_0 \left|_{HP} = \frac{\sqrt{G_1 G_2}}{C_2 C_3} \right.
\]

(4.36)

\[
Q_0 \left|_{HP} = \frac{\sqrt{G_1 C_2 C_3}}{\sqrt{G_2 (C_1 + C_2 + C_3)}} \right.
\]

(4.37)

\[
H_0 \left|_{HP} = -\frac{C_1}{C_3} \right.
\]

(4.38)

The BP response with proper admittance choices from Table 4.4 can be deduced as
\[ \frac{V_\theta}{V_{in}} \bigg|_{BP} = \frac{-sG_1/C_2}{s^2 + s(G_3(C_1 + C_2)/C_1 C_2) + (G_3(G_1 + G_2)/C_1 C_2)} \]  

(4.39)

The filter parameters \( \omega_0 \), \( Q_0 \) and \( H_0 \) can be expressed as

\[ \omega_0 \bigg|_{BP} = \sqrt{\frac{(G_1 + G_2)G_3}{C_1 C_2}} \]  

(4.40)

\[ Q_0 \bigg|_{BP} = \frac{\sqrt{C_1 C_2 (G_1 + G_2)}}{\sqrt{G_3 (C_1 + C_2)}} \]  

(4.41)

\[ H_0 \bigg|_{BP} = \frac{-G_1 C_1}{G_3 (C_1 + C_2)} \]  

(4.42)

Thus it is clear that no component matching / conditions on components are required for all the three responses.

### 4.3.2.1 Sensitivity Analysis

The passive sensitivities of \( \omega_0 \) and \( Q_0 \) for the LPF can be computed as

\[ S_{G_1}^{\omega_0} = 0, S_{G_2}^{\omega_0} = S_{G_3}^{\omega_0} = \frac{1}{2}, S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \]  

(4.43)

\[ S_{G_1}^{Q_0} = -1 + \frac{G_1}{(G_1 + G_2 + G_3)}, S_{G_2}^{Q_0} = \frac{1}{2} - \frac{G_2}{(G_1 + G_2 + G_3)}, S_{G_3}^{Q_0} = \frac{1}{2} - \frac{G_3}{(G_1 + G_2 + G_3)} \]

\[ S_{C_1}^{Q_0} = -S_{C_2}^{Q_0} = \frac{1}{2} \]  

(4.44)

The passive sensitivities of \( \omega_0 \) and \( Q_0 \) for the HPF can be expressed as

\[ S_{C_1}^{\omega_0} = 0, S_{C_2}^{\omega_0} = S_{C_3}^{\omega_0} = -\frac{1}{2}, S_{G_1}^{\omega_0} = S_{G_2}^{\omega_0} = \frac{1}{2} \]  

(4.45)
The passive sensitivities of $\omega_0$ and $Q_0$ for BPF can be computed as

$$S_{\omega_0} = S_{\omega_0}^0 = \frac{1}{2}, \quad S_{Q_0} = \frac{1}{2} - \frac{G_2}{2(G_1 + G_2)}, \quad S_{Q_0}^0 = \frac{1}{2} - \frac{G_1}{2(G_1 + G_2)}, \quad S_{Q_0}^0 = \frac{1}{2} \quad (4.47)$$

$$S_{C_1} = S_{\omega_0}^0 = \frac{1}{2} - \frac{G_2}{2(G_1 + G_2)}, \quad S_{C_1}^0 = \frac{1}{2} - \frac{C_2}{(C_1 + C_2)}, \quad S_{C_1}^0 = \frac{1}{2} - \frac{G_2}{(G_1 + G_2)}, \quad S_{C_1}^0 = \frac{1}{2} \quad (4.48)$$

It is clearly observed from (4.43) to (4.48) that the passive sensitivities for all the three responses are lower than $1/2$ in magnitude and therefore the filter configuration can be termed as insensitive.

### 4.3.2.2 Nonideality Analysis

The filter responses expressed by (4.31), (4.35) and (4.39) are derived considering the ideal behaviour of OTRA. In practice due to parasitics of OTRA the ideal responses get modified and can be represented as

$$\frac{V_{\omega}}{V_{in}}_{LP_n} = \frac{-G_1 G_2}{C_1(C_2 + C_p)} \cdot \frac{1}{s^2 + s\left(\frac{G_2 + G_3}{C_1}\right) + \left(\frac{G_2 G_3}{C_1(C_2 + C_p)}\right)} \quad (4.49)$$

$$\frac{V_{\omega}}{V_{in}}_{HP_n} = \frac{-s^2 C_1 / C_3}{s^2 + s\left(\frac{G_2 + s C_p}{C_2 C_3}\right) + \left(\frac{G_1 (G_2 + s C_p)}{C_2 C_3}\right)} \quad (4.50)$$

$$\frac{V_{\omega}}{V_{in}}_{BP_n} = \frac{-s G_1 / C_2}{s^2 + s\left(\frac{G_2 + s C_p}{C_1 C_2}\right) + \left(\frac{(G_2 + s C_p)(G_1 + G_2)}{C_1 C_2}\right)} \quad (4.51)$$

The effect of $C_p$ in case of LP response can be eliminated by pre-adjusting the value of $C_2$. **
thus achieving self-compensation. The $sC_p$ term appearing in parallel to $G_2$ in HP response and $G_3$ in BP response will introduce parasitic poles in the filter functions. The effect of the parasitic pole so introduced in HP and BP responses can be ignored by selecting the $C_2$ and $C_3$ to be much higher than the $C_p$ thereby keeping the parasitic pole frequency far off from the filter poles.

4.3.2.3 Simulation Results

The proposed filter circuits are verified through PSPICE simulations while using CMOS OTRA realization of Fig 2.9. The LP and HP filter configurations were designed to give Butterworth response for an $f_0$ of 1.5 MHz. For LP response the capacitors are chosen as $C_1 = 450 \text{ pF}$, $C_2 = 100 \text{ pF}$ and the conductances are computed as $G_1 = G_2 = G_3 = 2 \text{ mS}$. For HP response capacitances are chosen as $C_1 = C_2 = C_3 = 100 \text{ pF}$ and the conductances are calculated as $G_1 = 2 \text{ mS}$, $G_2 = 0.45 \text{ mS}$. The ideal and simulated LP and HP responses are shown in Fig. 4.7(a) and (b) respectively and it may be noted that both the responses the ideal and simulated curves are in close agreement.
Fig. 4.7 Frequency responses of SAB topology -II. (a) LP. (b) HP. (c) BP for \( f_0 = 1.6 \) MHz, \( Q_0 = 1 \). (d) BP for \( f_0 = 125 \) KHz, \( Q_0 = 2.3 \).

The BP response having center frequency \( f_0 = 1.6 \) MHz and \( Q_0 = 1 \) is shown in Fig. 4.7(c) for which \( C_1 \) and \( C_2 \) are chosen as 10 pF and accordingly the conductance are computed as \( G_1 = G_2 = 0.1 \) mS and \( G_3 = 0.03125 \) mS. The BP response for an \( f_0 = 125 \) KHz and \( Q_0 = 2.3 \) is shown in Fig. 4.7(d). It is observed that the theoretical and simulated behaviours of the filter are in close agreement.
4.4 OTRA BASED MULTIAMPLIFIER FILTERS

In this section multiple OTRAs based filters have been discussed which can realize biquads and higher order filter functions. In this section a multiamplifier biquadratic universal filter is proposed which is followed by the design of a multiamplifier higher order filter based on wave method.

4.4.1 Biquadratic Universal Filter

Many voltage-mode biquadratic universal filters using multiple OTRAs are available in open literature [36], [42], [44], [45], [47]. The structures available in [36], [44], [45] are SISO type and those presented in [42], [47] are MISO type and provide only one standard filter function at a time. Exhaustive survey suggests that though OTRA based SIMO [36], [46] and MIMO [48] structures providing multiple outputs are available in open literature yet none can provide more than three outputs simultaneously and do not qualify for universal filter. To fill this gap a SIMO universal filter which realizes all five standard responses is proposed and discussed in this section. The proposed universal filter is shown in Fig. 4.8. Using straightforward analysis of circuit of Fig. 4.8 the transfer functions at various nodes can be obtained as

\[
\frac{v_{o1}}{v_{in}} = \frac{s^2 G_4 C_1 C_2}{D(s)} \quad (4.52)
\]

\[
\frac{v_{o2}}{V_{in}} = \frac{s C_2 G_1 G_4}{D(s)} \quad (4.53)
\]

\[
\frac{v_{o3}}{v_{in}} = \frac{G_1 G_4 G_6}{D(s)} \quad (4.54)
\]

\[
\frac{v_{o4}}{v_{in}} = \frac{G_7 G_9 s^2 C_1 C_2 G_1 + G_8 G_9 G_1 G_4 G_6}{D(s)} \quad (4.55)
\]

\[
\frac{v_{o5}}{v_{in}} = \frac{G_7 G_11 G_9 s^2 C_1 C_2 G_1 - G_10 G_9 G_1 C_2 G_1 G_4 + G_8 G_9 G_11 G_1 G_4 G_6}{G_9 G_{12} D(s)} \quad (4.56)
\]
where \( D(s) = s^2C_1C_2G_3 + sC_2G_4G_5 + G_2G_4G_6 \) and \( G_i = \frac{1}{R_i} \) \( (4.57) \)

![Proposed biquadratic universal filter diagram](image)

**Fig. 4.8 Proposed biquadratic universal filter.**

Equations (4.52) – (4.56) clearly indicate that HP, BP, LP, BR and AP responses are available at \( v_{o1}, v_{o2}, v_{o3}, v_{o4} \) and \( v_{o5} \) respectively. BR and AP responses are obtained subjected to conditions given by (4.58) and (4.59) respectively.

\[
G_7 = G_9, \ G_1G_8 = G_2G_9
\]  \( (4.58) \)

\[
G_1G_{10} = G_5G_{12}, \ G_7G_{11} = G_9G_{12}, \ G_1G_8G_{11} = G_2G_9G_{12}
\]  \( (4.59) \)

Using (4.52) – (4.56), various filter gains can be computed as

\[
G_{HP} = G_{BR} = G_{AP} = \frac{G_1}{G_3}, \ G_{BP} = \frac{G_1}{G_5}, \ G_{LP} = \frac{G_1}{G_2}
\]  \( (4.60) \)

The \( \omega_0 \) and the \( Q_0 \) can be characterized by

\[
\omega_0 = \sqrt{\frac{G_2G_4G_6}{C_1C_2G_3}}
\]  \( (4.61) \)

\[
Q_0 = \frac{1}{G_5} \sqrt{\frac{C_1G_2G_3G_6}{C_2G_4}}
\]  \( (4.62) \)
This suggests that the $Q_0$ can be independently controlled by varying $G_5$ without affecting the $\omega_0$. It can be noted from (4.61) that simultaneous adjustment of $G_2$ and $G_4$ results in orthogonal tuning of $\omega_0$. Also the filter gain can be controlled through $G_1$ without affecting $\omega_0$ and $Q_0$. The sensitivities of $\omega_0$ and $Q_0$ with respect to each passive component are low and can be obtained as

\[ S_{G2}^{\omega_0} = S_{G4}^{\omega_0} = S_{G6}^{\omega_0} = \frac{1}{2}, S_{G3}^{\omega_0} = S_{C1}^{\omega_0} = S_{C2}^{\omega_0} = -\frac{1}{2} \]  \hspace{1cm} (4.63)

\[ S_{G5}^{Q_0} = 1, S_{C1}^{Q_0} = S_{G2}^{Q_0} = S_{G3}^{Q_0} = S_{G6}^{Q_0} = \frac{1}{2}, S_{G4}^{Q_0} = S_{C2}^{Q_0} = -\frac{1}{2} \]  \hspace{1cm} (4.64)

The expressions for sensitivity are simpler and independent of components used in contrast to SAB topologies of Fig. 4.1 and Fig. 4.5.

### 4.4.1.1 MOS–C Implementation

The proposed configuration is made fully integrated by implementing the resistors using matched transistors operating in linear region as explained in section 2.8. The MOS-C implemented universal filter configuration is shown in Fig. 4.9. The resistance value may be

![Fig. 4.9 OTRA MOS-C universal filter.](image-url)
adjusted by appropriate choice of gate voltages thereby making filter parameters electronically tunable. It also exhibits the feature of orthogonal controllability of $\omega_0$ and $Q_0$ through gate bias voltage.

### 4.4.1.2 Nonideality Analysis

The response of the filter may deviate due to nonideality of OTRA in practice. Taking this effect into account the transfer functions of the circuit of Fig. 4.8 modify to

$$\frac{V_{01}}{V_{in}} \bigg|_n = \frac{s^2 G_1 (C_1 + C_p) (C_2 + C_p)}{D(s)} \tag{4.65}$$

$$\frac{V_{02}}{V_{in}} \bigg|_n = \frac{s (C_2 + C_p) G_1 G_4}{D(s)} \tag{4.66}$$

$$\frac{V_{03}}{V_{in}} \bigg|_n = \frac{G_1 G_4 G_6}{D(s)} \tag{4.67}$$

$$\frac{V_{04}}{V_{in}} \bigg|_n = \frac{G_7}{(G_9 + s C_p)} s^2 (C_1 + C_p) (C_2 + C_p) G_1 + \frac{G_8}{(G_9 + s C_p)} G_1 G_4 G_6}{D(s)} \tag{4.68}$$

$$\frac{V_{05}}{V_{in}} \bigg|_n = \frac{G_7 G_{11}}{G_9 (G_{12} + s C_p)} s^2 (C_1 + C_p) (C_2 + C_p) G_1 - \frac{G_{10}}{(G_{12} + s C_p)} s (C_2 + C_p) G_1 G_4 + \frac{G_8 G_{11}}{(G_{12} + s C_p) (G_9 + s C_p)} G_1 G_4 G_6}{D(s)} \tag{4.69}$$

where $D(s) = s^2 (C_1 + C_p) (C_2 + C_p) G_3 + s C_p) + s (C_2 + C_p) G_4 G_5 + G_2 G_4 G_6 \tag{4.70}$

The effect of $C_p$ can be eliminated by pre-adjusting the value of capacitors $C_1$ and $C_2$ and thus achieving self-compensation. The $s C_p$ term appearing in parallel to $G_i$ for $i = 3, 9, 12$ will result in introduction of parasitic pole having radian frequency as $\omega = G_i / C_p$. By selecting the circuit components $C_1$ and $C_2 >> C_p$ the parasitic pole can be placed far away from the filter poles and thereby the effect of this additional pole can be eliminated.
4.4.1.3 Simulation Results

The working of the proposed SIMO biquadratic universal filter is verified through SPICE simulations. For simulations aspect ratio for all transistors used for resistance realization is taken as $W/L = 5\mu m/5\mu m$. The proposed SIMO biquadratic universal filter is designed for the resonant frequency ($f_0$) of 120 KHz and $Q_0 = 1$ with component values $C_1 = C_2 = 100$ pF and for which $R_i \approx 10.5$ KΩ for $i = 1, 2 \ldots 12$. The value of $R_i$ was set by taking the gate voltages as $V_{ai} = 1.4$ V and $V_{bi} = 0.75$ V for all $i = 1, 2, \ldots, 12$. Figure 4.10 shows the simultaneously available LP, HP, BP, BR and AP frequency responses. The ideal and simulated responses are found to be in close agreement. Simulated power consumption for the proposed universal filter is 4.04 mW. Orthogonal tunability of $Q_0$ with $R_5$ at $f_0 = 11.5$ KHz is shown in Fig. 4.11 for BP filter. This is obtained by selecting $C_1 = C_2 = 50$ pF, and $R_i = 272$ KΩ for $i = 1, \ldots, 4, 6, \ldots, 12$ for different values of $R_5$. The values of $Q_0$ as obtained and gate bias voltages used for tuning of $R_5$ are listed in Table 4.5.
Fig. 4.10 Simulated frequency responses of the proposed circuit (a) LPF and HPF.

(b) BPF and BRF. (c) APF.
Table 4.5: Component values used for orthogonal tunability of $Q_0$.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Bias Voltage $V_{a5}$ (V)</th>
<th>Bias Voltage $V_{b5}$ (V)</th>
<th>$R_5$ (KΩ)</th>
<th>$Q_0$</th>
<th>$f_0$ (KHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.76</td>
<td>0.75</td>
<td>$\approx$680</td>
<td>2.5</td>
<td>11.5</td>
</tr>
<tr>
<td>2.</td>
<td>0.80</td>
<td>0.75</td>
<td>$\approx$136</td>
<td>0.5</td>
<td>11.5</td>
</tr>
<tr>
<td>3.</td>
<td>0.85</td>
<td>0.75</td>
<td>$\approx$68</td>
<td>0.25</td>
<td>11.5</td>
</tr>
<tr>
<td>4.</td>
<td>0.95</td>
<td>0.75</td>
<td>$\approx$34</td>
<td>0.125</td>
<td>11.5</td>
</tr>
</tbody>
</table>

The $f_0$ is electronically tunable by varying the gate voltage and is verified through simulations as depicted in Fig. 4.12 for BR filter. Values of $f_0$, for $C_1 = C_2 = 50$ pF and $R_i \approx 23$ KΩ for $i = 1, 3, 5, \ldots, 12$, along with different gate voltages chosen for simultaneously varying $R_2$ and $R_4$ are listed in Table 4.6.

![Fig. 4.11 BP response for different $Q_0$ values.](image-url)
The time domain behavior of the BP filter is also studied by applying three sinusoidal frequency components, a low frequency signal of 1 KHz, a high frequency component of 100 KHz and the third component of 11.5 KHz which is $f_0$ of the BP filter. The transient response of the filter circuit is shown in Fig. 4.13. It may be noted that the frequency components other than $f_0$ are significantly attenuated.
Fig. 4.13 Simulated transient response of BP filter. (a) Input signal and its frequency spectrum (b) Output transient response and frequency spectrum.
To check the quality of the output of BP filter, the percentage total harmonic distortion (%THD) with the sinusoidal input signal is obtained as shown in Fig. 4.14. It is observed that the %THD remains considerably low [78] for input signal values till 60 mV. Simulated power consumption for the proposed universal filter is 4.04 mW.

![Graph of % THD variation with input signal amplitude.](image)

Fig. 4.14 % THD variation with input signal amplitude.

### 4.4.2 OTRA Based Higher Order Filters

There are many advantages of higher order filters using doubly terminated lossless ladders, like, low sensitivity to component tolerances, ample design information and design tables that can be readily applied. However, inductor realization in an integrated circuit is a challenging task. There are various techniques that circumvent these shortcomings like element replacement and operational simulation. In operational simulation signal flow graphs are used to emulate the relationship between various passive elements. These are then physically realized using lossy and lossless integrators [79]. Realization of lossless integrators is difficult because of non ideal characteristics of passive components used. Besides, floating capacitors are used in this topology, which are not very favorable in IC implementation. In the case of element replacement approach, inductors are replaced by gyrators. Although this practice leads to good results with low noise sensitivity, realizing high quality floating inductors proves to be difficult [79]. Another element replacement method using Frequency Dependent Negative Resistance (FDNR) was proposed by Bruton.
[80], which works well with low pass filters. LC ladder filters can also be emulated using Linear Transformation approach wherein every section of the original ladder prototype can be realized by using active elements individually [81]. The only drawback of this method is that it uses lossless integrators.

Apart from these approaches, the wave method [79], is also used for realizing higher order resistively terminated LC ladder filters. It uses wave equivalents for different passive elements which can be readily substituted to realize a filter. In this approach, the filter realization is based on modeling the forward and reflected voltage waves. The available wave active filters [79], [82]–[88] use various active blocks such as OTA [82], current amplifier [83], CMOS cascode current mirror [84], FPAA [85], OPAMP [79], [86], CFOA [87], and DVCCCTA [88] and operate in current [82]–[85] and voltage [79], [86]–[88] mode. In this section design approach for realization of OTRA based higher order wave filter is presented. First the concept of wave filter is elaborated which is then employed for OTRA based wave filter realization. Simulation results for a third order Butterworth filter realized using wave method, are shown in subsequent section.

### 4.4.2.1 Wave Filter Approach

The concept of wave filter is introduced in [79], [86]. This approach talks of applying scattering parameters to ladder filters. It uses voltage waves instead of power waves. Scattering matrix of a two port network is given as

\[
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}
\]

In a two port network having a series branch admittance \( Y \), as shown in Fig. 4.15, the scattering parameters, assuming the normalization resistance as \( R_n \), are obtained as

\[
S_{11} = \frac{1}{2R_nY+1}, \quad S_{12} = \frac{2R_nY}{2R_nY+1}, \quad S_{21} = \frac{2R_nY}{2R_nY+1}, \quad S_{22} = \frac{1}{2R_nY+1}
\]

For a series branch inductance \( L \), using (4.72), (4.71) reduces to
This can be further simplified to

\[ B_1 = A_1 - \frac{1}{(1+\tau L)} (A_1 - A_2) \quad (4.75) \]

\[ B_2 = A_2 + \frac{1}{(1+\tau L)} (A_1 - A_2) \quad (4.76) \]

Fig. 4.15 Series branch admittance Y.

In (4.76) \( \tau L = L/2R_n \) is the time constant. Figure 4.16(a) shows the symbolic representation of the wave equivalent of series branch inductor L. To calculate the S-matrix for series branch capacitance C, (4.71) reduces to

\[ B_1 = \frac{1}{2sCR_n+1} A_1 + \frac{2sCR}{2sCR_n+1} A_2 \quad (4.77) \]

\[ B_2 = \frac{2sCR_n}{2sCR_n+1} A_1 + \frac{1}{2sCR_n+1} A_2 \quad (4.78) \]

Simplifying further (4.77) and (4.78) can be expressed as

\[ B_1 = A_2 + \frac{1}{1+s\tau C} (A_1 - A_2) \quad (4.79) \]
\[ B_2 = A_1 - \frac{1}{1+s\tau_C} (A_1 - A_2) \]  

(4.80)

where \( \tau_C = 2CR_n \) is the time constant. It is observed that (4.75) and (4.76) are similar to (4.79) and (4.80) respectively, and can be obtained from each other by interchanging the output terminals \( B_1 \) and \( B_2 \). This result can be generalized to show that for a series branch admittance \( Y \), its dual admittance \( Y' \) can be obtained as

\[ Y' = \frac{1}{4R_n^2Y} \]  

(4.81)

Accordingly the wave equivalent symbol of series branch capacitance \( C \), can be drawn as shown in Fig. 4.16 (b).

![Wave equivalent of series branch elements](image)

**Fig. 4.16 Wave equivalent of series branch elements.**

(a) Inductance \( L \), \( \tau_L = L/2R_n \).

(b) Capacitance \( C \), \( \tau_C = 2CR_n \).

For an inductor \( L \) connected in series with capacitance \( C \) in a series arm the wave equivalent can be obtained by cascading the wave equivalents of \( L \) and \( C \). If the terminals are interchanged, the wave equivalent for a tank circuit connected in series branch can be obtained. Table 4.7 [79], [86] gives wave equivalents for all the series branch elements. Proceeding in a similar manner, wave equivalents for shunt branch elements can also be derived. Table 4.8 [79], [86] lists the results for shunt branch elements.
Table 4.7: Wave equivalents of series branch elements [79], [86].

<table>
<thead>
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<th>Series Branch Elements</th>
<th>Wave Equivalents</th>
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<tr>
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</tbody>
</table>

Table 4.8: Wave equivalents of shunt branch elements [79], [86].

<table>
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<th>Wave Equivalents</th>
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<tr>
<td><img src="image17" alt="Shunt Branch Elements" /></td>
<td><img src="image18" alt="Wave Equivalents" /></td>
</tr>
</tbody>
</table>
4.4.2.2 OTRA Based Wave Active Filter

In this section a systematic approach for realizing higher order resistively terminated LC ladder filters, using wave method, which employs OTRA has been discussed. A closer study of (4.75),

Using these blocks the wave equivalent for a series branch inductor, defined by (4.75) and (4.76) represented symbolically by Fig. 4.16 (a), can be obtained and is shown in Fig. 4.17. The dashed blocks indicate the OTRA based basic circuits constituting the wave equivalent of series branch inductor. The MOS-C equivalent of circuit of Fig. 4.17 is shown in Fig. 4.18 wherein all passive resistors are implemented using MOS transistors operating in non saturation region. This elementary block alongwith inverter can now be used to synthesize the wave equivalents for all the elements listed in Table 4.7 and Table 4.8. The circuit in Fig. 4.17 can be described by the following equations

\[
B_1 = A_1 - \frac{1}{1+sCR} (A_1 - A_2) \quad (4.82)
\]

\[
B_2 = A_2 + \frac{1}{1+sCR} (A_1 - A_2) \quad (4.83)
\]
Fig. 4.18 MOS-C equivalent of series branch inductor.

The actual value of inductance $L_A$ realized by circuit of Fig. 4.17 would be obtained by comparing (4.82) with (4.75) or (4.83) with (4.76). The realized value can be expressed as

$$L_A = 2R_n CR$$  \hfill (4.84)

Similarly, comparing (4.82) and (4.79) or (4.83) and (4.80), the capacitance value realized ($C_A$) can be written as

$$C_A = \frac{CR}{2R_n}$$  \hfill (4.85)

Similarly, the actual values of $L$ and $C$ for wave equivalents of shunt branch elements can be given by
For a filter if $L_n$ and $C_n$ are the normalized inductor and capacitor values respectively, $\omega_0$ be the normalizing pole frequency and $R_n$ is the normalizing resistance, then to de-normalize $L_n$ and $C_n$ following expressions can be used

\[
L_A = \frac{R_n C R}{2} \quad (4.86)
\]

\[
C_A = \frac{2 C R}{R_n} \quad (4.87)
\]

Using different notation of $C$ for $L_A$ and $C_A$, i.e. $C_L$ and $C_C$ respectively, (4.84) and (4.85) can be restated as

\[
L_A = \frac{R_n}{\omega_0} L_n \quad (4.88)
\]

\[
C_A = \frac{1}{R_n \omega_0} C_n \quad (4.89)
\]

From (4.88) - (4.91) $C_L$ and $C_C$ can be expressed as

\[
C_L = \frac{1}{2 \omega_0 R} L_n \quad (4.92)
\]

\[
C_C = \frac{2}{\omega_0 R} C_n \quad (4.93)
\]

The expression, for controlling $\omega_0$ using $R$, needs to be worked out for each circuit. A simple algorithm can be worked out to achieve the exact expression and range of tunability. For the frequency $\omega_0$, $C_L$ and $C_C$ are calculated in terms of $R$, as per the $L_n$ and $C_n$ values. For a suitable $R$ value, once $C_L$ and $C_C$ have been fixed, either equation (4.92) or (4.93) can be used to describe the relationship between $R$ and $\omega_0$ as

\[
\omega_0 = \frac{K}{R} \quad (4.94)
\]

$K$ can be expressed as
Substituting $R$ from (2.41) in (4.94) one may get

$$\omega_0 = KK_n(V_a - V_b)$$ \hspace{1cm} (4.96)

Equation (4.96) describes the electronic tunability of $\omega_0$ by varying gate voltages $V_a$ and/or $V_b$ of the transistors used to implement the linear resistor.

### 4.4.2.3 Simulation Results

To demonstrate the wave filter approach using OTRA, a doubly terminated third order Butterworth low pass filter (LPF), as shown in Fig. 4.19, has been implemented and simulated using SPICE. The normalized values of components are $L_{n1} = 2$, $C_{n1} = 1$ and $C_{n2} = 1$. The design specifications of LPF are taken to be $f_p = 200$ KHz and maximum attenuation in pass band $\alpha_{\text{MAX}}$ is 3 dB.

![3rd order low pass Butterworth filter](image)

The value of normalizing resistance $R_n$ is chosen to be 2.5 KΩ. De-normalizing the values of $L_{n1}$, $C_{n1}$ and $C_{n2}$, $L_A$ and $C_A$ can be computed as

$$L_A = 3.98 \text{ mH}, C_{A1} = C_{A2} = 318.31 \text{ pF}$$ \hspace{1cm} (4.97)

Setting $R$ initially to 12 KΩ, the value of $C_L$ for $L_{A1}$ can be calculated as 66.33 pF and that of $C_C$ for $C_{A1}$ and $C_{A2}$ as 132.66 pF. The value of $K$, as per (4.95) is $1.508 \times 10^{10}$. The W/L ratio for the transistors used for implementing linear resistors is taken as 10µm/2.5µm. For this simulation exercise, the value of $K_N$ was found to be $5.25 \times 10^{-4} \text{ A/V}^2$. The required $V_a$ and $V_b$ values for $R$ to be 12 KΩ were computed to be 0.908 V and 0.75 V. The wave
equivalent circuit of the LPF is shown in Fig. 4.20 in which the reflected waves are available at $v_{OL}$ and $v_{OH}$. These outputs complement each other by virtue of wave theory [86]. Thus as the $v_{OL}$ represents the LPF response; its complementary high pass output is available at $v_{OH}$. Fig. 4.21 shows the simulated LPF response of the circuit at $v_{OL}$. The complementary high pass output $v_{OH}$, as represented in Fig. 4.20, has been plotted in Fig. 4.22. The proposed circuit can be tuned to different cut-off frequencies by controlling the voltage difference ($V_a - V_b$) as described by (4.96). Comparison between theoretical and the observed frequencies obtained by variation of control voltage difference ($V_a - V_b$) is shown in Fig. 4.23.

Fig. 4.20 Wave equivalent of circuit of LPF of Fig. 4.19.

Fig. 4.21 LP response ($V_{OL}$).
The performance comparison of the proposed circuit with the previous voltage mode structures [79], [86] – [88] is summarized in Table 4.9. It may be noted that the topology presented in [78] shows best THD result, however the structure is not electronically tunable. Although the most recently reported work [88] is having better THD performance its simulated power consumption is higher as compared to the proposed one. The relevant data for structures of [79], [86] is not available in the literature, which are designed using commercially available op-amps.
Table 4.9: Comparison with the voltage mode filter structures.

<table>
<thead>
<tr>
<th>Ref</th>
<th>Active block and technology used</th>
<th>Filter structure</th>
<th>% THD</th>
<th>Power consumption (mW)</th>
<th>Output noise voltage (V/Hz (1/2))</th>
<th>Electronic tunability</th>
</tr>
</thead>
<tbody>
<tr>
<td>[78]</td>
<td>CFOA (Commercially available IC AD844 with ± 5V power supply)</td>
<td>3(^{\text{rd}}) order elliptic Low pass</td>
<td>1% for 1V pp signal</td>
<td>Not available</td>
<td>Not available</td>
<td>No</td>
</tr>
<tr>
<td>[79]</td>
<td>DVCCCTA (CMOS Technology-0.25(\mu)m, Power supply ± 1.25V)</td>
<td>4(^{\text{th}}) order Butterworth Low pass</td>
<td>Less than 5% up to 225 mV pp signal</td>
<td>59.2</td>
<td>8.36 (\times 10^{-8})</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Proposed work</td>
<td>3(^{\text{rd}}) order Butterworth Low pass</td>
<td>Less than 5% up to 125 mV pp signal</td>
<td>10.7</td>
<td>7.28 (\times 10^{-8})</td>
<td>Yes</td>
</tr>
</tbody>
</table>

4.5 CONCLUDING REMARKS

Single and multi amplifier based VM filter topologies are proposed in this chapter. SAB is a preferred choice when low power consumption is the primary design concern. To cater this design need two SAB topologies are presented which can be used to synthesize LP, HP and
BP filter functions with appropriate admittance choices. These topologies do not impose any matching constraints on components, in contrast to all existing structures. The first configuration is based on Sallen Key approach while the second is based on multiple feedback topology. High $Q_0$ realization with moderate component spread and independent adjustment of $\omega_0$ and $Q_0$ are the high points of the first topology. Passive sensitivities for both the configuration are quite low, though are dependent on components used.

Multiamplifier filters are superior to SABs in terms of passive sensitivity and versatility; basis enough to explore multiamplifier design and has led to the design of two; multiamplifier filter topologies. A universal biquadratic filter realizing all five standard filter functions simultaneously is proposed first. The filter parameters $\omega_0$ and $Q_0$ can be orthogonally tuned. This is succeeded by design details of higher order resistively terminated LC ladder using wave method and is implemented through multiple OTRAs. It uses wave equivalents for different passive elements which can be readily substituted in higher order resistively terminated LC ladder to realize a filter. MOS-C implementation of universal biquad and wave active filter configurations is also presented which makes filter parameters electronically tunable.

All proposed structures are validated through SPICE simulations. The effect of nonidealities of OTRA on filter responses is also analysed.