2 The Means of Exchange Function

2.1 Introduction

In this chapter we begin our study of intrinsically valueless money at the most primitive level—that of bilateral exchange. In section 2.2 we discuss the necessary technological and distributional assumptions that must be satisfied for there to even be the possibility for money to play a role in a particular economy. In section 2.3 we construct a simple model of explicit search and exchange which satisfies these conditions in order to examine how an intrinsically valueless money may still become acceptable in exchange and how the introduction of money affects economic welfare. We conclude by showing how even the primitive environments discussed in this chapter throw up some of the questions that must be answered by any theory of a monetary economy.
2.2 Necessary conditions for monetary exchange

2.2.1 Decentralization and control

What are the minimum properties that a hypothetical economy must satisfy for it to be a meaningful framework for discussing monetary questions? The first certainly is decentralization and some form of private property. It is only in a world of discrete agents each of whom has a prior claim on a part of the community's product that we can talk about the problems of exchange and their possible resolution through the institution of money. We believe that both these conditions would be fulfilled by any community provided we look at it at a sufficient degree of detail. The existence of decentralization follows from the discreteness of humanity itself. While our species has evolved remarkable abilities of communication and co-ordination, they are neither so rigid nor so costless that we can ignore the existence of individuals. Once the existence of decentralization is accepted, that of prior claims follows directly from the limited scope of control that can be directly exercised by one individual. Possession, after all, is the most direct form of property. This prior claim my be denied through force and fraud, or might be abdicated by conscious choice. But that such institutions of coercion or consent are required to enforce a particular system of property in itself demonstrates that the the prior individual claims that we have been talking about actually exist.

Our claim that every community can ultimately be decomposed into indi-
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Individuals with private claims should not be taken to mean that such atomistic analysis is always the most fruitful. We may choose to ignore the questions of allocation and coordination within certain groups and consider only the interactions between these groups in our analysis. This does not imply a fundamental change in method since we are still working in terms of individuals and their spheres of control—only it is the case that the individual agents of our analysis do not correspond to individual humans. Of course, this leaves us with the task of ascribing valid behavioural properties to these collective agents without having to resort to the atomistic analysis after all. On the other hand, empirical evidence may allow us to ascribe behaviour to collective agents even when we do not know or do not care how this behaviour emerges from the behaviour of the constituent individuals.

2.2.2 Specialisation

Even with decentralisation and private property, trade and money are not necessary unless there is heterogeneity among agents. This heterogeneity may come from various sources—inhherited endowments, intrinsic abilities, geographical distribution of resources, position in the life cycle. Even among identical agents there might be heterogeneity of outcomes. Thus we may imagine a community of identical agents living over a number of periods whose endowment in each period is a random quantities of some perishable good. If these agents are risk-averse then they will desire the opportunity for smoothing their income which is offered by intertemporal exchange, i.e.,
borrowing and lending.

More interesting than these exogenous sources of heterogeneity is endogenous heterogeneity which itself depends on the opportunities for exchange. This follows Adam Smith's original intuition about gains in productivity made possible by specialisation and the division of labour (Smith, 1904; Young, 1928). In this case, if the introduction of money reduces the costs of exchange then the result is not just a realisation of the productive potential that exists at the time money is introduced but also the initiation of a cumulative process of increase in productivity.

2.2.3 Information and utility constraints

Traditionally, the existence of decentralisation, private property and specialisation in an economy has been considered sufficient to produce an essential role for money. The argument works by contrasting monetary exchange with a system of direct barter where agents accept only those commodities that they wish to consume and offer only those commodities that they have themselves produced. Thus we have, in a classical statement by Jevons (1875):

Some years since, Mademoiselle Zélie, a singer of the Theatre Lyrique at Paris, made a professional tour round the world, and gave a concert in the Society Islands. In exchange for an air from Norma and a few other songs, she was to receive a third part of the receipts. When counted, her share was found to consist of
three pigs, twenty-three turkeys, forty-four chickens, five thousand cocoa-nuts, besides considerable quantities of bananas, lemons and oranges. At the Halle in Paris [...] this amount of live stock and vegetables might have brought four thousand francs, which would have been good renumeration for five songs. In the Society Islands, however, pieces of money were very scarce; and as Mademoiselle could not consume any considerable portions of the receipts herself, it became necessary in the mean time to feed the pigs and poultry with the fruit.

Such anecdotes, which are fairly commonplace in textbooks of monetary economics, are in one sense misleading. For while they certainly are effective in showing that direct barter is an inefficient exchange mechanism, it does not necessarily imply that monetary exchange is the only alternative. There may well be other mechanisms which can also support an efficient allocation.

The simplest such mechanism is that of indirect exchange where at least some agents accept commodities in trade which they do not intend to consume. Once we add differential storage, transport and quality-inspection costs to our model, it is likely that only a few commodities will participate in these indirect exchanges. This we might take as an explanation of the emergence of commodity money and then see fiat money as a further refinement of commodity money.
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But another possibility studied formally in the papers of Kocherlakota (1998) and Kocherlakota and Wallace (1998) shows that this may not be the most illuminating way to see the role of money in exchange. These papers study allocation mechanisms based on what they call 'memory'—the possibility of there being a public record of the trading history of the agents who make up a community. They are able to show that a community with 'memory' can not only achieve all those allocations which a monetary economy can achieve, but it can achieve even more. The way 'memory' can substitute for money is easy to see. Assume that in our community 'memory' is implemented as a huge scoreboard. Each time one agent delivers a commodity to another agent, the giving agent's score goes up and the receiving agent's score goes down by the amount of money that would otherwise have changed hands. Exchanges are constrained by the fact that scores can never be negative. Then this system tracks the monetary system exactly. Indeed it is analogous to our system of payments through cheques drawn on banks—the only difference being that unlike Kocherlakota's 'memory', the record of all commodity transfers are not publicly visible in a bank-based payments systems and therefore we need some additional reason to trust the bank.

But if we can actually implement something like 'memory' we need not constrain ourselves to *quid pro quo* either. With all transactions being public, agents may follow the policy of offering their commodity to anyone who needs it without demanding anything in return. Since givers in some pairings will be receivers in others, an optimal allocation can be achieved in this way. The question then arises of what prevents someone from receiving but
refusing to give. It is here that the assumption of a publicly visible ‘memory’ becomes necessary. In (Kocherlakota, 1998) a deviation by anyone from a strategy triggers the punishment of everyone refusing to participate in any trades from that point onwards. This threat is sufficient to ensure that no one actually deviates.

2.2.4 Conclusion

Thus we see that for money to be necessary for an economy, the economy must satisfy the assumption of decentralization, private property and heterogeneity. But even in such an economy, it is not the physical properties of the money object which are important. Rather, we can turn the result of (Kocherlakota, 1998) on its head to say that money is a way to economise on memory. Many allocations which would otherwise have required a costless centralised repository of trading records can now be achieved by passing useless tokens in a decentralized manner.

2.3 Valueless money and explicit search

While the previous section discussed situations in which money might be necessary, in this section we construct a model to show how in a simple world where money is necessary it can also be sufficient in making possible hitherto impossible exchanges. The model of this section is that of bilateral exchange between randomly-matched agents. There is an extensive litera-
ture on this class of models, starting with the work of Kiyotaki and Wright (1989, 1993). In the present study we shall limit ourselves to the simplest case for the sake of tractability. In particular, while the search-theoretic literature has also explored the emergence of commodity money and the coexistence of commodity and valueless money, in our model we shall exclude the direct exchange of commodities by assumption in order to restrict our attention to the process through which intrinsically valueless money circulates.\(^1\).

2.3.1 The model

Consider an island economy where the only commodity is coconuts. Coconuts are indivisible and come in three varieties—red, green and blue. The island is inhabited by three tribes of people—the Red, Green and Blue tribes. There is a continuum of agents on the island, with equal probabilities of an agent belonging to any of the three tribes.\(^2\).

Time is discrete and infinite and agents discount future payoffs at the rate \(\delta\).

As a proxy for intrinsically useless money we assume that a proportion \(M\) of agents are endowed with an unit each of an indivisible and useless object which we shall call ‘money’.

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\(^1\)Commodity-commodity trade does not take place in the model in (Kiyotaki and Wright, 1993). But there it is a consequence of other assumptions rather than a primitive assumption itself.

\(^2\)This is a technical assumption which, with the random meeting assumption we make below, means that the probability of two agents meeting repeatedly is zero and hence systems of exchange based on reputation and credit are ruled out.
An agent can be in two states. She can be carrying an object—coconut or money—or she can be trying to search for a coconut. We assume that the island is so fertile that search is instantaneous. So, an agent who has been able to consume a coconut in one period gains utility $U$ at the end of that period and enters at the beginning of the next period carrying a fresh coconut.

We introduce a double coincidence of wants problem with the following assumption. First, there is specialisation—red coconuts can be collected only by the people of the Red tribe, green coconuts by the Green tribe and blue coconuts by the Blue tribe. Second, there is a diversity of preferences—the people of the Red tribe consume only blue coconuts, people of the Green tribe consume only red coconuts and people of the Blue tribe consume only green coconuts.

In each period agents are randomly matched. They have to decide whether to exchange the goods or money that they are carrying. If they choose not to exchange they carry over their holdings to the next period. As mentioned above, consumption and search for new coconuts happens in between periods.

2.3.2 The optimization problem

We begin by limiting our attention to a situations which are symmetric across colours and tribes and stationary over time. Moreover, given our assumption of no commodity-commodity exchange, the only coconut-money
exchanges are possible.

The strategies can therefore be completely specified summarised by the probability $\pi_0$ of an agent offering a coconut offering in exchange for money and the probability $\pi_1$ of an agent offering money in exchange for a coconut. $\pi = \pi_0\pi_1$ is the probability that monetary exchange happens in situations where it is possible.

Consider a coconut-carrying agent. With probability $M/3$ she meets a money-carrying agent who desires coconuts of a colour that our agent carries. In this case the agent either sells her coconut with probability $\pi$. In all other cases she remains a coconut-carrying agent. If we take $V_c$ to be the lifetime expected utility of a coconut-carrying agent, then the Bellman equation is:

\[
(1 + \delta)V_c = \max_{n_0} \left\{ \frac{\pi_0\pi_1 M}{3} V_m + \left( 1 - \frac{\pi_0\pi_1 M}{3} \right) V_c \right\}
= \max_{n_0} \left\{ V_c + \frac{\pi_0\pi_1 M}{3} (V_m - V_c) \right\}
\]

The situation for the money-carrying agent is almost symmetrical. With probability $(1 - M)$ she meets a coconut-carrying agent. With further probability $1/3$ the coconut is of a colour of her choice. In this case she buys the coconut with a probability $\pi$, consumes it to gain an utility of $U$ and, given our assumption of instantaneous coconut search, emerges as a coconut-carrying
agent in the next period. In all other situations she remains a money-carrying agent. If \( V_m \) is the lifetime expected utility of this type of agent then the corresponding Bellman equation is:

\[
(1 + \delta)V_m = \max_{\pi_1} \left\{ \frac{\pi_0\pi_1(1 - M)}{3}(U + V_c) + \left(1 - \frac{\pi_0\pi_1(1 - M)}{3}\right)V_m \right\}
= \max_{\pi_1} \left\{ V_M + \frac{\pi_0\pi_1(1 - M)}{3}(U + V_c - V_m) \right\}
\]

\[
\delta V_m = \max_{\pi_1} \left\{ \frac{\pi_0\pi_1(1 - M)}{3}(U + V_c - V_m) \right\}
\]

(2.2)

2.3.3 Non-monetary equilibrium

Proposition 1. There is an equilibrium with \( \pi = 0 \).

Proof. It is immediately clear that \( \pi_0 = 0 \) is an equilibrium. If \( \pi_0 = 0 \) then from eq. (2.2) \( V_m = 0 \), which then in turn is consistent with \( \pi_0 = 0 \) being an optimal solution for eq. (2.1).

If money is not generally accepted, then it is not in the interest of any agent to accept money. This is a feature that we shall repeatedly meet in the models of intrinsically valueless money. In our opinion, just the existence of a non-monetary equilibrium is not enough to invalidate a model of a monetary economy. Rather, we take it as a confirmation that we have correctly modelled a money as intrinsically valueless, so that its value depends only on its acceptability. Instead, we shall evaluate different models on the basis of the monetary equilibria that they imply.
2.3.4 Monetary equilibrium

Proposition 2. There is an equilibrium with \( \pi > 0 \).

Proof. Let \( \hat{V}_c \) and \( \hat{V}_m \) be the values of \( V_c \) and \( V_m \) for some arbitrary value of \( \pi \) rather than optimal values. By referring to eq. (2.1) and eq. (2.2) we have,

\[
\delta \hat{V}_c = \frac{\pi M}{3} (V_m - \hat{V}_c) \quad (2.3)
\]

and

\[
\delta \hat{V}_m = \frac{\pi (1 - M)}{3} (U + \hat{V}_c - \hat{V}_m) \quad (2.4)
\]

By solving (2.3) and (2.4) simultaneously we have,

\[
\hat{V}_m - \hat{V}_c = \frac{(1 - M)\pi U}{3\delta + \pi} \geq 0 \quad (2.5)
\]

and,

\[
U + \hat{V}_c - \hat{V}_m = \frac{3\delta + M \pi}{3\delta + \pi} U > 0 \quad (2.6)
\]

By combining inequality (2.6) with eq. (2.2) and inequality (2.5) with eq. (2.1) we see that \( \pi_1 = 1 \) is the solution of the money-carrier’s optimisation problem whenever \( \pi_0 > 0 \) and \( \pi_0 = 1 \) is the solution of the coconut-carrier’s optimisation problem whenever \( \pi_1 > 0 \). Taken together this means that \( \pi_0 = \pi_1 = 1 \) is an equilibrium of our economy. Money buys goods and goods buy money. \( \Box \)
2.3.5 Welfare

Non-monetary equilibrium

**Proposition 3.** In the equilibrium with $\pi = 0$, we have $V_c = V_m = 0.$

*Proof.* This immediately follows from eq (2.2) and eq. (2.1).

Intuitively, this is a consequence of our assumptions of no commodity-commodity trade and the structure of preferences where agents desire commodities other than those they produce.

Monetary equilibrium

With $\pi = 1$ we have from the same equations,

$$V_c = \frac{M}{3\delta} \left[ \frac{(1 - M)}{3\delta + 1} \right] U > 0$$

and,

$$V_m = \frac{(1 - M)}{3\delta} \left[ \frac{3\delta + M}{3\delta + 1} \right] U > 0$$

More interestingly, since a proportion $M$ of agents carry money and a proportion $(1 - M)$ of them carry commodities, the expected social welfare\(^3\) of

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\(^3\)The expected social welfare may be interpreted as the expected utility of an agent who knows he will be born into our island community but who doesn’t know whether he will be cast into the role of a money-carrier or commodity-carrier initially. Hence it can be used as a metric of the desirability of our social arrangement.
an agent is,

\[ W = MV_m + (1 - M)V_c \]
\[ = \frac{U(1 - M)M}{3\delta} \]  

(2.9)

Thus we see that welfare is quadratic in the quantity of money, increasing till \( M = 1/2 \) and decreasing thereafter. This is a result of two contradictory tendencies. Beginning from \( M = 0 \), introduction of money increases welfare since it allows trade to take place where it had been impossible before. But since in our economy carrying money and carrying coconuts is mutually exclusive, more agents carrying money also means less agents carrying coconuts and hence reduces the possibility of finding something to consume. For \( M > 1/2 \) is is the latter effect which predominates.

While the details of this results depend a great deal on the specificities of this model, it can be taken to pointing to a contradiction between the role of money in providing liquidity and the transaction costs associated with running an efficient payments system.

\[ 2.4 \text{ Conclusion} \]

While the search-theoretic model of this chapter could demonstrate how an intrinsically valueless object could become acceptable in exchange, it did so under extremely restrictive assumptions. For our purposes, the most restrictive of these was that of stationarity, which prevented us from studying
the questions at the heart of monetary economics—why the value of money changes, how do those changes affect other economic variables and welfare, how might the changes be controlled? Indeed, the assumption of indivisibility means that there is no meaningful way to talk about the value of money within the context of this model at all.

There have been attempts to remove these restrictions while remaining within the framework of game theory and explicit search. However, gains in generality come at a high cost in tractability using this approach. In this study, we take the alternative path of ignoring the actual mechanism of exchange—assuming that it functions perfectly efficiently—and then seeing how changes in the value of money interact with the rest of the economy.

With this goal in mind, we turn to the study of the polar opposite case of a perfectly cashless economy in the next chapter.