CHAPTER 8

STOCHASTIC MULTI-OBJECTIVE HYDRO THERMAL SCHEDULING-MODELED USING WEIGHTING METHOD

8.1 INTRODUCTION

In various real life problems a decision maker is faced with multiple goals. This situation could be overcome by formulating a multi-objective optimization problem in which the goal is to maximize or minimize several objective functions simultaneously. Short range hydro thermal scheduling problem is concerned with optimization over a time horizon of a day or week. The main objective of this problem is to fully utilize the available water for hydro generation and to reduce the fuel cost of thermal generation to meet the load for every time period subject to various other constraints. Each hydro plant is constrained by the amount of water available for draw down in the interval.

In real time operation power system load is random. As there are many inaccuracies and uncertainties in the input information due to load forecasting errors and measurement errors the problem is formulated as stochastic multi-objective and the system is represented as a network characterized by random variables. This work proposes a simple robust method which makes use of particle swarm optimization technique to solve the multi-objective short-range fixed head hydrothermal scheduling problem.
8.2 PROBLEM FORMULATION

Consider an electric power system having $M$ total number of generating units, $N$ thermal generating plants and $M-N$ hydro plants. The basic problem is to find the active power generation of each plant in the system as a function of time over a finite time interval from 0 to $T$.

8.2.1 Stochastic Thermal Model

The objective function to be minimized is the total system operating cost represented by the fuel cost of thermal generation over the optimization interval.

$$F_1 = \int_0^T \left( \sum_{i=1}^{N} a_i P_i^2 + b_i P_i + c_i \right) dt$$  \hspace{1cm} (8.1)

where $F_1$ is the total cost

$a_i, b_i, c_i$ are the cost coefficients of the $i^{th}$ generator

$P_i$ the power generated by $i^{th}$ plant.

A stochastic model of function $F_1$ is formulated by considering errors in coefficient of input output characteristic and load demand during each subinterval as random variables. Any possible deviation in coefficient of input output characteristic and load demand from their expected values are manipulated through the randomness of generated power $P_i$. The random variables are assumed normally distributed and statistically independent. The expected value of operating cost is:

$$E(F_1) = E \left[ \int_0^T \left( \sum_{i=1}^{N} a_i P_i^2 + b_i P_i + c_i \right) dt \right]$$ \hspace{1cm} (8.2)
Which can be simplified like equation (5.4) and written as equation (8.3)

$$\bar{F}_i = \int \left[ \sum_{i=1}^{N} \bar{a}_i \bar{P}_i^2 + \bar{b}_i \bar{P}_i + \bar{c}_i + \bar{a}_i \var(P_i) \right] dt \quad (8.3)$$

where $\bar{F}_i$ is the expected total cost

$\bar{a}_i, \bar{b}_i, \bar{c}_i$ are the expected cost coefficients of the $i^{th}$ generator

$\bar{P}_i$ is the expected power generation of the $i^{th}$ generator.

The variance of power $P_i$ is given as:

$$\text{var} (P_i) = C_{P_i}^2 \bar{P}_i^2, \quad i = 1...N \quad (8.4)$$

where $C_{P_i}$ is coefficient of variation of random variable $P_i$.

The zero value of the coefficient of variation means no randomness or in other words there is complete certainty about the value of the random variable.

### 8.2.2 Stochastic Emission Model

Thermal power stations are the major cause of atmospheric pollution because of high concentration of $SO_2$, $CO_2$, and $NO_x$ emissions. Emission curves for a thermal plant can be directly related to the cost curve through emission rate per Mega Joule which is a constant for a given type of fuel. So $NO_x$, $CO_2$, and $SO_2$ emission curve are quadratic in terms of active power generation.
The NO\textsubscript{x} emission is estimated as:

\[ F_2 = \int_0^T \sum_{i=1}^N (d_{ii}P_i^2 + e_{ii}P_i + f_{ii}) \, dt \]  
(8.5)

where \( F_2 \) is the total NO\textsubscript{x} emission
\( d_{ii}, e_{ii}, f_{ii} \) are the NO\textsubscript{x} emission coefficients of the \( i\text{th} \) unit.

The SO\textsubscript{2} emission becomes:

\[ F_3 = \int_0^T \sum_{i=1}^N (d_{2i}P_i^2 + e_{2i}P_i + f_{2i}) \, dt \]  
(8.6)

where \( F_3 \) is the total SO\textsubscript{2} emission
\( d_{2i}, e_{2i}, f_{2i} \) are the SO\textsubscript{2} emission coefficients of the \( i\text{th} \) unit.

The CO\textsubscript{2} emission becomes:

\[ F_4 = \int_0^T \sum_{i=1}^N (d_{3i}P_i^2 + e_{3i}P_i + f_{3i}) \, dt \]  
(8.7)

where \( F_4 \) is the total CO\textsubscript{2} emission
\( d_{3i}, e_{3i}, f_{3i} \) are the CO\textsubscript{2} emission coefficients of the \( i\text{th} \) unit.

Stochastic model of the pollutants is formed by considering the power generated and load demand as random variables. Any possible deviation of NO\textsubscript{x}, CO\textsubscript{2}, and SO\textsubscript{2} emission coefficients and load demand from their expected values are managed through the random power generated \( P_i \). Presuming that random variables are normally distributed and statistically independent the expected values of emission are estimated.
Expected value of NO\textsubscript{x} emission is:

\[
\bar{F}_2 = \int \sum_{i=1}^{N} \left( d_{ii} \bar{P}_i^2 + e_{ii} \bar{P}_i + f_{ii} + \overline{d}_{ii} \text{var}(P_i) \right) dt
\]  
(8.8)

where \( \bar{F}_2 \) is the total expected NO\textsubscript{x} emission

\( d_{ii}, e_{ii}, f_{ii} \) are the expected NO\textsubscript{x} emission coefficients of the \( i^{th} \) unit.

The expected value of SO\textsubscript{2} emission:

\[
\bar{F}_3 = \int \sum_{i=1}^{N} \left( d_{2i} \bar{P}_i^2 + e_{2i} \bar{P}_i + f_{2i} + \overline{d}_{2i} \text{var}(P_i) \right) dt
\]  
(8.9)

where \( \bar{F}_3 \) is the total expected SO\textsubscript{2} emission

\( d_{2i}, e_{2i}, f_{2i} \) are the expected SO\textsubscript{2} emission coefficients of the \( i^{th} \) unit.

The expected value of CO\textsubscript{2} emission:

\[
\bar{F}_4 = \int \sum_{i=1}^{N} \left( d_{3i} \bar{P}_i^2 + e_{3i} \bar{P}_i + f_{3i} + \overline{d}_{3i} \text{var}(P_i) \right) dt
\]  
(8.10)

where \( \bar{F}_4 \) is the expected CO\textsubscript{2} emission

\( d_{3i}, e_{3i}, f_{3i} \) are the expected CO\textsubscript{2} emission coefficients of the \( i^{th} \) unit.

8.2.3 Stochastic Hydro Model

In a short-range hydrothermal scheduling problem an insignificant fuel cost is incurred in the operation of hydro units. The input output characteristic of a hydro generator is expressed by the variation of water discharge \( q(t) \) as a function of power output \( P_j \) and net head \( h \). Each hydro
The hydro plant has the amount of water available as constraint for the optimization interval.

\[
\int_0^T \dot{q}_j dt = R_j, \quad j = N + 1 \ldots M \tag{8.11}
\]

where \( q_j \) is the water discharge

\( R_j \) is the predefined volume of water available for the \( j^{th} \) hydro plant.

The performance of \( q_j \) is represented by:

\[
q_j = x_j P_j^2 + y_j P_j + z_j, \quad j = N + 1 \ldots M \tag{8.12}
\]

where \( x_j, y_j, z_j \) are the discharge coefficients of the \( j^{th} \) hydro plant

\( P_j \) is the power generation of \( j^{th} \) hydro plant

Since the thermal generations and load demand are random the hydro generations also become random in view of the load demand constraint given by equation (8.26). A stochastic model of function \( q_j \) is developed by considering the hydro generation and discharge coefficients during each subinterval to be random variables.

\[
\bar{q}_j = \bar{x}_j \bar{P}_j^2 + \bar{y}_j \bar{P}_j + \bar{z}_j + \bar{x}_j \text{Var}(P_j), \quad j = N + 1 \ldots M \tag{8.13}
\]

where \( \bar{x}_j, \bar{y}_j, \bar{z}_j \) are the expected discharge coefficients of the \( j^{th} \) hydro plant

\( \bar{P}_j \) is the expected power generation of \( j^{th} \) hydro plant
8.2.4 Expected Deviation

The solution will provide only the expected values of power generations. By virtue there will be a mismatch in load demand. The variance of a random variable quantifies the degree of uncertainty associated with the mean value of the random variable. The expected mismatch can be estimated through minimization of the squared error of the unsatisfied power demand. Correlations between generations exist because of the need to balance the active power demand of the system.

\[ \overline{P}_5 = \text{var} \left( \sum_{i=1}^{M} (P_i) \right) = E \left[ \sum_{i=1}^{M} \overline{P}_i - (\overline{P}_D + \overline{P}_L) \right]^2 \] (8.14)

where \( \overline{P}_D \) is the expected load demand

\( \overline{P}_L \) is the expected transmission losses

\( \sum_{i=1}^{N} \overline{P}_i \) is the total power generation.

Substituting power balance equation (8.26) in equation (8.14) we get

\[ \overline{P}_5 = E \left[ \sum_{i=1}^{M} P_i - \sum_{i=1}^{M} \overline{P}_i \right]^2 \] (8.15)

\[ \overline{P}_5 = E \left[ \sum_{i=1}^{M} (P_i - \overline{P}_i) \right]^2 \] (8.16)

\[ \overline{P}_5 = E \left[ \sum_{i=1}^{M} (P_i - \overline{P}_i)^2 + 2 \sum_{i=1}^{M} \sum_{j=1}^{M} (P_i - \overline{P}_i)(P_j - \overline{P}_j) \right] \] (8.17)

where \( E(\overline{P}_i - P_i)(\overline{P}_j - P_j) \) is the covariance of power.
\[ \bar{F}_5 = \int_0^T \left( \sum_{i=1}^M \text{var}(P_i) + \sum_{i=1}^M \sum_{j=1, j \neq i}^M 2 \text{cov}(P_i, P_j) \right) dt \]  

(8.18)

The variance of power \( P_i \) is given by equation (8.3).

Covariance of power \( P_i \) and \( P_j \) is given by equation (8.15) as

\[ \text{Cov}(P_i, P_j) = R_{P_i P_j} C_{P_i} C_{P_j} \bar{P}_i \bar{P}_j, \quad i = 1 \ldots M, \ j = 1 \ldots M, (i \neq j) \]  

(8.19)

where \( R_{P_i P_j} \) are correlation coefficient of random variables \( P_i \) and \( P_j \).

Substituting for variance and covariance, equation (8.18) can be written as equation (8.20)

\[ \bar{F}_5 = \int_0^T \left( \sum_{i=1}^M C_{P_i}^2 \bar{P}_i^2 + \sum_{i=1}^M \sum_{j=1, j \neq i}^M R_{P_i P_j} C_{P_i} C_{P_j} \bar{P}_i \bar{P}_j \right) dt \]  

(8.20)

### 8.2.5 Expected Transmission Losses

A common approach to model transmission losses in the system is to use krons approximated loss formula through B coefficients.

\[ P_L = \sum_{i=1}^M \sum_{j=1}^M P_i B_{ij} P_j \]  

(8.21)

where \( P_L \) is the transmission loss

\( B_{ij} \) is the B-coefficients.
The evaluation of B coefficients is very sensitive to operating conditions. So B-coefficients will become random because the generation levels are random. With normally distributed random variables the expected transmission losses are given by equation (8.25). Correlations between generations exist because of the need to balance the active power demand of the system.

\[
E(P_L) = E \left( \sum_{i=1}^{M} \sum_{j=1}^{M} P_i B_{ij} P_j \right) \quad (8.22)
\]

\[
\bar{P}_L = E \left( \sum_{i=1}^{M} P_i^2 B_{ii} \right) + E \left( \sum_{j=1}^{M} \sum_{j \neq i}^{M} P_i B_{ij} P_j \right) \quad (8.23)
\]

\[
\bar{P}_L = \sum_{i=1}^{M} \bar{P}_i^2 \bar{B}_{ii} + \sum_{i=1}^{M} \bar{B}_{ii} \text{ var} P_i + \sum_{i=1}^{M} \sum_{j \neq i}^{M} \bar{P}_i \bar{B}_{ij} \bar{P}_j + \sum_{i=1}^{M} \sum_{j \neq i}^{M} \bar{B}_{ij} \text{ cov}(P_i, P_j) \quad (8.24)
\]

where \( \bar{P}_L \) is the expected transmission loss

\( \bar{B}_{ij} \) is the expected loss coefficient.

\[
\bar{P}_L = \sum_{i=1}^{M} (1 + C_{P_i}^2) \bar{B}_{ii} \bar{P}_i^2 + \sum_{i=1}^{M} \sum_{j=1}^{M} (1 + R_{P_i, P_j} C_{P_i} C_{P_j}) \bar{P}_i \bar{B}_{ij} \bar{P}_j \quad (8.25)
\]
8.2.6 Equality and Inequality Constraints

The expected load demand equality constraint

\[ \sum_{i=1}^{M} \bar{P}_i = \bar{P}_D + \bar{P}_L \]  
(8.26)

The expected limits are imposed as

\[ \bar{P}_{i_{\text{min}}} \leq \bar{P}_i \leq \bar{P}_{i_{\text{max}}} \quad i = 1,...,M \]  
(8.27)

where \( \bar{P}_{i_{\text{min}}}, \bar{P}_{i_{\text{max}}} \) are the minimum and maximum power generation limits of \( i^{\text{th}} \) unit.

Multi-objective is formulated as:

Minimize

\[ F_1 = \int_{0}^{T} \left( \sum_{i=1}^{N} \left( a_i \bar{P}_{i_i}^2 + b_i \bar{P}_{i_i} + c_i + a_i \text{var}(P_i) \right) \right) dt \]

\[ F_2 = \int_{0}^{T} \left( \sum_{i=1}^{N} \left( d_{2i} \bar{P}_{i_i}^2 + e_{2i} \bar{P}_{i_i} + f_{2i} + d_{2i} \text{var}(P_i) \right) \right) dt \]

\[ F_3 = \int_{0}^{T} \left( \sum_{i=1}^{N} \left( d_{3i} \bar{P}_{i_i}^2 + e_{3i} \bar{P}_{i_i} + f_{3i} + d_{3i} \text{var}(P_i) \right) \right) dt \]

\[ F_4 = \int_{0}^{T} \left( \sum_{i=1}^{N} \left( d_{4i} \bar{P}_{i_i}^2 + e_{4i} \bar{P}_{i_i} + f_{4i} + d_{4i} \text{var}(P_i) \right) \right) dt \]

\[ F_5 = \int_{0}^{T} \left( \sum_{i=1}^{M} C_{P_i} \bar{P}_{i_i}^2 + \sum_{i=1}^{M} C_{P_j} \bar{P}_i \bar{P}_j \right) dt \]  
(8.28a)
Subject to

\[ \sum_{i=1}^{M} \bar{P}_i = \bar{P}_D + \bar{P}_L \]

\[ \int_{0}^{T} q_j dt = \bar{R}_j \quad j = N + 1 \ldots M \]

\[ \bar{P}_{i}\text{\textsuperscript{min}} \leq \bar{P}_i \leq \bar{P}_{i}\text{\textsuperscript{max}} \quad i = 1 \ldots M \]

\[ \sum_{k=1}^{S} w_k = 1 \quad w_k \geq 0 \quad (8.28b) \]

To generate the non-inferior solution of multi objective optimization problem, PSO is used. \( w_k \) are the levels of the weighting coefficients. The approach yields meaningful result to the decision maker when solved many times for different values of \( w_k \) (\( k = 1, 2 \ldots n \)). The values of weighting coefficients vary from 0 to 1 for each objective. For each combination of weight the results are obtained. This provides \( K \) number of non-inferior solution which are pareto optimal non dominating. To identify one optimal solution out of \( K \) solution fuzzy membership satisfaction index \( \mu_D^k \) can be used. This can be done off-line for all possible loads and the optimal set of weight for each load can be stored in memory. In real time application the optimal weight combination can be obtained from memory and included in the objective function to be minimized.

8.3 **PSO ALGORITHM FOR HYDROTHERMAL POWER DISPATCH**

The following steps are followed in PSO for obtaining the solution of deterministic and stochastic hydrothermal power dispatch.
**Step 1:** Read the total number of units $M$, the number of thermal units $N$, Number of hydro units $M-N$, total number of subintervals $T$, expected load for all sub interval, expected cost coefficients of thermal plant, expected emission coefficients of thermal plant, expected discharge coefficients of hydro plant, expected demand for each subinterval, pre specified available water, coefficient of variation, correlation coefficients of random variables, maximum number of iterations, population size, acceleration constants $c_1$ and $c_2$, inertia weight $w_{\text{min}}$ and $w_{\text{max}}$.

**Step 2:** Confine the search space. Specify the lower and upper limits of each decision variable.

**Step 3:** Initialize the individual of population. The velocity and position of each particle should be initialized with in the feasible decision variable space $X_i=[X_1,X_2,X_3,\ldots,X_N]$.

**Step 4:** Feed or generate the weight ($w_k=1,2\ldots n$) where $n$ is the number of objectives.

**Step 5:** Set iteration count $t=0$

**Step 6:** The discharge for each interval is scheduled based on the load for that interval and such that the sum of discharge for the whole period is equal to the pre specified water available.

**Step 7:** For each individual $X_i$ of the population the transmission loss $P_{L_i}$ is calculated by using B- coefficient.

**Step 8:** Evaluate the fitness of each individual $X_i$ in terms of pareto dominance.
**Step 9**: Record the non dominated solutions found so far and save them in archive.

**Step 10**: Initialize the memory of each individual where the personnel best position \( p^{(t)}_{\text{best id}} \) is stored.

**Step 11**: Find the best particle out of the population and store its position as \( g^{(t)}_{\text{best d}} \).

**Step 12**: Set iteration count \( t=1 \)

**Step 13**: Update the velocity of each particle \( X_i \) using the equation (4.1).

**Step 14**: Update the position of each particle \( X_i \) using the equation (4.2).

**Step 15**: Check whether the new particles are within the feasible region. If any element violates its inequality constraints then the position of the individual is fixed to its minimum/maximum operating points according to equation (4.5).

**Step 16**: Check the power balance and pre-specified water availability constraints. Penalty is added to particles violating constraints.

**Step 17**: Calculate the fitness function of the new individual.

**Step 18**: Update memory of each particle using the equation (4.6). Compare particles current fitness \( X^{(t+1)}_{\text{id}} \) with particles \( p^{(t)}_{\text{best id}} \). If the current value is better then the previous value then set \( p^{(t+1)}_{\text{best id}} \) to the new value and its location equal to current location in d dimensional space. Compare current
\(X_{id}^{(t+1)}\) with population’s overall best \(g_{best}^{(t)}\). If the current value is better than reset \(g_{best}^{(t+1)}\) to current particles array index and value.

**Step 19**: Update the archive which stores the non dominated solution.

**Step 20**: Iteration = Iteration +1

**Step 21**: The algorithm repeats Step 13 to Step 20 until a sufficient good fitness or a maximum number of iterations/epochs are reached. Once terminated, the algorithm outputs the points of \(g_{best}^{(t)}\) and \(f(g_{best}^{(t)})\) as its solution.

### 8.4 TEST SYSTEM AND RESULTS

A short-range hydrothermal load-scheduling problem for 24 hour duration is considered. The entire optimization period has been divided into 24 intervals each of one hour. The results obtained using PSO are compared with that obtained by Approximated Newton-Raphson iterative method which linearizes the coordination equation. Fuzzy decision index is proposed to provide the best compromise solution among all objectives and constraints. If the coefficient of variation is zero, random variables are uncorrelated to each other, than the problem is considered to be deterministic. One thermal plant and one hydro plant system is considered Dhillon et al (2002). The same algorithm can be used for higher order system also. The deterministic results are shown in Table 8.1. The deterministic results for each subinterval are presented in Table 8.2. Five different stochastic cases are considered with different coefficient of variation and correlation coefficient and the results are
tabulated in Table 8.3. Weight vector and $\mu^k_D$ for the cases considered in the test system is given in Table 8.4. The stochastic result’s for each subinterval (Case1) is given in Table 8.5.

Expected cost characteristics of thermal plant ($/h)$:

$$\bar{F}_{11} = 0.001991 \bar{P}_1 + 9.606 \bar{P}_1 + 373.7$$

Expected NOx, SO2, CO2 emission characteristics (kg/h)

$$\bar{F}_{21} = 0.006483 \bar{P}_1 - 0.79027 \bar{P}_1 + 28.82488$$

$$\bar{F}_{31} = 0.00232 \bar{P}_1 + 3.84632 \bar{P}_1 + 182.2605$$

$$\bar{F}_{41} = 0.084025 \bar{P}_1 - 2.9445484 \bar{P}_1 + 137.7043$$

Expected hydro plant characteristics (Mm3/h)

$$\bar{q}_2 = 2.19427 \times 10^{-5} \bar{P}_2 - 2.5709 \times 10^{-4} \bar{P}_2 + 1.742333$$

$$\bar{R}_2 = 72.4797 \text{ Mm}^3$$

Expected B-coefficients (MW)$^{-1}$

$$\bar{B}_{11} = 0.00005, \bar{B}_{12} = \bar{B}_{21} = 0.00001, \bar{B}_{22} = 0.00015$$

<table>
<thead>
<tr>
<th>Table 8.1 Deterministic results (24 hours)</th>
</tr>
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<tbody>
<tr>
<td>Cost ($$$)</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>95838.05</td>
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Table 8.2  Deterministic results for each subinterval

<table>
<thead>
<tr>
<th>Hour</th>
<th>$P_D$  (MW)</th>
<th>Thermal generation (MW)</th>
<th>Hydro generation (MW)</th>
<th>Water discharge (Mm$^3$/h)</th>
<th>$P_L$  (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>243.58</td>
<td>2.9565</td>
<td>12.5</td>
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<td>2</td>
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<td>239.28</td>
<td>195.25</td>
<td>2.5036</td>
<td>9.52</td>
</tr>
<tr>
<td>3</td>
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<td>203.41</td>
<td>2.5729</td>
<td>9.55</td>
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<td>254.28</td>
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<td>18.88</td>
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<td>25.03</td>
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<td>289.29</td>
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<td>299.31</td>
<td>3.6111</td>
<td>25.03</td>
</tr>
<tr>
<td>20</td>
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<td>272.90</td>
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<td>224.38</td>
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<td>22</td>
<td>585.0</td>
<td>354.81</td>
<td>247.42</td>
<td>3.0020</td>
<td>17.23</td>
</tr>
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<td>23</td>
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<td>312.83</td>
<td>242.41</td>
<td>2.9495</td>
<td>15.22</td>
</tr>
<tr>
<td>24</td>
<td>503.0</td>
<td>317.47</td>
<td>197.67</td>
<td>2.5289</td>
<td>12.16</td>
</tr>
</tbody>
</table>
Table 8.3 Comparison of stochastic results (24 hours)

<table>
<thead>
<tr>
<th>Test cases</th>
<th>Cost ($)</th>
<th>NO\textsubscript{x} emission (kg)</th>
<th>SO\textsubscript{2} emission (kg)</th>
<th>CO\textsubscript{2} emission (kg)</th>
<th>Risk (MW$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>C\textsubscript{Pi} = 0.01 R\textsubscript{PiPj} = 0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>96229.0</td>
<td>14470.17</td>
<td>44126.68</td>
<td>243585.25</td>
<td>462.87</td>
</tr>
<tr>
<td>NR</td>
<td>96386.43</td>
<td>14360.20</td>
<td>44156.03</td>
<td>242124.12</td>
<td>460.4</td>
</tr>
<tr>
<td>Case 2</td>
<td>C\textsubscript{Pi} = 0.05 R\textsubscript{PiPj} = 0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>96323.81</td>
<td>14525.01</td>
<td>44178.99</td>
<td>244355.71</td>
<td>11557.55</td>
</tr>
<tr>
<td>NR</td>
<td>96480.82</td>
<td>14442.96</td>
<td>44214.58</td>
<td>243249</td>
<td>11516.6</td>
</tr>
<tr>
<td>Case 3</td>
<td>C\textsubscript{Pi} = 0.10 R\textsubscript{PiPj} = 0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>96829.55</td>
<td>14728.8</td>
<td>44345.40</td>
<td>247196.9</td>
<td>46291.5</td>
</tr>
<tr>
<td>NR</td>
<td>96841.30</td>
<td>14686.73</td>
<td>44410.25</td>
<td>246600</td>
<td>46125.7</td>
</tr>
<tr>
<td>Case 4</td>
<td>C\textsubscript{Pi} = 0.10 R\textsubscript{PiPj} = 1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>96404.22</td>
<td>14822.69</td>
<td>44338.3</td>
<td>248658.7</td>
<td>88818.19</td>
</tr>
<tr>
<td>NR</td>
<td>96735.22</td>
<td>14726.10</td>
<td>44386.10</td>
<td>247042</td>
<td>88802.8</td>
</tr>
<tr>
<td>Case 5</td>
<td>C\textsubscript{Pi} = 0.10 R\textsubscript{PiPj} = -1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>96547.22</td>
<td>14710.28</td>
<td>44208.62</td>
<td>247457</td>
<td>3701.0</td>
</tr>
<tr>
<td>NR</td>
<td>96802.52</td>
<td>14687.49</td>
<td>44405.48</td>
<td>246601</td>
<td>3455.5</td>
</tr>
</tbody>
</table>

Table 8.4 Weight vector and $\mu^k_D$ for the cases considered in test system 1

<table>
<thead>
<tr>
<th>Test cases</th>
<th>PSO $\mu^k_D$</th>
<th>$\mu^k_D$ weight</th>
<th>NR $\mu^k_D$</th>
<th>$\mu^k_D$ weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.008264</td>
<td>0.6,0.1,0.1,0.1,0.1</td>
<td>0.008165</td>
<td>0.4,0.1,0.2,0.2,0.1</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.008279</td>
<td>0.5,0.1,0.2,0.1,0.1</td>
<td>0.008178</td>
<td>0.5,0.2,0.1,0.1,0.1</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.008224</td>
<td>0.4,0.1,0.2,0.2,0.1</td>
<td>0.008216</td>
<td>0.4,0.1,0.3,0.1,0.1</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.008958</td>
<td>0.5,0.1,0.2,0.1,0.1</td>
<td>0.009156</td>
<td>0.6,0.1,0.1,0.1,0.1</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.008269</td>
<td>0.5,0.1,0.2,0.1,0.1</td>
<td>0.008264</td>
<td>0.5,0.1,0.2,0.1,0.1</td>
</tr>
</tbody>
</table>
Table 8.5  Stochastic results for each subinterval (Case 1)

<table>
<thead>
<tr>
<th>Hour</th>
<th>$\overline{P}_D$ (MW)</th>
<th>Thermal generation (MW)</th>
<th>Hydro generation (MW)</th>
<th>Water discharge (Mm$^3$/h)</th>
<th>$\overline{P}_L$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>455.0</td>
<td>238.77</td>
<td>227.94</td>
<td>2.8239</td>
<td>11.73</td>
</tr>
<tr>
<td>2</td>
<td>425.0</td>
<td>210.00</td>
<td>225.16</td>
<td>2.7970</td>
<td>10.76</td>
</tr>
<tr>
<td>3</td>
<td>415.0</td>
<td>232.75</td>
<td>191.34</td>
<td>2.4966</td>
<td>9.09</td>
</tr>
<tr>
<td>4</td>
<td>407.0</td>
<td>228.73</td>
<td>187.02</td>
<td>2.4618</td>
<td>8.72</td>
</tr>
<tr>
<td>5</td>
<td>400.0</td>
<td>226.93</td>
<td>181.39</td>
<td>2.4178</td>
<td>8.33</td>
</tr>
<tr>
<td>6</td>
<td>420.0</td>
<td>218.64</td>
<td>211.36</td>
<td>2.6684</td>
<td>10.02</td>
</tr>
<tr>
<td>7</td>
<td>487.0</td>
<td>269.64</td>
<td>230.00</td>
<td>2.8440</td>
<td>12.81</td>
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<tr>
<td>8</td>
<td>604.0</td>
<td>350.00</td>
<td>273.11</td>
<td>3.3090</td>
<td>19.23</td>
</tr>
<tr>
<td>9</td>
<td>665.0</td>
<td>452.49</td>
<td>233.00</td>
<td>2.8738</td>
<td>20.49</td>
</tr>
<tr>
<td>10</td>
<td>675.0</td>
<td>464.25</td>
<td>231.53</td>
<td>2.8592</td>
<td>20.97</td>
</tr>
<tr>
<td>11</td>
<td>695.0</td>
<td>413.75</td>
<td>306.40</td>
<td>3.7238</td>
<td>25.18</td>
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<tr>
<td>12</td>
<td>705.0</td>
<td>461.32</td>
<td>266.44</td>
<td>3.2317</td>
<td>23.75</td>
</tr>
<tr>
<td>13</td>
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<td>338.50</td>
<td>258.98</td>
<td>3.1476</td>
<td>17.55</td>
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<tr>
<td>14</td>
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<td>258.27</td>
<td>3.1397</td>
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<td>15</td>
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<td>366.36</td>
<td>269.26</td>
<td>3.2641</td>
<td>19.56</td>
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<tr>
<td>16</td>
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<td>411.69</td>
<td>262.25</td>
<td>3.1842</td>
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<td>457.13</td>
<td>289.38</td>
<td>3.5056</td>
<td>25.66</td>
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<tr>
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<td>469.84</td>
<td>297.33</td>
<td>3.606</td>
<td>27.09</td>
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<td>479.79</td>
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<td>2.9710</td>
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<td>292.20</td>
<td>3.5409</td>
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<td>3.2435</td>
<td>20.10</td>
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<td>248.81</td>
<td>3.0369</td>
<td>17.24</td>
</tr>
<tr>
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<td>327.00</td>
<td>227.61</td>
<td>2.8207</td>
<td>14.61</td>
</tr>
<tr>
<td>24</td>
<td>503.0</td>
<td>303.00</td>
<td>212.70</td>
<td>2.6804</td>
<td>12.67</td>
</tr>
</tbody>
</table>
8.5 DISCUSSION

The covariance of bivariate random variables can be considered positive or negative. The covariance is represented by correlation coefficients which may vary from $-1.0$ to $+1.0$. Coefficient of variation of each plant is assumed to vary from 0 to 10%. In order to study the effect of variation of coefficient of variation and correlation coefficient on the objective function five different cases are considered. The values of coefficient of variation and correlation coefficient are assumed. In real system the power system operator can find the optimal solution for the actual values of coefficient of variation and correlation coefficient.

Figure 8.1 shows the percentage deviation of expected cost, expected NO$_x$ emission, expected SO$_2$ emission and expected CO$_2$ emission from their respective deterministic values. The weight $W_1$, $W_2$, $W_3$, $W_4$ and $W_5$ are taken as 0.6, 0.2, 0.1, 0.1, and 0 respectively. Figure 8.1 indicates that

- There is an increase in the percentage relative deviation in total expected cost $\bar{F}_1$ as the value of $R_{P_i P_j}$ ($i \neq j$) is changed from a negative value to a positive value.

- There is a decrease in the percentage deviation in total expected NO$_x$ emission as the value of $R_{P_i P_j}$ ($i \neq j$) is changed from negative to positive value.

- There is a very small change on SO$_2$ emission and CO$_2$ emission emission
Figure 8.1 Percentage deviation against coefficient of correlation

In order to find the effect of variance on an objective, full weightage is given to that objective neglecting others. The variance is represented by coefficient of variation. To find the effect of variance, coefficient of variation is varied. In this test system coefficient of variation is varied from 0 to 10%. The percentage deviation of expected cost, expected NO\textsubscript{x} emission, expected SO\textsubscript{2} emission, expected CO\textsubscript{2} emission from their respective deterministic values with respect to the coefficient of variation is shown in Figure 8.2. The Figure 8.2 indicates that

- Relative percentage deviation of all the objectives increases as the variance increase.

- The effect of variation on operating cost is more compared to other objectives.
Figure 8.2 Percentage deviations against coefficients of variations

8.6 CONCLUSION

Deterministic and stochastic model of multi-objective hydrothermal power dispatch is solved using PSO. From the obtained results the following points are summarized:

- Stochastic model helps to calculate cost accurately considering the variation in power due to inaccuracies and uncertainties.

- When the coefficient of variation increases there is increase in objective function value. It is essential to keep the values of coefficient of variation as small as possible. The expected risk depends on the value of coefficient of variation.
• Among the five cases case 1 is considered as best case as the coefficient of variation is less and the risk associated is less.

• PSO is capable of finding a solution which decreases cost, SO$_2$ emission. There is an increase in CO$_2$, NO$_x$ emission and risk compared to Approximated Newton-Raphson iterative method. The average of the results obtained using PSO is tabulated in Table 8.6.

Table 8.6 Average results obtained using PSO for all five cases

<table>
<thead>
<tr>
<th>In PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost Decreases by</td>
</tr>
<tr>
<td>0.188 %</td>
</tr>
</tbody>
</table>

• The stochastic costs on an average for the cases considered in this problem are 0.5% higher than deterministic case and it requires consideration for the benefit of the utility. Even a small percentage increase in cost will result in huge amount when considered for the whole year.

• The execution time is 185 seconds for 24 hours in Pentium IV 3 GHz system.