Analysis and Modeling of Coplanar Waveguide

5.1 Introduction

Coplanar waveguides (CPW) are a type of planar transmission line used in MICs as well as in MMICs. The CPW is uniplanar in construction and it facilitates easy shunt, as well as series, surface mounting of active and passive devices. It also eliminates the need for wraparound and via holes, and further, it reduces radiation loss. Its characteristic impedance is determined by the ratio of central conductor width and gap between central conductor and ground. Its structure has influence on losses. The CPW is a low dispersion line, so it is suitable for development of wide-band circuits and components [60, 73, 105, 123, 134]. The CPW structure has several variations, like asymmetrical CPW (ACPW), the CPW with finite ground planes, conductor-backed CPW, CPW with top shield etc. These structures are also used with multilayer substrates [43, 64, 105]. Furthermore, the CPW structures have also been constructed on non-planar substrates like circular cylindrical and elliptical cylindrical structures [19, 52, 55, 84, 90, 105, 130].

Several rigorous methods, including the spectral domain analysis (SDA), and experimental investigations have been used to determine the line parameters of the standard CPW line [73, 105]. The commercial EM-simulators, such as Ansoft HFSS, Sonnet, CST Microwave Studio etc., are in use to analyze the CPW based circuits. However, these are not convenient for interactive design CAD tool of the CPW structures. The conformal mapping method, discussed in Chapter-3, is used to get design oriented expressions to compute the static characteristic impedance and static effective relative permittivity of varieties of CPW structures, both planar and non-planar, single-layer substrate and multilayer substrate. However, for most of the CPW structures
dispersion is not considered through closed-form expressions, except in the case of standard planar CPW on finite thickness single-layer substrate with infinite extent ground planes [18,73,105]. Likewise, losses have been computed only for this standard planar CPW [21,48,96]. Even the dispersion expression of the standard planar CPW does not account for finite thickness of the strip conductors. Thus there is need to extend the dispersion expression with finite strip conductor thickness to the planar and non-planar CPW under multilayer environment. The methods to compute dielectric loss and conductor loss are also to be extended for these more general CPW structures.

In this chapter, we present the analysis and modeling of conductor thickness and frequency dependent line parameters of different configurations of CPW. Their design and analysis are discussed in detail. Two different methods for conductor loss computation i.e. Wheeler’s incremental inductance formulation and perturbation method with the concept of stopping distance have been used. The effects of strip asymmetry, top shield and conductor backing on the characteristics of CPW are also analyzed. All the developed models are further extended to the non-planar (elliptical and circular cylindrical) CPW. The models are used with the multilayered substrates, both planar and non-planar, with the help of single layer formulation (SLR), summarized in Chapter-4. Lastly, the circuit models are developed to account for low frequency features. All developed models are verified against available experimental results and the results obtained using the EM- simulators over wide range of parameters.

![Cross section of coplanar transmission line.](image)
5.2 Effect of Conductor Thickness on Characteristics of CPW

Fig. (5.1) shows the CPW with central strip width $2a = s$, ground-to-ground inner spacing $2b = (s + 2w)$, ground-to-ground outer spacing $2c$, substrate thickness $h$ and conductor thickness $t$ with relative permittivity $\varepsilon_r$. The conductor thickness independent expressions for the effective relative permittivity $\varepsilon_{\text{eff}}$ and the characteristic impedance $Z_0$ of the CPW with $c \to \infty$, using the conformal mapping method are obtained from equations-(3.15) and (3.16) respectively.

\[
\varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k_0^{'})}{K(k_0)} \frac{K(k_1^{'})}{K(k_1)} \quad (5.1)
\]

\[
Z_0 = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0^{'})}{K(k_0)} \quad (5.2)
\]

where modulus $k_0$ and $k_1$, along with their complementary modulus $k_0^{'}$ and $k_1^{'},$ are defined in equations-(3.10) -(3.12). However, we modify these expressions empirically to take into account the finite strip conductor thickness $t$ [73]. The equivalent slot-width $w_{eq}$ is reduced slot-width with zero conductor thickness; whereas the equivalent strip-width $s_{eq}$ is enlarged one.

\[
s_{eq} = s + \Delta s \quad (i) \quad w_{eq} = w - \Delta s \quad (ii)
\]

\[
\Delta s = \frac{t}{2\pi\varepsilon_r} \left[1 + \ln \left(\frac{8\pi w}{t}\right)\right] \quad (iii) \quad \text{Alternatively}
\]

\[
\Delta s = \frac{t}{\pi} \left[1 + \ln(4) - 0.5\ln \left(\frac{t}{h} + \left(\frac{t}{2\pi s}\right)^2 \frac{8\pi w}{t}\right)\right] \quad (iv)
\]

The aspect-ratio $k_0$ and $k_1$ are replaced by the modified aspect ratio $k_0,t$ and $k_1,t$, respectively, that takes care of the strip conductor thickness.
Analysis and Modeling of CPW

\[ k_{0,t} = \frac{s_{eq}}{s_{eq} + 2w_{eq}} \quad (i) \quad k_{0,t}^\prime = \sqrt{1 - k_{0,t}^2} \quad (ii) \] (5.4)

\[ \begin{align*}
  k_{1,t} &= \frac{\sinh \left[ \frac{\pi s_{eq}}{4h} \right]}{\sinh \left[ \frac{\pi}{4h}(s_{eq} + 2w_{eq}) \right]} \quad (i) \\
  k_{1,t}^\prime &= \sqrt{1 - k_{1,t}^2} \quad (ii) \end{align*} \] (5.5)

The effective relative permittivity of the CPW with conductor thickness is computed from the following empirical relation [73]

\[ \varepsilon_{\text{eff}}(t) = \varepsilon_{\text{eff}}(t = 0) - \frac{0.7[\varepsilon_{\text{eff}}(t = 0) - 1]t/w}{[K(k_0)/K(k_0')] + 0.7(t/w)} \] (5.6)

The characteristic impedance of the CPW with conductor thickness \( t \) is computed from the equation-(5.2) after using conductor thickness dependent effective relative permittivity and modified aspect-ratio [18]:

\[ Z_0(t) = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}(t) K(k_0,t) K(k_0,t) / K(k_0,t) K(k_0,t)}} \] (5.7)

The conductor thickness has more effect on \( Z_0 \) as compared to its effect on \( \varepsilon_{\text{eff}} \) [22].

5.3 Dispersion in CPW

Frankel et. al. [91] verified the analytic dispersion expressions for the CPW experimentally that take into account the transmission line geometry and the substrate thickness. However, thickness of the strip conductor has not been considered. Moreover the dispersion expression does not account for the dispersion at low frequency due to the field penetration in the finite conductivity strip conductors. Gevorgian et. al. [150] has
suggested such expression to include the finite conductivity and finite conductor thickness in propagation characteristics. In this section we summarize these results and incorporate conductor thickness in the dispersion relation.

5.3.1 Effective Relative Permittivity

Gevorgian et al. [150] has adopted dispersion relation [91] to the shielded and conductor-backed CPW. Further, they have considered the finite thickness and also the finite conductivity of the strip conductors to get the improved dispersion model. The total inductance of a CPW is a sum of external and internal inductances

\[ L = L_{\text{Ext}} + L_{\text{Int}} \]  

(5.8)

The internal inductance is due to the penetration of magnetic fields in the strip conductors given by the skin depth, \( \delta_s = 1/\sqrt{\mu_0 \pi \sigma} \), where \( \sigma \) is conductivity of the strip conductors. The magnetic field penetration of \( \delta_s/2 \) depth in the conductor is shown in Fig (5.2) for a thin film strip conductor, \( t \leq 3\delta_s \).

![Fig.(5.2): Geometry of CPW with magnetic field penetration](image)

The total line inductance \( p.u.l. \) is obtained from equation-

\[ L = \frac{\mu_0}{4} \frac{K(k'_\delta)}{K(k_\delta)} \]

where, \( k_{0,\delta} = \frac{s - \delta_s}{s + 2w + \delta_s} \quad k'_{0,\delta} = \sqrt{1 - k^2_{0,\delta}} \)  

(5.9)
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On using above concept and dispersion relation due to Frankel *et. al.* and Gevorgian *et. al.* we obtained dispersion relation that accounts for the low frequency dispersion due to the internal inductance. They used one multiplying factor \((S)\) with dispersion relation to account for the skin-depth effect at lower frequency. Their expression is not accurate. We have incorporated skin-depth dependent factor \((S)\) with static effective relative permittivity. Further, they did not account for effect of conductor thickness on dispersion at higher frequency. We also incorporated conductor thickness in dispersion expression.

The final modified closed-form expressions for CPW dispersion with skin-depth dependent factor \((S)\) and finite strip thickness is summarized below:

\[
\varepsilon_{\text{eff}}(f, t, \delta) = \left( \sqrt{S \times \varepsilon_{\text{eff}}(f = 0, t)} + \frac{\sqrt{\varepsilon_{r} - \sqrt{S \times \varepsilon_{\text{eff}}(f = 0, t)}}}{1 + m(\frac{f}{f_{TE}})^r} \right)^2 \quad (a)
\]

\[
where \quad S = \left\{ \frac{K(k_{0,\delta}) K(k_{0,f})}{K(k_{0,\delta}) K(k_{0,f})} \right\} \quad (b)
\]

where \(r = 1.8\). However ‘\(r\)’ could be improved by comparing the results against the EM-simulator. In this expression, \(\varepsilon_{\text{eff}}(f = 0, t)\) is the conductor thickness dependent static effective relative permittivity which is computed using equation-(5.6), \(f\) is the frequency as independent variable and \(f_{TE}\) is the surface wave \(TE_{1}\)-mode cut-off frequency, given by

\[
f_{TE} = \frac{c}{4h \sqrt{\varepsilon_{r} - 1}} \quad (5.11)
\]

where \(c\) is the velocity of the EM-wave in the free space and \(h\) is the substrate thickness.

The parameter \(m\) of equation-(5.10) is given by
Fig.(5.3): Comparisons of $\varepsilon_{eff}(f,t)$ computed by the closed-form model against the EM-simulators and the measurement data as a function of: (a) & (b) Frequency, (c) Conductor thickness, and (d) $s/(s+2w)$ ratio for CPW on various substrates.

$$\log(m) = u \log(s_{eq}/w_{eq}) + v$$  \hspace{1cm} (i)

$$u = 0.54 - 0.64q + 0.015q^2$$  \hspace{1cm} (ii)

$$v = 0.43 - 0.86q + 0.54q^2$$  \hspace{1cm} (iii), where $q = \log(s_{eq}/h)$

The above model is valid for frequency, $f > 1/(\mu_0\sigma ts)$. Below this frequency, the current distribution is almost uniform across the strip and the line inductance has to be
computed as the DC inductance [134]. At high frequency, factor $\frac{K(k_{0,\delta})}{K(k_{0})}$ tends to unity as the field penetration is very small and internal inductance role is insignificant.

Fig.(5.3a) shows characteristics of $\varepsilon_{\text{eff}}(f,t)$ in CPW on different substrates with $\varepsilon_r = 2.5, 9.8, 12.9$ and $37$; $s = 9.68 \, \mu$m, $w = 22 \, \mu$m, $h = 635 \, \mu$m and $t = 5 \, \mu$m. It compares results of three EM-simulators- HFSS, Sonnet and CST and closed-form model against the experimental results over frequency range 20 GHz -200 GHz for $\varepsilon_r = 12.9$ [91]. At lower end of frequency, $\varepsilon_{\text{eff}}(f,t)$ increases with decrease in frequency that is due to the internal inductance caused by the penetration of magnetic fields in the strip conductors. A minimum value of $\varepsilon_{\text{eff}}(f,t)$ is observed and then $\varepsilon_{\text{eff}}(f,t)$ increases with frequency as usual. When compared against the EM-simulators in the frequency range 0.1 GHz – 200 GHz, the closed-form model has 5.1% average deviation and 15% maximum deviation at 0.1GHz for $\varepsilon_r = 2.5$. The present closed-form model shows better agreement with experimental results as computed against the results of EM-simulator.

Fig.(5.3b) shows further comparison of the model and the EM-simulators against experimental results of Papapolymerou et. al. [64] over frequency range 2 GHz- 118 GHz. At frequencies below 40 GHz, EM-simulators show better agreement with experimental results. However above 40 GHz, the model shows better agreement. The % average and % maximum deviation in the results obtained from the closed-form model, HFSS, Sonnet and CST against both sets of the experimental results are summarized in Table-5.1a.

Fig.(5.3c) and Fig.(5.3d) show variation in $\varepsilon_{\text{eff}}$ with conductor thickness and the slot gap. The model follows results of the CST more closely. Thus for $2.5 \leq \varepsilon_r \leq 20$, $0.25 \mu$m $\leq t \leq 9 \, \mu$m and $0.2 \leq s/(s+2w) \leq 0.6$, the closed-form model has % average and %
maximum deviation of (4.2%, 7.2%), (5.4%, 8.8%) and (3%, 6.5%) against HFSS, Sonnet and CST respectively.

The electric field distribution associated with CPW in the air and in the substrate are approximately independent of geometry and frequency. Therefore, unlike microstrip, $\varepsilon_{eff}$ is not particularly sensitive to the geometry of the structure in CPW. The slope of $\varepsilon_{eff}(f,t)$ in low frequency range is negative as noted above. However, for $t/\delta \to \infty$, the current flows on surface of the metal and the $\varepsilon_{eff}(f,t)$ increases with increasing frequency. Thus, when $t$ and $\delta$ are of the same order, the internal inductance plays its role and the slope of $\varepsilon_{eff}(f,t)$ becomes negative at the lower frequency end. The dispersion in the CPW is less for a thicker substrate.

### 5.3.2 Characteristic Impedance

The dispersion in the characteristic impedance of the CPW is estimated from the following expression which is based on voltage-current definition:

$$Z_0(f,t) = \frac{30\pi}{\sqrt{\varepsilon_{eff}(f,t)}} \times \frac{1}{S} \left( a \right) \text{ where } S = \frac{K(k_{0,\delta}) K(k_{0,t})}{K(k_{0,\delta}) K(k_{0,t})} \left( b \right)$$

The above expression accounts for the low frequency dispersion in characteristic impedance due to field penetration. It causes increase in characteristic impedance at low frequency due to increase in the internal inductance. Fig.(5.4a) - Fig.(5.4d) compares results on the characteristic impedance of the CPW as obtained by the model, various EM-simulator and also experimental results [131] on different substrates with $\varepsilon_r = 2.5$, 9.8, 20 and 37; $s = 9.68 \mu m$, $w = 22 \mu m$ $h = 635 \mu m$ and $t = 5 \mu m$. Fig.(5.4a) shows significant variation in $Z_0$ for the CPW on $\varepsilon_r = 2.5$, as computed by various simulators.
The closed-form model is within 5.8% average deviation against the results obtained from EM-simulators in the frequency range 0.1 GHz – 200 GHz. The model has 12% maximum deviation in the lower frequency range. Fig.(5.4b) shows comparison of the model and the EM-simulators against SDA-based results of Kitazawa et. al. [120]. It shows that $Z_0(f,t)$ computed by the model decreases with increasing frequency. However, based on the power-voltage relation; somewhat increases with frequency and then it decreases with frequency [110]. At lower frequency end, the characteristic
impedance computed by the model is much higher than the SDA results. However, even
CST results are much less.

Table - 5.1: % Average and maximum deviation of closed-form model and EM-simulators for
CPW on GaAs substrate with $\sigma=3.33\times10^7$ S/m.

<table>
<thead>
<tr>
<th></th>
<th>Closed-form Model</th>
<th>HFSS</th>
<th>Sonnet</th>
<th>CST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. [64]</td>
<td>5.4</td>
<td>15.3</td>
<td>4.9</td>
<td>13.9</td>
</tr>
<tr>
<td>Exp. [91]</td>
<td>5.8</td>
<td>14.8</td>
<td>5.5</td>
<td>15.5</td>
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</tbody>
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<tr>
<th></th>
<th>Closed-form Model</th>
<th>HFSS</th>
<th>Sonnet</th>
<th>CST</th>
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</thead>
<tbody>
<tr>
<td>SDA. [120]</td>
<td>5.7</td>
<td>13.7</td>
<td>1.9</td>
<td>2.1</td>
</tr>
<tr>
<td>Exp. [131]</td>
<td>3.1</td>
<td>7.7</td>
<td>6.7</td>
<td>15.9</td>
</tr>
<tr>
<td>MMM [133]</td>
<td>4.8</td>
<td>6.8</td>
<td>3.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Fig.(5.4c) and Fig.(5.4d) show effect of the conductor thickness and slot-gap on the
characteristic impedance. For $2.5 \leq \varepsilon_r \leq 20$, $0.25 \mu m \leq t \leq 9 \mu m$ and
$0.2 \leq s/(s+2w) \leq 0.6$, the closed-form model has % average and % maximum deviation
of (5.3%,16.9%), (6.2%,12.9%) and (5.6%,17%) against HFSS, Sonnet and CST respectively.
The average and maximum % deviation of the closed-form model and EM-simulators against the SDA [120], experiment [131] and MMM [133] based results are summarized in Table-5.1b. The model is closer to experimental results as compared to the results of the EM-simulators.
The present closed-form models for both \( \varepsilon_{\text{eff}} \) and \( Z_0 \) are accurate to within 5% for the following range of parameters [105]: \( 0.1 < s/w < 5 \), \( 0.1 < s/h < 5 \), \( 1.5 < \varepsilon_r < 50 \) and \( 0 < f/f_{te} < 10 \). Overall, the results of the model follow the full-wave and experimental results, as well as the results of EM-simulators faithfully for both \( \varepsilon_{\text{eff}} \) and \( Z_0 \) in CPW.

5.4 Computation of Losses in CPW

The conductor loss due to the finite conductivity of conducting strip conductors is the main source of power loss in a CPW. The dielectric loss is small due to low loss-tangent of the substrate material. The radiation loss is ignored for most of the practical applications of CPW at the microwave and lower end of the mm-wave. This section presents computation of dielectric loss and conductor loss using the closed-form expressions suitable for the CAD applications.

5.4.1 Dielectric Loss

The dielectric loss in a planar transmission line is due to the EM-wave propagating in a lossy dielectric substrate. The CPW lines are fabricated on the low-loss substrates. The following standard expression is used to compute the dielectric loss [147] of the CPW structures:

\[
\alpha_d = 27.29 \frac{\varepsilon_r}{\sqrt{\varepsilon_{\text{eff}}(f,t)}} \left[ \frac{\varepsilon_{\text{eff}}(f,t)-1}{\varepsilon_r-1} \right] \tan \delta \frac{dB}{unit \ length}
\]  

(5.14)

where, \( \lambda_0 \), \( \tan \delta \) and \( \varepsilon_r \) are free-space wavelength, loss tangent and relative permittivity of the substrate respectively. The frequency and conductor thickness dependent \( \varepsilon_{\text{eff}}(f,t) \) of CPW is computed using equation-(5.10). Fig.(5.5) compares the computation
of $\alpha_d$ by the closed-form model against the EM-simulators w.r.t. conductor thickness over the range 0.25 $\mu$m – 9 $\mu$m. The $\alpha_d$ computed by the model is higher than the results of the EM-simulators. However the model has less than 4.4% average deviation against results of EM-simulators.

![Dielectric loss as a function of conductor thickness for CPW.](image)

**Fig.(5.5): Dielectric loss as a function of conductor thickness for CPW.**

### 5.4.2 Conductor Loss

The conductor loss is caused by the finite conductivity of the strip conductor. This loss is dependent on the skin-depth or the surface resistivity of the conductor, geometry of the structures and on the thickness of strip conductors for $t \leq 3 \delta_s$, where $\delta_s$ is the skin-depth.

The conductor loss is usually a predominant factor to the wave attenuation as the current density near the edge of CPW is quite high. An accurate computation of conductor loss of the CPW is important for the design and development of CPW based devices and circuits.

The field theoretic methods such as quasi-TEM method [134], spectral domain analysis (SDA) [72,120] and mode matching methods etc. [133] are adopted to compute the conductor loss of a CPW. The EM-softwares also help to extract the conductor loss of a
CPW over wide range of parameters. Haydl et. al. [131,136], Ponchak et. al. [49] and Papapolymerou et. al. [64] have experimentally explored the conductor loss of CPW structure in the frequency range 2 GHz - 118 GHz. Some closed-form expressions to compute the conductor losses have also been reported [13, 21, 49, 73].

In this work, two different closed-form models for conductor loss computation in CPW are used:

(i) Wheeler’s incremental inductance formulation
(ii) Improved Holloway and Kuester model (IHK)

The above mentioned closed-form models and the EM-simulators have been tested against the available extensive experimental results in this section.

- **Wheeler’s Incremental Inductance Formulation**

The incremental inductance rule developed by Wheeler [73,103] formulation avoids calculation of current density on the surface. It provides closed-form expression for the conductor loss in terms of the physical parameters of line. This is the greatest strength of this method. However, the method is only applicable to thick strip conductors \( t \geq 1.1 \delta_r \). Thus it does not meet the requirement of present day MMIC technology. Besides microstrip and coupled microstrip line under shielded and layered conditions [66], this method has also been extended to the CPW on the finite thickness substrate in terms of fractional characteristic impedance given below [13]:

\[
\alpha_c = \frac{\pi}{\lambda_0} \sqrt{\epsilon_{\text{eff}}(\epsilon_r, w, s, h, f, t)} \ \frac{A Z(\epsilon_r = 1, w, s, h, f, t, \delta_r)}{Z_0(\text{weq}, \text{seq}, h, f, t, \epsilon_r = 1)} \quad \text{Np/m} \quad (5.15)
\]
where, $\lambda_0$ is free space wavelength. The expressions for the parameters $\varepsilon_{eff}(\varepsilon_r, w, s, h, f, t)$, $Z_0\left(\varepsilon_{eq}, s_{eq}, h, f, t, \varepsilon_r = 1\right)$ and $\Delta Z(\varepsilon_r = 1, w, s, h, f, t, \delta_e)$ are obtained from Verma et. al. [13].

**Improved Holloway and Kuester model**

Holloway and Kuester [21] presented their quasi closed-form model to compute conductor loss of a CPW with infinitely wide ground conductors $c \to \infty$, based on the perturbation method and concept of the stopping distance i.e. edge singularity of current distribution. Fig.(5.6) shows the CPW structure with the stopping distance $\Delta$ measured from the edge of the strip conductors. Holloway and Kuester provided tabular form of results for the stopping distance that is not suitable for the CAD applications. In this section, we give a closed-form expression in terms of stopping distance to compute the conductor loss of a CPW with finite width ground conductors. We also give closed-form expressions for the stopping distance.

![Fig.(5.6): Infinitely thin CPW conductors with stopping distance (\Delta).](image)

The current distribution on the infinitely thin conductor is described by [8, 13, 14]:

$$J = \frac{A}{\sqrt{(a^2 - x^2)(b^2 - x^2)}} \quad |x| < a$$  \hspace{1cm} (a) \hspace{1cm} \frac{A}{\sqrt{(x^2 - a^2)(x^2 - b^2)}} \quad |x| > b$$  \hspace{1cm} (b)$$

where, $A = \frac{bl}{2K(k)}$; $k = \frac{a}{b}$ \hspace{1cm} (c)

\[ 5.16 \]
In the above expression $I$ is the total longitudinal current on the central strip conductor, $k$ is the aspect-ratio of the structure and $K(k)$ is the elliptic integral. The conductor loss of a strip conductor, using the perturbation method is computed by the following expression [21,38]

$$\alpha_c = \frac{R_{sm}}{4Z_0(f,t)} \int \left( \frac{J}{I} \right)^2 dl$$

(5.17)

The above equation takes care of the modified Horton surface impedance both at the top and back sides of the strip conductors. The real part of the surface impedance on both sides of a strip conductor with finite thickness ($t$) is given by [21]

$$R_{sm} = \omega \mu_c t \Im \left( \frac{\cot(k_c t) + \csc(k_c t)}{k_c t} \right)$$

where, $k_c = \omega \sqrt{\mu_0 \varepsilon_0} \left[ 1 - j \frac{\sigma_c}{\omega \varepsilon_0} \right]^{1/2}$

(5.18)

where, $\sigma_c$ is the conductivity of strip conductors in S/m and $k_c$ is the wave number in conductor. Equation-(5.17) as applied to a CPW structure of Fig.(5.6) is reduced to

$$\alpha_c = \frac{R_{sm}}{4Z_0(f,t)} \left[ \int_{-a+\Delta}^{a-\Delta} \left( \frac{J}{I} \right)^2_{\text{central strip}} dl + 2 \int_{b1+\Delta}^{c-\Delta} \left( \frac{J}{I} \right)^2_{\text{lateral gnd}} dl \right]$$

(5.19)

The surface current density $J$ is given by equation- (5.16). The stopping distance $\Delta$ appears in the limits of the integral. The conductor loss diverges logarithmically if the above integration is carried out for $\Delta = 0$ i.e. over the complete width of the strip conductors [14]. Using equations-(5.16) and (5.19), the conductor loss of a CPW structure is
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\[ \alpha_c = \frac{\text{Re}(Z_s + Z_m)}{4Z_0(f,t)} \left\{ s \int_0^{a-A} \frac{A^2}{a^2 - x^2} dx + s \int_{b+\Delta}^{c-A} \frac{A^2}{c^2 - b^2} dx \right\} \]  \hspace{1cm} (5.20)

On using the integration by parts, the conductor loss of a CPW structure is

\[ \alpha_c = \frac{R_{sm}b^2}{16Z_0(f,t)K^2(k)} \left\{ \frac{1}{a} \ln \left( \frac{2a - b - a + A}{A} \right) \left( b + a + A \right) \left( c + a - A \right) \right\} + \left\{ \frac{1}{b} \ln \left( \frac{2b - a + A}{b + a - A} \right) \left( c - b + A \right) \right\} \]  \hspace{1cm} (5.21)

For \( \Delta \ll a, b, (b - a) \), the above expression is reduced to

\[ \alpha_c = \frac{R_{sm}b^2}{16Z_0(f,t)K^2(k)} \left\{ \frac{1}{a} \ln \left( \frac{2a}{A} \right) + \frac{c + a - A}{c - a} \right\} + \frac{1}{b} \ln \left( \frac{2b}{A} + \frac{c - b + A}{c + b - A} \right) \]  \hspace{1cm} (5.22)

where, we have \( a = s/2, b = s/2+w \) and \( Z_0 \) of a CPW structure with finite width ground conductors is computed from equation-(3.29).

Ponchak et. al. [49] have taken \( 2c = (2w + 9s) \) for a practical CPW. The elliptic integral ratio \( K(k)/K(k') \) is evaluated by using the closed-form expressions summarized by Collin [38]. Holloway and Kuester have taken a CPW structure with infinite width ground conductors. In that case by taking \( c \rightarrow \infty \), equation- (5.22) is reduced to the Holloway – Kuester (HK) model [21]

\[ \alpha_c = \frac{R_{sm}b^2}{16Z_0(f,t)K^2(k)} \left\{ \frac{1}{a} \ln \left( \frac{2a}{A} \right) + \frac{1}{b} \ln \left( \frac{2b}{A} \right) \right\} \]  \hspace{1cm} (5.23)

We have curve-fitted normalized reciprocal stopping distance \((t/\Delta)\) tabular data of Holloway and Kuester with 90° and 45° conductor edges of isolated strip conductor [24]
as a function of the normalized conductor thickness \((t/2\delta_p)\). The empirical expressions are summarized below:

- **Expressions for rectangular 90° cross-section:**
  \[
  y = \begin{cases}
    -60.89x^4 + 282.17x^3 - 27.764x^2 + 0.5103x + 9.1907, & \text{for } 0.03 \leq x < 0.64 \\
    711.01x^5 - 2120.2x^4 + 1038.3x^3 + 9881.7x^2 - 12798x^2 + 6935.8x - 1372.7, & \text{for } 0.64 \leq x < 1.5 \\
    -183.05x^4 + 1583.9x^3 - 4981.9x^2 + 6618.6x - 2807.6, & \text{for } 1.5 \leq x < 2.76 \\
    0.2953x^4 - 7.7733x^3 + 73.991x^2 - 293.88x + 581.98, & \text{for } 2.76 \leq x < 8.0 \\
    -0.0198x^3 + 0.9848x^2 - 12.104x + 240.01, & \text{for } 8.0 \leq x < 16.0
  \end{cases}
  \]

- **Expressions for trapezoidal 45° cross-section**
  \[
  y = \begin{cases}
    -234.52x^5 + 231.86x^4 + 140.04x^3 - 20.231x^2 + 1.1217x + 6.5491, & \text{for } 0.03 \leq x < 0.64 \\
    589.31x^6 + 2175.7x^5 + 1049x^4 + 4836.5x^3 - 7660.5x^2 + 4439.8x - 905.16, & \text{for } 0.64 \leq x < 1.5 \\
    -206.38x^4 + 1848.3x^3 - 6025.2x^2 + 8396.8x - 3888.4, & \text{for } 1.5 \leq x < 2.76 \\
    -1.0366x^4 + 24.614x^3 - 210.34x^2 + 776.85x - 726.6, & \text{for } 2.76 \leq x < 8.0 \\
    0.4653x^3 - 10.904x^2 + 113.16x - 63.032, & \text{for } 8.0 \leq x < 16.0
  \end{cases}
  \]

Fig. (5.7): Frequency dependence of stopping distance for an isolated strip conductor.

Fig. (5.7) compares the stopping distance for the 90° and 45° conductor edges computed from expressions – (5.24) and (5.25) respectively against the tabular data of Holloway.
and Kuester. The expressions have average and maximum deviations (0.26\%, 1.28\%) and (0.26\%, 1.03\%) for the rectangular 90° and trapezoidal 45° cross-sections respectively. The equations-(5.23) to (5.25) form the HK model to compute the conductor loss of a CPW structure and are suitable for the CAD application.

We also note from Fig.(5.7) that theoretical stopping distance does not follow experimentally extracted stopping distance [21] as the theoretical stopping distance does not account for proximity of two ground conductors near the central strip conductor of a CPW structure. The experimentally extracted data on the stopping distance, from the experimental results of conductor loss of CPW given by Haydl et al. [131,136] for the frequency range 1 GHz to 60 GHz, is summarized in Appendix-A. The curve-fitted expressions for the experiment based inverse normalized stopping distance \( y = (t/\Delta) \) with respect to \( x = t/(2\delta) \) are given below

For \( 0.10 \leq s/2b \leq 0.4 \)

\[
y = 1668 x^4 - 1504.1 x^3 + 595.85 x^2 - 18.823 x + 0.6874, \quad \text{for} \ 0.045 \leq x < 0.7 \quad (5.26)
\]

For \( 0.4 < s/2b \leq 0.73 \)

\[
y = \begin{cases} 
- 2308.6 x^4 + 2314.6 x^3 - 595.39 x^2 + 65.242 x - 1.6152, & \text{for} \ 0.045 \leq x < 0.35 \\
- 8702.8 x^4 + 18054 x^3 - 13118 x^2 + 4109.8 x - 462.55, & \text{for} \ 0.35 \leq x < 0.7 \\
- 43937 x^4 + 170137 x^3 - 233814 x^2 + 138995 x - 30454, & \text{for} \ 0.7 \leq x < 1.4 
\end{cases} \quad (5.27)
\]

Fig.(5.7) shows that the experimentally extracted results of \((t/\Delta)\) for \( t/(2\delta) \leq 1.3 \). For \( t/(2\delta) \leq 0.2 \), \((t/\Delta)\) is structure independent while for \( t/(2\delta) > 0.2 \); i.e. at higher frequency, it is significantly structure dependent. It is large for the wide slot-gap. We note that around \( t/(2\delta) = 1.3 \) experimentally extracted \((t/\Delta)\) is significantly different from the theoretical results of Holloway and Kuester. This is due to the presence of
ground conductors in a CPW structure. The maximum deviation of computed stopping distance from the curve-fitted expressions is within 0.81 % of the experimental data.

Fig.(5.8) further compares the stopping distance as obtained from Holloway and Kuester equation-(5.24) and experimentally generated equation-(5.26) and equation-(5.27). It is obvious that Holloway and Kuester theoretical results are not valid in lower frequency range. At high frequency, the stopping distance of a CPW structure gradually becomes identical to the stopping distance of the isolated strip. At high frequency the slot-gap increases in terms of the wavelength and the effect of presence of ground conductors on the stopping distance diminishes.

Fig.(5.8): Comparison of stopping distances obtained theoretically and experimentally for s/2b = 0.73, t = 0.5µm.

Fig.(5.9) compares four models - HK model, IHK model, Wheeler’s model and Ponchak, Matloubian, and Katehi (PMK) model [49] and also results of HFSS, CST, Sonnet against the experimental results of Haydl et. al. [131], Ponchak et. al. [49] and Papapolymerou et. al. [64]. Haydl et. al. have provided 336 experimental data in 7 set of figures for the conductor loss of the CPW with finite substrate thickness h and large width (2c) of ground conductors. They have considered the CPW on GaAs (εr =12.9) and InP (εr =12.6) substrates of thickness 0.5 mm, conductor thickness t = 0.25, 0.5, 1.0 µm.
and $s/d = 0.13, 0.4, 0.73$ in the frequency range $1 \text{ GHz} – 60 \text{ GHz}$. The comparison is done for the conductor thickness $0.25 \mu m$ in the frequency range $1 \text{ GHz} – 60 \text{ GHz}$.

The second source of 36 experimental data is extracted from one graphical figure of Ponchak et. al. [49] for the CPW on GaAs, InP and Si substrates with characteristic impedances $35 \Omega$, $50 \Omega$, $65 \Omega$. The graphical data are enlarged by 8 times in order to reduce the reading errors. However, it is very difficult to extract accurate data from this source. The third source of experimental is from Papapolymerou et. al. [64] for the CPW on GaAs and quartz in the frequency range $1 \text{ GHz}$-$120 \text{ GHz}$.

![Graphical data comparison](image)

**Fig.(5.9):** Comparison of conductor losses in CPW against experimental results of: (a) Haydl [131] for $s/2b = 0.13$ and $s/2b = 0.73$, $t = 0.25 \mu m$,(b) Ponchak et. al.[49], (c) Papapolymerou et. al. [64] for quartz ($\varepsilon_r = 3.8$) substrate, and (d) Haydl et.al.[136].
Table-5.2 shows details of the CPW structures used by three groups of investigators. Table-5.3 presents the average in four models against the experimental results. Table-5.3 also shows a summary of overall deviation and maximum deviation in three models and two EM-simulators against the experimental results of three groups of investigators. The present improved HK (IHK) model has better average accuracy (4.4%-5.4%) against three groups of experimental results. The model of Ponchak et. al. has high deviation (17.07%) against the experimental results of Haydl et. al. It has small deviation (5.52%) only against its own experimental data that has been used for the development of their empirical expression. The IHK model gives maximum deviation 18% for some rare cases. The maximum deviations in HK model and PMK models are 62.5% and 26.8% respectively. HFSS has average deviation 7.8%, maximum deviation 25.2% and Sonnet has average deviation 10.3%, maximum deviation 33.6%. Thus the IHK model is more accurate than any of the existing closed-form models as compared against the experimental results from three independent sources [49, 64, 131, 136].

So far we have compared above accuracy of the IHK model against the experimental results for the conductor thickness between 0.25 µm – 1.58 µm. However, present MMIC technology also uses thick strip conductors in the range of 3 µm - 9 µm in order to get high-\(Q\) planar inductors [57]. Therefore accuracy of the IHK- model for the conductor loss of CPW with thick conductor is also examined against the results of EM-simulators. In Fig.(5.9d), the IHK model and three EM-simulators are compared for the range 0.25 µm \(\leq t \leq\) 9 µm, against the experimental results of Haydl et. al. [136]. The % average and % maximum deviation of the model, HFSS, Sonnet and CST w.r.t. Haydl’s results are (4.9%, 8.8%), (5.9%, 9.7%), (5.4%, 8.6%) and (5.6%, 8.9%) respectively. The present IHK model results are closer to experimental results as compared to the results of EM-simulators.
Table - 5.2: CPW structures

<table>
<thead>
<tr>
<th>S.No</th>
<th>Sets</th>
<th>$s/2b$</th>
<th>$t$ (µm)</th>
<th>$\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Haydl [136]</td>
<td>0.13</td>
<td>0.25</td>
<td>$InP$ (12.6)</td>
</tr>
<tr>
<td>2</td>
<td>Haydl [131]</td>
<td>0.4</td>
<td>0.25</td>
<td>$InP$ (12.6)</td>
</tr>
<tr>
<td>3</td>
<td>Haydl [136]</td>
<td>0.73</td>
<td>0.25</td>
<td>$InP$ (12.6)</td>
</tr>
<tr>
<td>4</td>
<td>Haydl [131]</td>
<td>0.13</td>
<td>0.5</td>
<td>$InP$ (12.6)</td>
</tr>
<tr>
<td>5</td>
<td>Haydl [131]</td>
<td>0.4</td>
<td>0.5</td>
<td>$InP$ (12.6)</td>
</tr>
<tr>
<td>6</td>
<td>Haydl [131]</td>
<td>0.73</td>
<td>0.5</td>
<td>$InP$ (12.6)</td>
</tr>
<tr>
<td>7</td>
<td>Haydl [131]</td>
<td>0.73</td>
<td>1</td>
<td>$GaAs$ (12.9)</td>
</tr>
<tr>
<td>8</td>
<td>Ponchak [49]</td>
<td>0.10-0.11</td>
<td>1.58</td>
<td>$InP$ (12.4), $GaAs$ (12.85)</td>
</tr>
<tr>
<td>9</td>
<td>Papapolymerou [64]</td>
<td>0.28</td>
<td>1</td>
<td>Quartz (3.8), $GaAs$ (12.85)</td>
</tr>
</tbody>
</table>

Table- 5.3: Average and overall deviations of four models and two EM-simulators

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
<tr>
<td>HK [21]</td>
<td>8.4</td>
<td>13.4</td>
<td>22.4</td>
<td>12.9</td>
<td>15.5</td>
<td>11.2</td>
<td>12.1</td>
<td>13.7</td>
<td>34.3</td>
</tr>
<tr>
<td>Wheeler [13]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>32.1</td>
<td>32.2</td>
<td>15.1</td>
<td>21</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ponchak [49]</td>
<td>35.1</td>
<td>25</td>
<td>10.2</td>
<td>18.3</td>
<td>16.4</td>
<td>6.1</td>
<td>8.3</td>
<td>17.1</td>
<td>26.8</td>
</tr>
<tr>
<td>IHK</td>
<td>3.3</td>
<td>4.3</td>
<td>5.1</td>
<td>4.2</td>
<td>5.8</td>
<td>6.1</td>
<td>4.0</td>
<td>4.4</td>
<td>17.5</td>
</tr>
<tr>
<td>HFSS</td>
<td>4.0</td>
<td>6.1</td>
<td>3.1</td>
<td>6.1</td>
<td>7.9</td>
<td>7.2</td>
<td>8.0</td>
<td>6.1</td>
<td>23.1</td>
</tr>
<tr>
<td>Sonnet</td>
<td>5.8</td>
<td>9.8</td>
<td>13.1</td>
<td>6.1</td>
<td>10.6</td>
<td>12.9</td>
<td>6.9</td>
<td>9.3</td>
<td>27.5</td>
</tr>
</tbody>
</table>

The total loss of the CPW structures is computed by adding the conductor loss and dielectric loss together, i.e.

$$\alpha_T = \alpha_c + \alpha_d \quad dB / unit \ length \quad (5.28)$$

The variation in $\alpha_T$ w.r.t. frequency in CPW, on different substrates with $\varepsilon_r = 3.78, 9.8$ and 20; $s/w = 0.44, h = 635 \, \mu m, \ tan \delta = 0.002$ and $t = 5 \, \mu m$, is shown in Fig.(5.10a). The closed-form model for $\alpha_c$ (IHK model) and $\alpha_d$ (equation-(5.14)) are used for the computation of $\alpha_T$ and compared against the EM- simulators within 7.4% average deviation. Fig.(5.10b) shows the comparison of computed $\alpha_T$ by the model against the EM-simulators as a function of $s/(s+2w)$ ratio with 5.7% average deviation. For thick
Analysis and Modeling of CPW

conductors, the closed-form model shows close agreement with the softwares. Fig.(5.10c) shows the variation of unloaded $Q$ factor ($Q_u$) in CPW on different substrates with $\varepsilon_r = 3.78, 9.8$ and $20$; $s/w = 0.44, h = 635 \, \mu m$, $\tan \delta = 0.002$ and $t = 5 \, \mu m$ in the frequency range $0.1 \, \text{GHz} – 200 \, \text{GHz}$. Fig.(5.10d) compares computation of $Q_u$ for $\varepsilon_r = 3.78$ and $20; f = 20\text{GHz}$ as a function of $s/(s+2w)$ ratio. Overall, % average and % maximum deviation in the closed-form model against EM-simulators are 6.7% and 9% respectively.

Fig.(5.10): Total loss, as a function of (a) Frequency and (b) $s/(s+2w)$ ratio and $Q$ factor, as a function of (c) Frequency and (d) $s/(s+2w)$ ratio for CPW on various substrates.
5.5 Effect of Asymmetry in Characteristics of CPW

The structures of asymmetric CPW (ACPW) shown in Fig. 5.11(a) provide additional degree of freedom to control characteristic impedance and effective permittivity. Hanna et al. [151], using conformal mapping method, reported analytical closed-form expressions to compute quasi-static values of $\varepsilon_{\text{eff}}$ and $Z_0$ of the ACPW with infinite or finite dielectric thickness. They have shown experimental results on six asymmetric CPWs, fabricated on an alumina substrate ($\varepsilon_r = 9.9$ and $h = 0.635$ mm) metalized with gold of thickness 4 µm. Karpuz and Görür [20] have reported another analytical closed-form expression, for the quasi-TEM parameters of ACPW using conformal mapping technique. However, both the authors have not considered effect of conductor thickness and dispersion in their models. Only a few publications present loss computation of ACPW using conformal-mapping technique [48, 83]. We present below computation of line parameters of ACPW.

![Fig. 5.11](image)

**Fig.5.11:** Different configurations of asymmetrical CPW (ACPW) structures
Analysis and Modeling of CPW

- Effective relative permittivity and Characteristic impedance

In our study, we have used the conductor thickness independent expressions for \(\varepsilon_{\text{eff}}\) and \(Z_0\) of the ACPW obtained from equations-(3.38) and (3.39) respectively:

\[
\varepsilon_{\text{eff}} = 1 + \frac{1}{2} (\varepsilon_r - 1) \frac{K(k_4') K(k_5')}{K(k_4) K(k_5)} \quad (a) \quad Z_0 = \frac{60\pi}{\sqrt{\varepsilon_{\text{eff}}} K(k_4')} \quad (b) \quad (5.29)
\]

where, modulus \(k_4\) and \(k_5\) along with their complementary modulus \(k_4'\) and \(k_5'\) are defined in equations-(3.33) and (3.36) respectively. However, these expressions are modified empirically to take into account the finite strip conductor thickness and the skin-depth penetration using equation-(5.10) and (5.13).

Fig.(5.12): Comparison against experimental results of Hanna et. al. with three cases of asymmetry: (a) Effective relative permittivity and (b) Characteristic impedance

Fig. (5.12a) compares \(\varepsilon_{\text{eff}}\) computed by the closed-form model on the substrate with \((\varepsilon_r = 9.9\) and \(h = 0.635\) mm) and gold conductor of thickness 4 \(\mu\)m. \(\varepsilon_r = 9.9\) against two
EM-simulators for three cases of asymmetry \( i.e. w_1/w_2 = 0.1, 1 \) and 2.5 for frequency range 1 GHz - 60 GHz. The model is showing closer agreement with HFSS and has % average and % maximum deviation of (0.89%, 2.7%) and (1.53%, 4.19%) w.r.t. HFSS and Sonnet respectively. However the closed-form model does not account for the low frequency dispersion due to the skin-effect penetration. The results of the model are in between results of Sonnet and HFSS. Fig. (5.12b) compares results of the closed-form model and simulated results of HFSS against the experimental results of Hanna et al. [151] for \( Z_0 \). For computation of characteristic impedance, the model has average accuracy of 1.95% and maximum deviation of 3.85% over whole frequency range. The model and EM-simulator results are nearly identical. It can be concluded that for a given shape ratio \( s/(s+w_1+w_2) \), the line asymmetry leads to decrease of its \( Z_0 \) and to an increase in \( \varepsilon_{\text{eff}} \).

- **Dielectric loss**

The dielectric loss of ACPW is computed using equation-(5.14) in which \( \varepsilon_{\text{eff}}(f,t) \) is used from equation-(5.29a).

- **Conductor loss**

The conductor loss of ACPW is computed with the help of Wheeler’s incremental inductance rule [13] and using the perturbation method. Equation-(5.29b) of characteristic impedance accounts for the asymmetry in the CPW structure in Wheeler’s method:

\[
\alpha_c = \frac{\pi}{\lambda_0} \sqrt{\varepsilon_{\text{eff}}(\varepsilon_r,w_1,w_2,s,h,f,t)} \frac{\Delta Z(\varepsilon_r = 1,w_1,w_2,s,h,f,t,\delta_s)}{Z_0(w_{eq1},w_{eq2},s_{eq},h,f,t,\varepsilon_r = 1)} \quad \text{Np/m} \quad (5.30)
\]
Holloway and Kuster have not accounted asymmetry in conductor loss computation of CPW [21]. Our improved Holloway and Kuster (IHK) closed-form expression presented below accounts for the asymmetry in CPW for computation of the conductor loss of ACPW. Due to asymmetry, shown in Fig.(5.11a) the longitudinal current \( I \) and the current density \( J \) over the strip conductors from equation-(5.16), is defined by the following function:

\[
J = \begin{cases} 
\frac{A}{\sqrt{(a^2 - x^2)b_1 - x(b_2 + x)}} & |x| < a \\
\frac{A}{\sqrt{(x^2 - a^2)(x - b_1)(x + b_2)}} & |x| > b 
\end{cases} 
\]

where, \( A = \frac{I}{4K(k')} \sqrt{(b_1 + a)(b_2 + a)} \) (c)

where \( a = \frac{s}{2} \); \( b_1 = w_1 + a \); \( b_2 = w_2 + a \); \( b = b_1 + b_2 \).  

(5.31)

The ratio of current density \( J \) to the longitudinal current \( I \) is integrated along whole of the strip as follows:

\[
\int \left( \frac{J}{I} \right)^2 dl = \int_{-\infty}^{-b_2 - A} \left( \frac{J}{I} \right)^2_{\text{lateral gnd}} dl + \int_{-a + A}^{a - A} \left( \frac{J}{I} \right)^2_{\text{central strip}} dl + \int_{b_1 + A}^{\infty} \left( \frac{J}{I} \right)^2_{\text{lateral gnd}} dl 
\]

(5.32)

On integrating to the whole range, the final expression for the conductor loss of the ACPW structure is

\[
\alpha_c = \frac{R_{\text{sm}}}{16Z_0(f,t)k^2(k')} \left\{ \begin{array}{c} 
\frac{1}{2a} \left[ \left( \frac{b_1 + a}{b_1 - a} \right) \ln \left( \frac{2a}{\Delta} - 1 \right) + \ln \left( \frac{b_1 - a + \Delta}{b_2 + a + \Delta} \right) \right] \\
\frac{1}{b_1 + b_2} \left[ \left( \frac{b_1 + a}{b_2 - a} \right) \ln \left( \frac{2a}{\Delta} - 1 \right) + \ln \left( \frac{b_2 - a + \Delta}{b_1 + a + \Delta} \right) \right] \\
\end{array} \right\} 
\]

(5.33)
Fig. (5.13): Total loss of ACPW as function of: (a)-(c) line impedance for three different values of the asymmetry parameter $w_1/w_2$, and (d) conductor thickness.

Fig. (5.13a) - Fig. (5.13c) compare the results for computed total loss $\alpha_f$ of ACPW by IHK, Wheeler [13], HFSS, Sonnet and Ghione [48] against the SDA-based results of Kitazawa [120] for three cases of asymmetry. The ACPW is considered on a semi-conductor substrate with $\varepsilon_r = 12.8$, $\tan \delta = 0.0006$, $f = 60$ GHz, $h = 100 \mu$m, $\sigma = 5.88 \times 10^7$ S/m and $t = 3 \mu$m. The inner spacing of two ground is maintained as $b_1 + b_2 = 300 \mu$m. We have computed the total loss for three cases of asymmetry i.e. $w_1/w_2 = 1$, 2 and 4 by varying the line impedance from $30 \Omega - 130 \Omega$. As the $b_1 + b_2$ is maintained at $300 \mu$m,
the slot-width becomes narrower with increase in the width of the central conductor, thus changing the line impedance accordingly. We have also computed the loss of the structure using HFSS and Sonnet. It is observed that with increase in asymmetry ratio \(w_1/w_2\) from 1 to 4, the loss increases. Outcome of the comparison in terms of % average and % maximum deviation is summarized in Table-5.4. The IHK model has the highest accuracy amongst all with % average deviation of 2.3%.

For \(w_1/w_2 = 4\) and \(s = 160 \mu\text{m}\), Fig.(5.13d) shows variation of \(\alpha_T\) in ACPW w.r.t. conductor thickness in the range 0.25 \(\mu\text{m}\) - 9 \(\mu\text{m}\). As expected, the conductor loss decreases with increase in conductor thickness. The results of IHK, HFSS and Sonnet follow results of Kitazawa closely with average deviation of 1.18%. Whereas results of Wheeler’s incremental inductance formulation deviates much and it fails to compute the loss for the conductor thickness less than the skin-depth. Otherwise, the IHK model, HFSS and Sonnet are in close agreement with each other with % average deviation of 1.18%.

<table>
<thead>
<tr>
<th>(w_2/w_1)</th>
<th>% Deviation</th>
<th>IHK</th>
<th>Wheeler [13]</th>
<th>Sonnet</th>
<th>HFSS</th>
<th>Ghione [48]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Av. 2.5</td>
<td>12.4</td>
<td>5.5</td>
<td>5.4</td>
<td>3.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max. 5.3</td>
<td>17.7</td>
<td>34.3</td>
<td>37.8</td>
<td>11.9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Av. 2.1</td>
<td>15.3</td>
<td>5.1</td>
<td>5.9</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max. 5.5</td>
<td>21.6</td>
<td>33.5</td>
<td>33.4</td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Av. 2.2</td>
<td>28.8</td>
<td>5.1</td>
<td>4.7</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max. 4.8</td>
<td>37.9</td>
<td>20.2</td>
<td>18.3</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>Av. 2.3</td>
<td>18.9</td>
<td>5.6</td>
<td>5.4</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max. 5.6</td>
<td>37.9</td>
<td>34.3</td>
<td>37.8</td>
<td>11.9</td>
<td></td>
</tr>
</tbody>
</table>

Table - 5.4: % Deviation of models against analytical results of Kitazawa [120]
[Data range: \(t= 3 \mu\text{m}\); \(f= 60 \text{GHz}\); \(\varepsilon_r= 12.8\); \(\tan\delta= 0.0006\); \(\sigma= 5.88 \times 10^7 \text{S/m}\)]
5.6 Effect of Top Shield and Conductor Backing

The top shield is used to protect a CPW against the environment. It is also useful for the post fabrication adjustment of the line parameters. The conductor-backed CPW provides mechanical strength and improves the average power-handling capacity of the structure for thin and fragile semiconductor and quartz substrates. These types of structures are explored and used to implement a band-reject filter and end-coupled filters [60,117]. Further, upper shielding is almost always present in MMIC applications since the conductor-backed configuration put inside a metallic enclosure offers protection from the environment [20]. We consider three different types of configurations of ACPW, shown in Fig.(5.11b)-(5.11d):

- Conductor – backed ACPW (CBACPW)
- ACPW with upper shielding (ACPWUS)
- Conductor-backed ACPW with upper shielding (CBACPWUS)

The thickness of the substrate, having relative permittivity \( \varepsilon_r \), is \( h \) and the top shield is located at the height \( h_1 \). The medium between the strip conductors and the top shield is air with \( \varepsilon_r = 1 \). Karpuz et. al. [20] reported static analytic expressions for \( \varepsilon_{eff} \) and \( Z_0 \) of the CBACPW, ACPWUS and CBACPWUS using the conformal mapping techniques. They have ignored conductor thickness. Their expressions are summarized below:

\[
\begin{align*}
\varepsilon_{eff} &= 1 + (\varepsilon_r - 1) \left[ \frac{K(k_6)}{K(k'_6)} \right] \quad (a) \\
Z_0 &= \frac{120\pi}{\sqrt{\varepsilon_{eff}}} \frac{1}{\sqrt{\frac{K(k_4)}{K(k'_4)} + \frac{K(k_6)}{K(k'_6)}}} \quad (b)
\end{align*}
\]

(5.34)
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- **ACPWUS:**

\[ \varepsilon_{\text{eff}} = 1 + (\varepsilon_r - 1) \left[ \frac{K(k_{5})}{K(k'_{5})} \right] \]

(a) \[ Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{1}{K(k_{4}) + K(k_{7})} \]

(b) \[ 1 \]

\[ \begin{bmatrix} K(k_{5}) / K(k'_{5}) \\ K(k_{4}) / K(k'_{4}) \\ K(k_{7}) / K(k'_{7}) \end{bmatrix} \]

\[ (5.35) \]

- **CBACPWUS:**

\[ \varepsilon_{\text{eff}} = 1 + (\varepsilon_r - 1) \left[ \frac{K(k_{6})}{K(k'_{6})} \right] \]

(a) \[ Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{1}{K(k_{6}) + K(k_{7})} \]

(b) \[ 1 \]

\[ \begin{bmatrix} K(k_{6}) / K(k'_{6}) \\ K(k_{6}) / K(k'_{6}) \\ K(k_{7}) / K(k'_{7}) \end{bmatrix} \]

\[ (5.36) \]

where modulus \( k_4 \) and \( k_5 \) along with their complementary modulus \( k'_4 \) and \( k'_5 \) are defined in equations-(3.33) and (3.36) respectively, and modulus \( k_6 \) and \( k_7 \) are given below

\[ k_6 = \sqrt{1 - k_5^2} \]

\[ (b) \]

\[ (5.37) \]

\[ k_7 = \sqrt{1 - k_6^2} \]

\[ (b) \]

\[ (5.38) \]

with

\[ s' = e^{-\pi \sqrt{2h}} \left( e^{\pi \sqrt{h_1}} - 1 \right) \]

\[ (a) \]

\[ s'' = e^{-\pi \sqrt{h_1}} \left( e^{\pi \sqrt{h_1}} - 1 \right) \]

\[ (d) \]

\[ w_1' = e^{-[s + w_1] \pi / \sqrt{2h_1}} \left( e^{w_1 \pi / \sqrt{h_1}} - 1 \right) \]

\[ (b) \]

\[ w_1'' = e^{-[w_1 + s] \pi / \sqrt{2h_1}} \left( e^{w_1 \pi / \sqrt{h_1}} - 1 \right) \]

\[ (e) \]

\[ w_2' = e^{-[s + w_2] \pi / \sqrt{2h_1}} \left( e^{w_2 \pi / \sqrt{h_1}} - 1 \right) \]

\[ (c) \]

\[ w_2'' = e^{-[w_2 + s] \pi / \sqrt{2h_1}} \left( e^{w_2 \pi / \sqrt{h_1}} - 1 \right) \]

\[ (f) \]

\[ (5.39) \]
In case asymmetry ratio \( \frac{w_1}{w_2} \) is 1, the above mentioned equations can be used to study the effect of conductor backing and upper shielding on symmetric CPW as well.

The SLR formulation (discussed in section-4.4.2) is used to extend the closed-form models for ACPW to compute line parameters of CBACPW, ACPWUS and CBACPWUS. Over the equivalent single-layer substrate with an equivalent relative permittivity \( \varepsilon_{eq} \), equivalent loss tangent \( \tan \delta_{eq} \) and equivalent substrate thickness \( h_{eq} \), the empirical expressions from equation-(5.10) and (5.13) along with equations-(5.34) - (5.36) are used to compute \( \varepsilon_{eff}(f,t), Z_0(f,t), \alpha_d \) and \( \alpha_c \) of all the three structures.

Fig. (5.14) – (5.16) shows comparison of computed line parameters of all the three structures - CBACPW, ACPWUS and CBACPWUS respectively, against two EM-simulators – HFSS and Sonnet. In Fig.(5.14a) and (5.14b), \( \varepsilon_{eff} \) and \( Z_0 \) of CBACPW for \( G/h = 1, 2 \) and 4, where, shown in Fig.(5.11b), \( G \) is ground-to-ground width \( (G = w_1 + w_2 + s) \); asymmetry ratio \( \frac{w_1}{w_2} = 0.4 \), \( \varepsilon_r = 2.5 \), \( t = 4 \) \( \mu \)m, \( h = 635 \) \( \mu \)m for frequency range 1 GHz - 60 GHz are shown. In Fig.(5.14a), computed \( \varepsilon_{eff} \) by the model shows better agreement with Sonnet, while results of HFSS are marginally on the higher side. Like EM-simulators, the present model shows low frequency dispersion. In Fig.(5.14b), computed \( Z_0 \) by the model is in close agreement with both the EM simulators, having almost identical results except for \( f < 10 \) GHz. The results of simulators show increase in characteristic impedance as it is based on power – voltage definition. However model shows decrease in characteristic impedance as the model uses \( V-I \) definition. It results into deviation in the results. The computation of \( \varepsilon_{eff} \) and \( Z_0 \) by the model has % average and % maximum deviation of \( (2.17\%, \ 5.53\%) \) and \( (3.07\%, \ 5.83\%) \) respectively against both the EM simulators. It is observed that both \( \varepsilon_{eff} \) and \( Z_0 \) increase with increase in \( G/h \) ratio from 1 to 4.
Fig. (5.14): CBACPW: (a) Effective relative permittivity, (b) Characteristic impedance, (c) Conductor loss, and (d) Dielectric loss.

In Fig. (5.14c) and (5.14d) show $\alpha_c$ and $\alpha_d$ of CBACPW for $G/h = 1$ and 4 with $w_1/w_2 = 2$, $\varepsilon_r = 16$, $\tan \delta = 0.002$, $h = 635 \mu m$, $\sigma = 4.1 \times 10^7$ S/m and $t = 4 \mu m$ for frequency range 1 GHz – 30 GHz. It is observed that $\alpha_c$ increases with decrease in ground plane spacing from $G=4h$ to $G=h$. The close spacing of conductors increases the current density at the edges. On comparing against HFSS; IHK, Wheeler [13] and Sonnet have % average and % maximum deviation of (2.47%, 4.87%), (5.16%, 7.93%) and (3.25%, 6.17%) respectively. The improved perturbation method is more accurate as
compared to Wheeler’s model. It is observed that $\alpha_d$ increases as $G/h$ ratio increases from 1 to 4, as shown in Fig. (5.14d). The increase in separation between conductors for the conductor-backed CPW permits more concentration of electric field in the lossy substrate. It increases the dielectric loss. The model is in close agreement with both the EM simulators with % average deviation of only 0.13%.

**Fig.(5.15):** ACPWUS: (a) Effective relative permittivity, (b) Characteristic impedance, (c) Conductor loss, and (d) Dielectric loss.

Fig.(5.15a) and (5.15b) show $e_{eff}$ and $Z_0$ of the asymmetric CPW with upper shield
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(ACPWUS). The structure is shown in Fig. (5.11c). The results are presented for $G/h = 1$ and 4; $h_1/h = 2$ and 5 with $\varepsilon_r = 2.5$, $t = 4 \mu m$, $h = 635 \mu m$ for frequency range 1 GHz – 60 GHz. The computation of $\varepsilon_{eff}$ and $Z_0$ by the model has % average and % maximum deviation of (4.14%, 6.75%) and (4.26%, 6.83%) respectively against both the EM simulators. Both $\varepsilon_{eff}$ and $Z_0$ increase with increasing $h_1/h$ ratio, whereas $\varepsilon_{eff}$ decreases and $Z_0$ increases with increasing $G/h$ ratio.

In Fig. (5.15c) and (5.15d), $\alpha_c$ and $\alpha_d$ of ACPWUS for $G/h = 2$ and 4; $h_1/h = 2$ and 10 with $w_1/w_2 = 2$, $\varepsilon_r = 16$, $\tan \delta = 0.002$, $h = 635 \mu m$, $\sigma = 4.1 \times 10^7$ S/m, $t = 4 \mu m$ and frequency range 1 GHz – 30 GHz, are shown. It is observed that $\alpha_c$ increases when upper shielding is nearer to the substrate and it increases as $G/h$ ratio decreases from 4 to 2, as shown in Fig. (5.15c). The IHK, Wheeler [13] and Sonnet have % average and % maximum deviation of (4.16%, 6.93%), (5.98%, 8.9%) and (2.89%, 5.96%) respectively w.r.t. HFSS. It is also observed that $\alpha_d$ decreases as $h_1/h$ ratio decreases from 10 to 2 and it increases as $G/h$ ratio increases from 2 to 4, as shown in Fig. (5.15d). The closed-form model has 1.8% average deviation against both the EM simulators.

In Fig. (5.16a) and (5.16b), $\varepsilon_{eff}$ and $Z_0$ of CBACPWSH for $G/h = 1$ and 4; $h_1/h = 2$ and 5 with $\varepsilon_r = 2.5$, $t = 4 \mu m$, $h = 750 \mu m$ for frequency range 1 GHz – 60 GHz, are shown. The computation of $\varepsilon_{eff}$ and $Z_0$ by the model has % average and % maximum deviation of (2.4%, 5.82%) and (2.84%, 3.65%) respectively against both the EM simulators. It is observed that both $\varepsilon_{eff}$ and $Z_0$ increases with increasing $h_1/h$ ratio and $G/h$ ratio.

In Fig. (5.16c) and (5.16d), $\alpha_c$ and $\alpha_d$ of CBACPWSH for $G/h = 1$ and 2; $h_1/h = 2$ and 10 with $w_1/w_2 = 2$, $\varepsilon_r = 16$, $\tan \delta = 0.002$, $h = 750 \mu m$, $\sigma = 4.1 \times 10^7$ S/m, $t = 4 \mu m$ and frequency range 1 GHz – 30 GHz, are shown. It is observed that the presence of both
conductor backing and upper shielding increases $\alpha_c$ when $G/h$ ratio is nearer to 1 and height of upper shielding is nearer to the substrate. The IHK, Wheeler [13] and Sonnet have an average and maximum deviation of (2.68%, 4.71%), (4.11%, 7.57%) and (3.19%, 6.9%) respectively w.r.t. HFSS. It is observed that $\alpha_d$ decreases when $h_1/h$ ratio decreases from 10 to 2 and when $G/h$ ratio increases from 1 to 2. The closed-form model has 0.75% average deviation against both the EM simulators.

Fig.(5.16): CBACPWUS: (a) Effective relative permittivity, (b) Characteristic impedance, (c) Conductor loss, and (d) Dielectric loss.
5.7 Closed-form Dispersion and Loss Models for Multilayer CPW

The combined strength of the CPW and multilayer MMIC technology provides an attractive field for the design of compact low cost MMICs for affordable wireless communications [60]. The cross-sectional view of a CPW on multilayer dielectric substrates is shown in Fig.(5.17).

Svačina [66] has applied conformal mapping technique to obtain general expressions for capacitance of multilayer unshielded and without conductor-backed CPW. Others have adopted these existing conformal mapping results for the multilayer cases. However their
schemes do not consider more than two numbers of lower or upper layers. In our study we have computed $\varepsilon_{\text{eff}}$ and $Z_0$ of multilayer CPW, shown in Fig.(5.17), using conformal mapping based technique [105] giving equations:

\[
\varepsilon_{\text{eff}} = 1 + \frac{K(k_0')}{K(k_0)} \left\{ \frac{\varepsilon_{r1} - \varepsilon_{r2}}{2} K \left( k_{1H1u} \right) + \frac{\varepsilon_{r2} - \varepsilon_{r3}}{2} K \left( k_{1H2l} \right) + \frac{\varepsilon_{r3} - 1}{2} K \left( k_{1H3l} \right) \\
+ \frac{\varepsilon_{r1} - \varepsilon_{r2}}{2} K \left( k_{1H1u} \right) + \frac{\varepsilon_{r2} - \varepsilon_{r3}}{2} K \left( k_{1H2u} \right) + \frac{\varepsilon_{r3} - 1}{2} K \left( k_{1H3u} \right) \right\} 
\]

\[
Z_0 = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0')}{K(k_0)} 
\]

where, modulus $k_{1H1u/l}^u$ and $k_0$ along with their complements can be computed using equation-(3.10) and (3.12) respectively, in which $h$ will be replaced by $H_{iu/l}(i=1,2,3)$ accordingly, given by:

\[
H_{1u} = h_1^u 
\]

\[
H_{2u} = h_1^u + h_2^u 
\]

\[
H_{3u} = h_1^u + h_2^u + h_3^u 
\]

\[
H_{1l} = h_1^l 
\]

\[
H_{2l} = h_1^l + h_2^l 
\]

\[
H_{3l} = h_1^l + h_2^l + h_3^l 
\]

The method employed is generalized enough to handle any number of lower or upper layers efficiently. Fig.(5.17) also shows CPW on two and three layered substrates. We have computed the conductor thickness dependent $\varepsilon_{\text{eff}}(f,t), Z_0(f,t), \alpha_c$ and $\alpha_d$ of these two multilayer SLR discussed in Chapter-4.
The dispersive effective relative permittivity, characteristic impedance and losses are computed as follows:

- Effective relative permittivity
  \[
  \varepsilon_{\text{eff}}(f,T) = \left( \sqrt{S \times \varepsilon_{\text{eff}}(f=0,T)} + \frac{\left[ \varepsilon_{\text{req}} - \sqrt{S \times \varepsilon_{\text{eff}}(f=0,T)} \right]}{1 + m(f/f_{\text{TE}})^{-r}} \right)^2
  \]
  where Skin-effect factor, \( S = \left( \frac{K(k_{0,\delta})}{K(k_{0,T})} \right)^2 \)

- Characteristic impedance
  \[
  Z_0(f,T) = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}(f,T)}} \frac{1}{S}
  \]

- Wheeler’s incremental inductance rule
  \[
  \alpha_c = \frac{27.29}{A_0} \sqrt{\varepsilon_{\text{eff}}(\varepsilon_{\text{req}},w,s,h_{\text{eq}},f,T)} \frac{\Delta Z(\varepsilon_{\text{req}}=1,w,s,h_{\text{eq}},f,T,\delta_s)}{Z_0(w_{\text{eq}},s_{\text{eq}},h_{\text{eq}},f,T,\varepsilon_{\text{req}}=1)} \text{ dB/m}
  \]

- Improved Holloway and Kuester (IHK)
  \[
  \alpha_c = \frac{R_{\text{sm}} b^2}{16Z_0(f,T) K^2(k)(b^2-a^2)} \left\{ \ln \left( \frac{2a}{\Delta b + a c} \right) + \frac{1}{a} \ln \left( \frac{2b c}{\Delta b + a c} \right) \right\} \text{ Np/m}
  \]
Dielectric loss

\[ \alpha_d = 27.29 \frac{\varepsilon_{req}}{\sqrt{\varepsilon_{eff}(f,t)}} \left( \frac{\varepsilon_{eff}(f,t) - 1}{\varepsilon_{req} - 1} \right) \frac{\tan \delta_{eq}}{\lambda_0} \] dB/m (e) (5.43)

where \( h_{eq} \) is the total substrate thickness between strip conductors and bottom layer of the multilayer substrate. \( \varepsilon_{req} \) and \( \tan \delta_{eq} \) of the equivalent single-layer substrate CPW.
are obtained from equation-(4.28). The dielectric loss and conductor loss of multilayer CPW had been computed by Verma et. al. [13-14] using SLR technique.

Fig.(5.19): Three layered composite substrate CPW: (a) Effective relative permittivity, (b) Characteristic impedance, (c) Conductor loss, and (d) Dielectric loss.

Fig. (5.18a) – (5.18d) show comparison and validity of SLR-based computed line parameters of two layered composite substrate CPW against results from experiment,
SDA and softwares, for different line structures. Fig. (5.19a) – (5.19d) show such comparisons for three layered composite substrate CPW. The computed $\varepsilon_{\text{eff}}$ and $Z_0$ by the model has average deviation of 1.63% and 4.6% respectively, against SDA-based results of Bedair et al. [110]. The computed $\varepsilon_{\text{eff}}$ and $Z_0$ by the model has average and maximum deviation of (1.8%, 3.7%) and (5.1%, 12%) respectively against both the EM simulators for frequency range 1 GHz – 65 GHz. The variation in $\alpha_c$ and $\alpha_d$ of multilayer CPW are shown in Fig. (5.18) and Fig. (5.19). Papapolymerou et al. [64] results of $\alpha_c$ for frequency range 2 GHz – 118 GHz on polyimide Pyralin PI2545 ($\varepsilon_r=3.5$) is taken with two GaAs wafers, in order to get a thickness of 3 µm. The results for both closed-form models and EM-simulators are compared against this experimental result.

Overall, the average and maximum deviations in IHK, Wheeler, HFSS and Sonnet are (3.4%,9.8%), (4.7%,10.1%), (5.3%,9.8%) and (3.8%,8.6%) respectively. The computed $\alpha_d$ by the model is in close agreement with both the EM simulators and MoM based LINPAR, for frequency range 1 GHz – 60 GHz, with % average deviation of 4.6%.

### 5.8 Closed-form Dispersion and Loss Models for Non-Planar CPW

There is a demand for non-planar CPW i.e. CPW on elliptical surface (ECPW) and CPW on cylindrical surface (CCPW) in order to produce compact devices applicable to aircraft, missiles and mobile communication. Such structures are needed to feed wrapped around printed antennas on different geometric surfaces [54, 84-85, 90]. To date, many authors [19, 52, 55, 85, 90, 130] have investigated the electrical parameters of ECPW and CCPW using full-wave approach incorporating a moment-method calculation and conformal mapping technique. However, effect of conductor thickness and dispersion on line
parameters of non-planar CPW has not been investigated yet using closed-form expressions.

In this section, we will present the CAD oriented closed-form models for the line parameters of different configurations of single-layered and multilayered non-planar CPW, shown in Fig.(5.20) and (5.23). The available static closed-form models are modified empirically, to take into account the finite strip conductor thickness and dispersion on the circular and elliptical cylindrical surfaces, by adopting the expressions used for the planar single-layered and multilayered CPW.

5.8.1 Single-Layer Case

The four configurations of non-planar CPW with finite ground plane width – on elliptical, circular, semi-ellipsoidal and semi-circular cylindrical surfaces, are shown in Fig.(5.20). The structural parameters of the SC - transformed ECPW/ CCPW into the corresponding planar CPW are given by equation-(3.54):

\[ s = 2\Psi \quad (i) \quad w = \theta - \Psi \quad (ii) \quad h = \ln \frac{a_2 + b_2}{a_1 + b_1} \quad (iii) \quad t = \ln \frac{a_3 + b_3}{a_2 + b_2} \quad (iv) \quad (5.44) \]

The detailed definition of the above mentioned parameters are given in Chapter-3.

- **Effective relative permittivity and Characteristic impedance**

We have used available static expressions for \( \varepsilon_{eff} \) and \( Z_0 \) of the non-planar CPW without conductor thickness to compute the strip conductor thickness dependent dispersive \( \varepsilon_{eff} \) and \( Z_0 \) of the non-planar CPW. The static expressions for \( \varepsilon_{eff} \) and \( Z_0 \) of the non-planar CPW obtained from equations-(3.48) and (3.49) are given below.
\[ \varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k_0')}{K(k_0)} \frac{K(k_1')}{K(k_1)} \quad (a) \]
\[ Z_0 = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0')}{K(k_0)} \quad (b) \quad (5.45) \]

where, modulus \( k_0 \) and \( k_1 \) along with their complementary modulus \( k_0' \) and \( k_1' \) for finite ground plane widths \((2\pi - 2\theta)\) and \((\pi - 2\theta)\), are obtained from equations - (3.42) - (3.44) and equations - (3.55) - (3.56) respectively.

Fig.(5.20): CPW with finite ground plane on the curved surfaces: (a) ECPW, (b) CCPW, (c) SECPW and (d) SCCPW.

In order to account the strip conductor thickness for these line structures, we have to modify equation- (5.3) to equation-(5.7) by using equation-(5.44). The equivalent slot width \( (\theta_{eq}) \) and equivalent strip width \( (\psi_{eq}) \) for the ECPW / CCPW are given as
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\[ \psi_{eq} = 2\psi + \Delta \psi \quad (i), \quad \theta_{eq} = \theta - \psi - \Delta \psi \quad (ii) \]

where \( \Delta \psi = \frac{1}{2\pi \epsilon_r} \left[ \ln \frac{a_3 + b_3}{a_2 + b_2} \right] + \ln \left[ \frac{8\pi (\theta - \psi)}{\ln \frac{a_3 + b_3}{a_2 + b_2}} \right] \quad (iii) \]

The modulus \( k_0 \) and \( k_1 \) are modified into \( k_{0f} \) and \( k_{1f} \) along with their complementary modulus, in which \( \psi \) and \( \theta \) is replaced by \( \psi_{eq} \) and \( \theta_{eq} \) respectively. The above equation - (5.46) can be inserted in equations - (5.10) and (5.13) to compute the conductor thickness and frequency dependent \( \epsilon_{eff}(f,t) \) and \( Z_0(f,t) \) of the ECPW and CCPW lines.

- **Dielectric and Conductor losses**

The dielectric loss \( \alpha_d \) is computed by using equation-(5.14) in which the frequency and conductor thickness dependent \( \epsilon_{eff}(f,t) \) of ECPW and CCPW are used. Both Wheeler’s incremental inductance formulation and perturbation method \( i.e. \) IHK model are used to compute the conductor loss \( \alpha_c \) of the non-planar CPW. Wheeler’s incremental inductance formulation [13] is modified using equation-(5.44):

\[
\alpha_c = \frac{\pi}{\lambda_0} \left[ \epsilon_{eff}(\theta-\psi)2\psi,\ln \frac{a_2+b_2}{a_1+b_1},f,\ln \frac{a_3+b_3}{a_2+b_2} \right] \left[ \frac{1}{\epsilon_{eff}(\theta-\psi)2\psi,\ln \frac{a_2+b_2}{a_1+b_1},f,\ln \frac{a_3+b_3}{a_2+b_2}} \right] \left[ \frac{1}{\epsilon_{eq}(\theta-\psi)2\psi,\ln \frac{a_2+b_2}{a_1+b_1},f,\ln \frac{a_3+b_3}{a_2+b_2}} \right] \left[ Z_0^2(\theta_{eq},\epsilon_{eq},\ln \frac{a_2+b_2}{a_1+b_1},f,\ln \frac{a_3+b_3}{a_2+b_2},\epsilon_{eq}=1) \right] \] \quad (5.47)

Then IHK model for ECPW/CCPW with ground plane width \( (2\pi - 2\theta) \) is obtained by using equation – (5.44) with equation – (5.22):
$\alpha_c = \frac{R_{sm} \theta^2}{16 Z_0 (f, t) K^2 (k) \theta^2 - \psi^2} \left\{ \frac{1}{\theta} \ln \left( \frac{2 \psi \theta - \psi \pi + \psi}{\Delta \theta + \psi \pi - \psi} \right) + \frac{1}{\theta} \ln \left( \frac{2 \theta \theta - \psi \pi - \psi}{\Delta \theta + \psi \pi + \psi} \right) \right\}$ Np/m \quad (5.48)

When $\pi$ is replaced by $\pi/2$ in the above equation, IHK model for non-planar CPW with ground plane width $(\pi - 2\theta)$ is obtained.

We have tested the accuracy of the closed-form models developed for propagation characteristics of non-planar CPW, for both elliptical/circular i.e. $(2\pi - 2\theta)$ and semi-elliptical/circular i.e. $(\pi - 2\theta)$ ground plane width, against the results obtained from EM-simulators- HFSS and CST, as shown in Fig.(5.21) and Fig.(5.22). Fig.(5.21) presents comparisons of performances of CPW on the circular and semi-circular cylindrical surfaces in respect of effective relative permittivity, characteristic impedance and losses. Fig.(5.22) presents such comparisons for the CPW on the elliptical and semi-ellipsoidal cylindrical surfaces. The results are obtained over the frequency range, 1 GHz – 60 GHz.

For simulation, we have taken the substrate with $\varepsilon_r = 2.5$, $\theta = 40^\circ$, $\psi = 25^\circ$, $h = 500 \mu$m and $t = 3 \mu$m. For ECPW, the ellipticity $c = 0.7$ is considered. With increase in the ellipticity and decrease in the ground width, there is increase in $\varepsilon_{\text{eff}}$; whereas with decrease in both ellipticity and ground width, $Z_0$ increases.

We note that effective relative permittivity increases for the semi-circular and semi-ellipsoidal CPW as compared against the CPW on the circular and elliptical surfaces. The characteristic impedances of semi-circular and semi-ellipsoidal cases are also higher as compared to the CPW on the circular and elliptical surfaces. The losses are almost identical for both the cases. The closed-form model follows closely the results of both HFSS and CST with average deviation of 2.3%. The nature of dispersion is identical for both cases. The results are summarized in Table-(5.5).
Fig. (5.21): Comparison of different line parameters of non-planar CPW on circular and semi-circular surfaces.
Fig.(5.22): Comparison of different line parameters of non-planar CPW on elliptical and semi-ellipsoidal surfaces.
Fig.(5.21) and Fig.(5.22) also show the effect of conductor thickness on $\varepsilon_{\text{eff}}$ and $Z_0$ of CCPW and ECPW with varying dimensional parameter ratio $\psi/\theta$ for $\varepsilon_r=9.8$, $\psi = 25^\circ$, $h = 500 \, \mu m$, $c = 0.5$ and $f = 30 \, \text{GHz}$. The two conductor thicknesses $t = 3 \, \mu m$ and $6 \, \mu m$ are used in the investigation. Both $\varepsilon_{\text{eff}}$ and $Z_0$ decreases with increase in the conductor thickness.

**Table - 5.5:** % Change in characteristics of models against simulators on non-planar CPW with different ground-plane widths

(a): % change in characteristics of semi-ellipsoidal CPW over elliptical CPW.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Closed-form Model</th>
<th>HFSS</th>
<th>CST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% av. change</td>
<td>% max. change</td>
<td>% av. change</td>
</tr>
<tr>
<td>Increase in Effective relative permittivity</td>
<td>1.6</td>
<td>1.8</td>
<td>3.1</td>
</tr>
<tr>
<td>Increase in Characteristic impedance</td>
<td>7.3</td>
<td>10.5</td>
<td>4.8</td>
</tr>
<tr>
<td>Decrease in Total loss</td>
<td>2.6</td>
<td>4.3</td>
<td>1.6</td>
</tr>
</tbody>
</table>

(b): % change in characteristics of semi-circular CPW over circular cylindrical CPW.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Closed-form Model</th>
<th>HFSS</th>
<th>CST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% av. change</td>
<td>% max. change</td>
<td>% av. change</td>
</tr>
<tr>
<td>Increase in Effective relative permittivity</td>
<td>6.3</td>
<td>6.5</td>
<td>2.3</td>
</tr>
<tr>
<td>Increase in Characteristic impedance</td>
<td>4.4</td>
<td>4.7</td>
<td>4.7</td>
</tr>
<tr>
<td>Decrease in Total loss</td>
<td>2.8</td>
<td>4.9</td>
<td>1.7</td>
</tr>
</tbody>
</table>
The total losses $\alpha_T$ are compared for non-planar CPW with both cases of finite ground plane between frequency ranges 1 GHz – 60 GHz. We have taken $t = 3\mu m$, $\theta = 40^\circ$, $\psi = 35^\circ$, $c = 0.7$, $h = 500 \mu m$, $\tan \delta = 0.0015$ and $\sigma = 4.1 \times 10^7$ S/m on substrate with $\varepsilon_r = 12.9$. On comparison, the closed-form model # 1 ($= \text{Wheeler} + \alpha_d$) and closed-form model # 2 ($= \text{IHK} + \alpha_d$) have average and maximum deviation of (7.4%, 15.6%) and (4.7%, 16.3%) respectively against both the EM-simulators, excluding results at 1 GHz due to large deviations. The closed-form model # 2 is in close agreement with HFSS at higher frequencies.

Fig.(5.21) and Fig.(5.22) further compare $\alpha_c$ of the structures computed by Wheeler and IHK for $\varepsilon_r=12.9$, $\theta = 28^\circ$, $\psi = 25^\circ$, $f = 60$ GHz, $h = 500 \mu m$, $\sigma = 3 \times 10^7$ S/m with conductor thickness range $0.25 \mu m – 9 \mu m$. Both the models are in close agreement with HFSS. The average and maximum deviation of Wheeler and IHK against results of Duyar et. al. [84] are (5.8%, 9.6%) and (4.4%, 14.4%) respectively.

5.8.2 Multilayer Case

In this section, we have extended the improved closed-form models for the line parameters of single-layered, equations-(5.45)-(5.48), to multilayer non-planar CPW by applying SLR technique and using equation-(5.43). The SLR is presented in Chapter-4. The structural parameters of multilayer structure obtained from SC-transformation (discussed in Chapter-3) are:

\[
\begin{align*}
  s &= 2\psi \quad (i) \\
  w &= \theta - \psi \quad (ii) \\
  t &= \ln \frac{a_4 + b_4}{a_3 + b_3} \quad (iii) \\
  h_1 &= \ln \frac{a_3 + b_3}{a_2 + b_2} \quad (iv) \\
  h_2 &= \ln \frac{a_3 + b_3}{a_1 + b_1} \quad (v) \\
  h_3 &= \ln \frac{a_5 + b_5}{a_3 + b_3} \quad (vi)
\end{align*}
\]
For the sake of convenience, we have considered up to three layers only, as shown in Fig.(5.23). The partial capacitance technique is used for computation of $\varepsilon_{\text{eff}}$ and $Z_0$ of the multilayer non-planar CPW. The expressions are summarized below and other details are given in Chapter-3:

\[
\varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_{r1} - \varepsilon_{r2}}{2} \frac{K(k_0')} {K(k_1')} + \frac{\varepsilon_{r2} - 1} {2} \frac{K(k_0')} {K(k_2')} + \frac{\varepsilon_{r3} - 1} {2} \frac{K(k_0')} {K(k_3')}
\]

\[
Z_0 = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0')} {K(k_0)}
\]

where modulus $k_0$ and $k_i (i=1,2,3)$ along with their complements can be computed using equation-(3.42) and (3.44) respectively for finite ground plane widths $(2\pi - 2\theta)$ and $(\pi - 2\theta)$. The parameter $H$ in equation–(3.45) is replaced by $h_i (i=1,2,3)$ accordingly.
Fig. (5.24): Multilayer non-planar CPW: (a) Effective relative permittivity, (b) Characteristic impedance, and (c) Total loss.

Fig. (5.24) shows comparison and validity of SLR-based computed line parameters of multilayer non-planar CPW with $\varepsilon_{r1} = 3.8$, $\varepsilon_{r2} = 12.9$, $\varepsilon_{r3} = 1$, $\theta = 28^\circ$ and $\psi = 25^\circ$ against EM-simulators for frequency range 1 GHz – 60 GHz. The computation of $\varepsilon_{\text{eff}}$ and $Z_0$ by the model has average and maximum deviation of (4.7%, 10.8%) and (5.4%, 8.3%) respectively against both the EM simulators. Fig.(5.24) also shows the variation in $\alpha_T$ of multilayer non-planar CPW for two conductor thicknesses $t = 0.5 \mu m$ and 3 $\mu m$ with finite ground plane width $(2\pi - 2\theta)$. The average and maximum deviation of closed-
form model #1 and closed-form model #2 against EM-simulators are (8.5%, 19.8%) and (5.3%, 17.4%) respectively. Outcome of the comparison in terms of % average and % maximum deviation is summarized in Table-5.6 for different multilayer configurations. The closed-form model #2 is used for the computation of total loss.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \varepsilon_{r2} = 20, \varepsilon_{r1} = 10; h_1=h_2=635 , \mu m; \tan \delta_1=0.0015; \tan \delta_2=0.002 )</th>
<th>( \varepsilon_{r2} = 12.9, \varepsilon_{r1} = 10; h_1=h_2=635 , \mu m; \tan \delta_1=0.0015; \tan \delta_2=0.002 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \varepsilon_{eff} )</td>
<td>( Z_0(\Omega) )</td>
</tr>
<tr>
<td>HFSS</td>
<td>Av. 6.2 Max. 11.2 Av. 5.4 Max. 11.6</td>
<td>Av. 11.4 Max. 15.1</td>
</tr>
<tr>
<td>CST</td>
<td>4.2</td>
<td>15.4</td>
</tr>
</tbody>
</table>

### Table - 5.6: % Deviation of models against simulators on different dielectric interfaces

[Data range: \( t = 0.25 \, \mu m - 9 \, \mu m; f = 1 \, GHz - 60 \, GHz; \theta = 28^\circ; \psi = 25^\circ; \varepsilon_r=1; h_3=500 \, \mu m; \sigma=4.1 \times 10^7 \, S/m \)]

5.9 Circuit Model of CPW

We have presented models to compute dispersive effective relative permittivity, characteristic impedance, dielectric loss and conductor loss of CPW, CPS and slotline structures - both planar and non-planar structures, on single-layer substrates and also on multilayer substrates. The dispersion at low frequency, due to field penetration, has also been accounted in the modeling. However, we know that losses have influence on the propagation parameter and to some extent the propagation parameter also influence the loss computation. Moreover, a lossy line has complex characteristic impedance. This information is missing in our modeling presented so far. These missing information can be computed with the help of \( R (f) \), \( L (f) \), \( C (f) \), and \( G (f) \) circuit model of any of these line on assumption that they support quasi-TEM mode of propagation. We can extract \( R (f) \), \( L (f) \), \( C (f) \), and \( G (f) \) from models of dispersive effective relative permittivity, characteristic impedance, dielectric loss and conductor loss of transmission lines.
discussed so far. It discussed in Chapter-2. Chapter-2 also presents method to extract these primary line parameters from the $S$-parameters obtained using the EM-simulators.

In this section, we first extract the primary and secondary line parameters of the CPW using closed-form models. We compare accuracy of our parameters extraction against the extracted line parameters using EM-simulators—HFSS, Sonnet, CST and MOM-based software LINPAR. We have taken 0.01 GHz – 10 GHz frequency range and the CPW is considered on the substrate with $\varepsilon_r = 9.8$, $s = 45 \, \mu m$, $w = 50 \, \mu m$, $h = 165 \, \mu m$ and $t = 1 \, \mu m$. The extracted frequency dependent $RLCG$ line parameters are shown and compared in Fig.(5.25).

![Fig.(5.25): Extraction of RLCG parameters of lossy planar CPW using circuit model and EM-simulators.](image)
The line resistance $R$, presented in Fig.(5.25a) shows increase with frequency and at low frequencies resistance $R$, approaches a constant. As frequency increases, the skin-depth becomes smaller than the conductor thickness and surface resistance increases as the square-root of frequency. The results obtained from circuit model are in between the results of HFSS and CST and are comparably closer to CST and Sonnet. The results of LINPAR have more deviations from the EM-simulators. The line inductance $L$, presented in Fig.(5.25b), shows decrease in its value with increase in frequency from 0.01 GHz to 1 GHz. It is due to decrease in the internal inductance of the CPW. Above 1 GHz change in line inductance is insignificant. The results of the circuit model follow results of EM-simulators closely, closer to Sonnet.

The line capacitance $C$, presented in Fig.(5.25c), also shows decrease in its value with increase in frequency from 0.01 GHz to 10 GHz. However decrease is faster in the frequency range from 0.01 GHz to 0.1 GHz and then decrease is slower. The circuit model closely follows results of HFSS and Sonnet. It follows physical behavior of a capacitor. We have taken both relative permittivity and loss-tangent constant of substrate material frequency independent which is not correct from Krammer - Kronig point of view. Thus our circuit model is non-causal. The results of circuit model and EM-simulators, presented in Fig.(5.25d), show increase in line conductance $G$ with frequency. The circuit model closely follows EM-simulators.

Finally we obtain frequency dependent effective relative permittivity, characteristic impedance - real and imaginary parts, and total loss from the $RLCG$ parameters. These results obtained from the circuit models are presented in Fig.(5.26) and compared against the results of EM-simulators. The results of circuit model follow results of EM-simulators; whereas results of our previous closed-form individual models do not follow simulators, especially at lower end of frequency inspite of our accounting for the internal inductance effect. It is true for all the parameters - effective relative permittivity, losses and characteristic impedance.
Table-5.7 consolidates the comparison of the models against HFSS for line parameters of CPW on the alumina substrate. All the EM-simulators are within average deviation of 1.8% amongst themselves while LINPAR has deviation w.r.t. the simulators with 5.2% average deviation. The circuit model accurately predicts and improves the dispersive nature of the CPW at the lower frequency range for computation of $e_{eff}(f,t)$, $\alpha_f(f,t)$, $\text{Re}(Z_0(f,t))$ and $\text{Im}(Z_0(f,t))$ with average and maximum deviation of (3.6%, 7.5%), (2.3%, 4.5%) and
Analysis and Modeling of CPW

(2.1%, 5.2%) respectively. The circuit model is applicable to all the line structures presented in this chapter. However for sake of brevity, we are not presenting the results.

Table - 5.7: % Deviation of models against HFSS
[Data range: t =1 µm; f = 0.01 GHz - 10 GHz; ε_r=9.8; tanδ=0.0002; σ=4.1x10^7 S/m]

<table>
<thead>
<tr>
<th>Model</th>
<th>ε_{eff}</th>
<th>Z_0 (Ω)</th>
<th>α_T (Np/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LINPAR</td>
<td>5.8</td>
<td>17.2</td>
<td>5.1</td>
</tr>
<tr>
<td>Sonnet</td>
<td>2.2</td>
<td>5.4</td>
<td>1.2</td>
</tr>
<tr>
<td>CST</td>
<td>2.5</td>
<td>5.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Closed-form Model</td>
<td>9.6</td>
<td>51.8</td>
<td>7.7</td>
</tr>
<tr>
<td>Circuit Model</td>
<td>3.6</td>
<td>7.5</td>
<td>2.3</td>
</tr>
</tbody>
</table>