Chapter-3

Warranty Analysis & Two Dimensional Modeling in Marketing

A warranty is a manufacturer’s assurance to a buyer that a product or service is or shall be as represented. It may be considered to be a contractual agreement between the buyer and manufacturer entered into upon the sale of the product or service. A warranty may be implicit or it may be explicitly stated. In broad terms, the purpose of a warranty is to establish liability among the two parties (manufacturer and buyer) in the event that an item fails. An item is said to fail when it is unable to perform satisfactorily its intended function when properly used. The contract specifies both the performance that is to be expected and the redress available to the buyer if a failure occurs. Warranties can be considered for both new and used products. New products can be divided into the following three categories:

1. Consumer durables (e.g., household appliances, cars) bought by individual households as a single item.
2. Industrial and commercial products bought by businesses for the production of services (e.g., equipment used in a hospital to provide medical care, aircraft’s used by airline operators) or products (e.g., components bought by a manufacturer).
3. Government acquisitions (e.g., new fleet of tanks or jet fighters) involving new and evolving technologies. As such, they are characterized by a high degree of uncertainty in the product development process.

Used products can be either consumer durables or industrial and commercial products and these are in general bought individually although sometimes they can also be

This chapter is based on the following papers:

bought in lots. Another related concept is that of an “extended warranty” or a “service contract”. The difference between a warranty and a service contract is that the latter is entered into voluntarily and is purchased separately – the buyer may even have a choice of terms, whereas a warranty is part of product purchase and integral to the sale.

Warranties are an integral part of nearly all commercial and many government transactions that involve product purchases. The buyer (individual, corporation, or government agency) point of view of a warranty is different from that of the manufacturer (or distributor, retailer, and so forth). Another is the societal point of view and this includes legislators, consumer affairs groups, the courts, and public policy decision-makers.

- **Buyer’s point of view**
  From the buyer’s point of view, the main role of a warranty in these transactions is protectional – it provides a means of redress if the item, when properly used, fails to perform as intended or as specified by the seller. Specifically, the warranty assures the buyer that a faulty item will either be repaired or replaced at no cost or at reduced cost. A second role is informational. Many buyers infer that a product with a relatively longer warranty period is more reliable and long lasting than one with a shorter warranty period.

- **Manufacturer’s point of view**
  One of the main roles of warranty from the manufacturer’s point of view is also protectional. Warranty terms may, and often do, specify the use and conditions of use for which the product is intended and provide for limited coverage or no coverage at all in the event of misuse of the product. The manufacturer may be provided further protection by specification of requirements for care and maintenance of the product. A second important purpose of warranties for the manufacturer is promotional. Since buyers often infer a more reliable product when a long warranty is offered, this has been used as an effective advertising tool. This is often particularly important when marketing new and innovative products, which may be viewed with a degree of uncertainty by many potential consumers. In addition, warranty has become an instrument, similar to product performance and price, used in competition with other manufacturers in the marketplace.
Warranty is seller assurance to a buyer that a product will carry out as stated and this promise works as a confidence for the buyer in purchasing the product. Whether a firm is selling commercial products or marketing to the Government, warranties is an imperative ingredient to competitive victory. Warranties are also an effective means of promoting a product in the market when the company or product is not well known. In other words warranty serves as a source for spread of an innovation in the market. But servicing a warranty engages additional costs to the manufacturer and this has an effect on the profit levels. In the first section, we have formulated an optimization problem that determine the optimal adoption time and sale price of product that taking into consideration the cost of warranty. The factors like fixed cost, production cost and inventory cost also considered in the problem of maximization of the profit. In the later section of the chapter, a scientific approach to meticulously examine the significance of warranty as a key marketing tool to promote sales of goods by virtue of warranty length optimization, which presumably results into an increase in the overall profit for an organization is proposed. A two-dimensional technology diffusion innovation model which combines the adoption time of technological diffusion and price of the technology product has been considered. In the model classical Cobb–Douglas production function is used that takes into account the technological adoptions and other dimensions explicitly.

The performance and application of the proposed models has been demonstrated using real data set. Soft Computing Technique like Genetic Algorithm and softwares like SAS, Maple and SPSS have been used to estimate the unknown parameters.

3.1 PROFIT MAXIMIZATION AND DETERMINATION OF OPTIMAL ADOPTION TIME AND PRICE FOR A PRODUCT UNDER WARRANTY COST

Product warranty is a major factor that influences the buyer’s decision to purchase the product in a high competitive marketing environment. Michael Porter said product differentiation is a tool for competitive advantage in the highly competitive markets. These widespread differentiation parameters may perhaps be the product or service offerings i.e. the product expansion through the value added services or offerings through product extensions. And one of the most influencing differentiating parameters is Product warranty. It is used as a instrument to catch the attention of the customer and
construct faith relationship between the buyer and seller. Many marketers employ warranty as improved service to its basic product offerings. It is used as a promotional tool by various well established companies for selling the product. Khetan was the first ceiling fan manufacturer in India to put forward seven years warranty. The rivals followed the steps in. Samsung CTVs buyers entitled to get additional three-year warranty as against the one year normally offered. That means overall warranty of four years was put on the purchase of the Samsung CTV product. LG refrigerators are covered under seven year’s product warranty. Likewise, numerous companies’ names can be referred to from every marketing segment in this list. Warranties are also one of successful means of selling a product in the market when the company or product is not well branded. Or in other terms warranty serves as a foundation for broadening of an innovation in the market.

The spread of an innovation in the market is termed as diffusion. An accepted definition of diffusion of innovations is “the process by which an innovation is communicated over time among the members of a social system” (Rogers 1995). Diffusion processes evoke a variety of possible research questions like one might wonder what drives growth, what will be the shape of an innovation’s life-cycle curve, and what will be the relationship between individual adoption decisions and aggregate market growth. These issues lead to significant implementation related questions such as the influences of marketing mix variables on the growth pattern of the industry and brand, the interactions between growth processes in various countries wherein the firm has presence and the effect of competition on growth. Diffusion research is the branch of marketing that aims to answer such questions through modeling the life cycle of new products. Since its start in the 1960s, diffusion research has been, and still is, the only modeling framework in marketing that is targeted at modeling the entire life-cycle course of an innovation from the perspective of communications and consumer interactions.

The Bass Model, its features, application has been extensively discussed in detail in Section 1.5.3, Chapter 1, some of its limitations can be stated as:

- The model doesn't take in the direct influence of any marketing-mix variable such as price or advertisement. Thus the effects of other diffusion parameters that may influence the growth of new product are not captured by Bass Model

- Bass model like most of the other diffusion models studies the time path of adoption or examines the individual's adoption of an innovation and doesn't spotlight on any other dimensions of marketing.

In today's scenario of marketing the sales rate is dependent not only on time but price is another one most important aspect that governs the pace of adoption. The application of time and price in managing sales and has significantly improved the landscape of management. This management in both manufacturing and service organizations have evolved tremendously over the years with the change in market requirements, Gunasekaran and Ngai (2012). Also, servicing a warranty involves additional costs to the manufacturer and this has an impact on the profit levels. Therefore the marketing diffusion model must incorporate marketing variables like price and marketing decisions must be made jointly so that the ultimate goal, should be the result of the improved effectiveness with an increase in the net profits to the firm.

In this section, we formulate an optimization problem that determines the optimal adoption time and sale price of product that taking into consideration the cost of warranty. The factors like fixed cost, production cost and inventory cost also considered in the problem of maximization of the profit. In the proposed optimization problem, a two-dimensional technology diffusion innovation model which combines the adoption time of technological diffusion and price of the technology product has been considered. In the model classical Cobb–Douglas production function (discussed in Section 1.6.7, Chapter 1) is used that takes into account the technological adoptions and other dimensions explicitly. The profit maximization problem is solved using genetic algorithm.

### 3.1.1 Warranty-Definition and Role

Whether a firm is selling commercial products or marketing to the Government, warranties are an important ingredient to competitive success. Where on one hand effective warranty planning can guarantee success, their lack of attention to cost analyses can spell disaster. Warranty is seller assurance to a buyer that a product will carry out as stated and this promise works as a confidence for the buyer in purchasing the product. To participate in the race of competition, the manufacturers may need to
recommend a better warranty so as to provide the customer with better assurance against the financial risk associated with a crash. However, this improvement generally involves incurring additional costs and as a result the manufacturing cost increases, which may imply a higher selling price. Price and warranty are two major commercial variables apart from time that influence sales decisions and ultimately the profit levels. To find out the optimal price and warranty Glickman and Berger (1976) presented a model that maximized a manufacturer's profit for a failure-free warranty policy. Nguyen and Murthy (1988) presented a model for achieving the optimal reliability allocation taking into consideration the effect of and warranty costs. A model to acquire the optimal price, warranty period and product reliability was proposed by Murthy (1990) that maximized a firm's profit. The cost of manufacturing and the price was assumed to be constant in all these models over the product life cycle. Further, Teng and Thompson (1996) measured the optimal price and quality strategy for the introduction of a new product. They examined the relationship between price and quality for the new product. Teng and Thompson research was modified by Lin and Shue (2005) and Wu et al. (2006) into a price-warranty decision model where the the quality level replaced the warranty length. A game-theoretic model representing in an oligopoly that chose warranty, reliability and price levels for their goods and examination of the Nash equilibria for the game was presented by DeCroix (1999). In the model proposed here we considered warranty cost as a factor which affects the objective of profit maximization of a product for a firm.

3.1.2 Building Blocks for Proposed Maximization Problem

The optimization problem proposed in later section of the chapter lays its foundation on the following building blocks:

1. Two Dimensional Product Sales Model
2. Warranty Policy

3.1.2.1 Two Dimensional Product Sales Model

Though most of the traditional diffusion model gave more emphasis on one dimension of diffusion (i.e. time), however the diffusion of an innovation can take place simultaneously due to other marketing dimensions also (Mahajan et al.(1990); Kalish, (1985)). The interest of this model is to focus on factors and dimensions of time and
price that affect the diffusion of an innovation. To make accurate sales forecast for a new innovation it is important to determine the factors affecting a product's diffusion process. Moreover, if the variables affecting the diffusion process are such that they could be adjusted then the whole diffusion process of the product may be controlled. Thus it becomes strategically important to explicitly identify the different drivers of the decision process. Van den Bulte and Lilien (2001) have identified two such factors: marketing efforts and word-of-mouth, and suggested that the initial knowledge of a new innovation occurs mainly through commercial sources such as salespeople and direct mailings, whereas personal contacts with colleagues gain importance in later stages. When a new technology takes the form of a product in the market it is desired that it gets the maximum exposure. The promoter runs promotional campaigns to advertise it among target consumers. Word also spread through interpersonal communication. It is expected that within a short period a large segment of target population become aware of the product. But often, this does not translate into immediate sales figures. One major reason can be the time taken by each individual buyer in making a purchase decision where pricing of the products can play an important role in decision making (Kalish, 1985).

In the optimization problem proposed we consider a two-dimensional technology diffusion innovation model proposed by Kapur et al. (2010) which combines the adoption time of technological diffusion and price of the technology product. The formulation of the functional relationship between the product continuation time in the market and the price of the product is given by Cobb and Douglas' (1928) production function. The framework has been discussed in Section 1.6.7 of Chapter 1. Refer figure 1.13 to check how the total production is influenced due to change in the proportion of labor and capital. The two dimensional model considered in the paper is given by differential equation (1.45) of chapter 1, i.e.

\[ \bar{n}(x) = \frac{d}{dx} \bar{N}(x) = p(m-\bar{N}(x)) + q \frac{\bar{N}(x)}{m}(m-\bar{N}(x)) = (p+q) \frac{\bar{N}(x)}{m} (m-\bar{N}(x)) \]  

(3.1)
Note that here

$\bar{N}(x)$: Cumulative number of adopters due to the value of the product $x$.

$n(x)$: The number of adopters due to the value of the product $x$.

$m$: Initial market size.

$p$ and $q$: Innovation and imitation coefficients respectively.

$x(T, Pri) = T^\alpha Pri^{1-\alpha}$: Cobb–Douglas functional form that represents the value of the product,

$x$: the value of the product,

$T$: continuation time of the product in the market,

$Pri$: the price of the product,

$\alpha$: the output elasticity.

with the initial condition $x(0,0)=0$ and $N(0,0)=0$, on solving Eq. (3.1) we have

$$\bar{N}(T, Pri) = mF(T, Pri) = m \frac{1-e^{-(p+q)T^\alpha Pri^{1-\alpha}}}{1 + \frac{q}{p} e^{-(p+q)T^\alpha Pri^{1-\alpha}}}$$

(3.2)

### 3.1.2.2 Warranty Policy

When a product is purchased and used by a customer then it might be possible that a difference between the product performance and customer expectation crops in. As a result the insurance policy of warranty comes into the picture. Warranty cost is a result of the conflict between product performance and customer expectation as depicted in Figure 3.1. The model proposed here assumes the warranty cost where there is disparity between product performance and customer expectation (Sharma et al., 2008).

For determining the warranty cost distribution of product performance and customer expectation needs to be determined. Product performance is calculated in terms of some characteristics which lie within a certain interval that convince the customer. A producer must decide a particular fixed target value within a certain range for a particular product; of course some variation is unavoidable. Hence, it is essential to reflect on the distribution of product performance. Moreover, a product is purchased by many customers who have different choices. The population of customers does not
have a permanent expected value for the performance of the product. Therefore, it is also indispensable to take into account the distribution of customer expectation.

Some Useful Definitions for studying our problem are as follows:

- **Warranty probability**

If $T_p$ denotes the total number of products and $S$ denotes the potential number of complaints, then the warranty probability is defined as: the ratio of potential number of complaints to total number of products. It is represented mathematically as

$$W_p = \frac{S}{T_p}$$

(3.3)

- **Actual number of complaints**

Complaint can come from those customers for whom product performance is smaller than customer expectation. From these potential complainers those who actually lodges the complaints comes into the population of actual number of complaints. We denote this number by (act _ n) in the paper.
• **Complaint factor**

The ratio of actual number of complaints to potential number of complaints denoted by \( \omega \) is termed as complaint factor in this research. Mathematically, it is of the form:

\[
\omega = \frac{\text{act}_n}{S}
\]

or

\[
\text{act}_n = \omega S
\]

Substituting value of S from (3.3) we get

\[
\text{act}_n = \omega T_p W_p
\]

• **Calculation of warranty probability dependent on product performance and customer expectation**

If it is assumed that customer expectation (CE) and product performance (PP) are independent of each other, then we can suppose that the distribution functions of both can be taken as are normal distribution, i.e.

\[
CE \sim N\left(\mu_{CE}, \sigma_{CE}^2\right)
\]

\[
PP \sim N\left(\mu_{PP}, \sigma_{PP}^2\right)
\]

Further, assume \( Y^* \) is a parameter that determines customer satisfaction which can be put mathematically as:

\[
Y^* = PP - CE
\]

If \( y^* \) is the maximum value for which the customer is satisfied; then there will be complaint from customer when \( Y^* > y^* \), i.e. the customer is dissatisfied.

Since PP and CE both follow normal distribution, therefore from statistics, \( Y^* = PP - CE \) will also be distributed normally as:
\[ Y^* = PP - CE \sim N \left( \mu_{pp} - \mu_{ce}, \sigma_{pp}^2 + \sigma_{ce}^2 \right) \]  

(3.10)

And, the probability of the customer not complaining will be:

\[ P \left( Y^* \leq y^* \right) = P[Z \leq z] = \Phi \left( \frac{y^* - \left( \mu_{pp} - \mu_{ce} \right)}{\sqrt{\sigma_{pp}^2 + \sigma_{ce}^2}} \right) \]  

(3.11)

The probability of the customer complaining will be:

\[ P \left( Y^* > y^* \right) = 1 - \Phi \left( \frac{y^* - \left( \mu_{pp} - \mu_{ce} \right)}{\sqrt{\sigma_{pp}^2 + \sigma_{ce}^2}} \right) \]  

(3.12)

If we suppose that the cost of making repair, say A, is proportional to the square of distance from customer satisfaction parameter to maximum value for which the customer is satisfied; then this cost will be given

\[ A = k \left( y^* - y^* \right)^2 \]  

(3.13)

where, \( k \) is a proportionality constant.

Therefore, warranty cost (WC) for all the products will be:

\[ WC = act \_ n.A = k \left( y^* - y^* \right)^2 \omega T_p \left( 1 - \Phi \left( \frac{y^* - \left( \mu_{pp} - \mu_{ce} \right)}{\sqrt{\sigma_{pp}^2 + \sigma_{ce}^2}} \right) \right) \]  

(3.14)

By setting \( T_p = 1 \), in above equation, we can get the unit warranty cost as:

\[ WC = act \_ n.A = k \left( y^* - y^* \right)^2 \omega \left( 1 - \Phi \left( \frac{y^* - \left( \mu_{pp} - \mu_{ce} \right)}{\sqrt{\sigma_{pp}^2 + \sigma_{ce}^2}} \right) \right) \]  

(3.15)

### 3.1.3 Profit Maximization under Warranty cost

Main aim of any kind of marketing activity is earning profit. Profit is the measuring technique to understand the efficiency of the marketing concern. Profit maximization is also the traditional and narrow approach which aims at maximizing the profit of the concern. The following important points are in support of the profit maximization objectives of the marketing firms:
• Profit is the parameter of the marketing operation.
• Profit reduces risk of the marketing firm
• Profit is the main source of finance.
• Profitability meets the social needs also.

This paper aims at maximizing the profit for a product when the time and price are the two factors that govern the sales rate. The notations used are given in Table 3.1.

### Table 3.1: Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>Fixed cost</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Production cost</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Holding cost</td>
</tr>
<tr>
<td>$\bar{N}(T, Pri)$</td>
<td>Potential Number of Adopters</td>
</tr>
<tr>
<td>$Q(N, Pr_i) = \theta\bar{N}^{\eta} Pri^{1-\eta}$</td>
<td>Production Function</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Minimum Demand of Product</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>Maximum Production of Product</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Complaint factor</td>
</tr>
</tbody>
</table>

#### 3.1.4 Model Development

The optimization problem takes into the factors like fixed cost, production cost and inventory cost and most importantly the warranty cost. Mathematically the problem is proposed as following:

$$
\text{Max } \text{profit}(T, Pri) = \text{Pri.}\bar{N}(T, Pri) - \left[ C_0 + C_1Q(T, Pri) + C_2(Q(T, Pri) - \bar{N}(T, Pri)) \right]
$$

$$
- k \left( Y^* - y^* \right)^2 \omega \left( 1 - \Phi \left( \frac{y^* - (\mu_{pp} - \mu_c)}{\sqrt{\sigma_{pp}^2 + \sigma_{CE}^2}} \right) \right) \bar{N}(T, Pri)
$$

Subject to

$$
\bar{N}(T, Pri) \geq N_0
$$

$$
Q(T, Pri) \leq Q_0
$$

$$
Q(T, Pri) - \bar{N}(T, Pri) \geq 0
$$
\[ Q(T, Pri) \geq 0 \]
\[ N(T, Pri) \geq 0 \]

In the problem given by equation (3.16); first term in the objective function is the sales price of \( N(T, Pri) \) products. Second term is the inventory cost which includes fixed cost, production cost, and inventory holding cost. And third term represents the warranty cost. For maximizing profit the inventory cost and warranty cost are subtracted from sales price of \( N(T, Pri) \) products.

The first constraint of the problem denotes the market coverage of the product. Second restriction is on the production of the product. Third constraint states that the Production should always be greater or equal to demand, so that firm does not go out of stock that creates a bad effect on its goodwill in long terms.

The above problem is solved using meta-heuristic soft computing method of genetic algorithm. Genetic algorithm (GA) has the ability to solve the optimization problem when the structure comprising of a product in a multi segmented market is complex. Genetic Algorithms stands up as a powerful tool for solving search & optimization problems. GA is an evolutionary algorithm where automation is the key objective which says the way our human body system is automated, the same way we want our computer software systems to operate automatically once the failure occurs. GA always considers a population of solutions that offers a lot of advantages. There is no particular requirement on the problem before using GA’s, as it can be applied to solve any kind of problem. GA has been used to solve many difficult engineering problems and is particularly effective for combinatorial optimization problems with large, complex search spaces.

### 3.1.5 Parameter Estimation and Numerical Illustration

For the profit maximization problem we have taken the sales data of cumulative adopters for 4 K, DRAM datasets (The data is available online http://phe.rockefeller.edu/LogletLab/DRAM.). DRAMs are the highest volume commodity semiconductors built today, with about 11% of the total semiconductor
market. It has shown clear discrete innovations in its product characteristics, especially in memory density. Also due to the PC boom and the growing need for memory in all information appliances, the DRAM sector became the lead product in the overall integrated circuit (IC) market in about 1990 (Victor & Ausubel, 2001). The parameters of the adoption sales model are estimated using nonlinear least squares (NLLS) (Srinivasan & Mason, 1986) by software package SAS (SAS User's Guide, 2004). The values of the parameters used are given in Table 3.2. Further, the parameters used in GA evaluation are: Population size, number of generations and are given in Table 3.3.

**Table 3.2: Parameter Values Used in Profit Maximization Problem**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>4</td>
</tr>
<tr>
<td>$C_1$</td>
<td>2</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.5</td>
</tr>
<tr>
<td>$p$</td>
<td>0.003</td>
</tr>
<tr>
<td>$q$</td>
<td>0.994</td>
</tr>
<tr>
<td>$m$</td>
<td>315</td>
</tr>
<tr>
<td>$A$</td>
<td>0.67</td>
</tr>
<tr>
<td>$k$</td>
<td>0.0592</td>
</tr>
<tr>
<td>$Y^* - y^*$</td>
<td>0.001</td>
</tr>
<tr>
<td>Complaint factor</td>
<td>0.05</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>30.7</td>
</tr>
<tr>
<td>$\mu_{CE}$</td>
<td>30</td>
</tr>
<tr>
<td>$\sigma_{pp}$</td>
<td>18.1</td>
</tr>
<tr>
<td>$\sigma_{CE}$</td>
<td>3.33</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.603</td>
</tr>
<tr>
<td>$\theta$</td>
<td>4</td>
</tr>
<tr>
<td>$N_0$</td>
<td>250</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>300</td>
</tr>
</tbody>
</table>
Table 3.3: Initial Values Used

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Population Size</th>
<th>Number of generations</th>
<th>Crossover Probability</th>
<th>Mutation Probability</th>
</tr>
</thead>
</table>

On solving the profit maximization problem as formulated in equation (3.16) through the technique of Genetic Algorithm and using the above parameters, the optimal adoption time comes out to be 7.78 units and the optimal price of the product was obtained as 6.4 units.

The two dimensional model discussed here helps the marketing manager to figure out the factors and dimensions that affect the diffusion of an innovation separately under the effect of goodwill and price. Warranty policies have been explicitly studied for several decades; with the study of guarantees against random occurrences (in the form of insurance) going back even further. With the emphasis on product quality and manufacturer cost cutting, the study of warranties has taken on new importance. The optimization problem formulated takes into account the factors like fixed cost, production cost, inventory carrying cost and most importantly the warranty cost that can help the managers to appropriately determine the profit for the firm.

3.2 OPTIMAL WARRANTIES AND TWO DIMENSIONAL INNOVATION DIFFUSION

Marketing is a concept that is not easy to define, as there are several different competing views of the role of marketing. It can be viewed as creating and managing markets, with market defined as an outcome of the interaction between manufacturers and consumers, leading to a transfer of goods (products and/or services). Product warranty plays an important role in the creation of market for the product and for managers to manage the market. This is because of consumer uncertainty regarding product performance over its useful life. Warranties serve as persuasive marketing tools: (i) promotional and (ii) protectional. As a promotional tool, warranties serve to
promote the reliability and quality of a product with longer and better warranty terms implying a more reliable product. As a protectional tool, warranties provide assurance to consumers against defective products that fail to perform satisfactorily over the warranty period. This assurance reduces the risks associated with purchase of the product.

For most products (such as consumer durables, industrial and commercial products), a manufacturer will have several competitors who are producing similar products and attempting to sell them to a given set of consumers, so that the market (for the product) is competitive. For some specific products (mainly industrial and commercial products), the manufacturer has no competitor so that the market is monopolistic rather than competitive. The market outcome depends on the interaction between several variables. On the manufacturer side, the variables include price, promotion, warranty etc. whereas on the consumer side, product choice (between no purchase/purchase; which of the competing brands to purchase) depends on several variables such as product features, perceived risk, brand, reputation, etc.

The initial point of any analysis is the consumer purchase decision. A microeconomic analysis of these issues considers market outcomes and resulting implications with regard to social welfare. In marketing the focus is on either total sales or sales over time. This information is used to manage other activities such as production rates, logistics, product diffusion etc.

Diffusion research has been broadly discussed in chapter 1 as well as in section (3.1) of this chapter. The framework used in this section is also dependent on utilizing the Cobb-Douglas production function as done Kapur et al (2010). Using the mathematical structure of Cobb-Douglas (1928);

$$x(t, r_s) = t^\alpha r_s^{1-\alpha}$$  \hspace{1cm} (3.17)

where,

- $x$ is the value of the product;
- $t$ is the continuation time of the product in the market;
- $r_s$ is the revenue per unit;
- $\alpha$ is the output elasticity.
The two dimensional model considered in the modeling framework is given by the following differential equation

\[ n(x) = \frac{dN(x)}{dx} = p(m - N(x)) + \frac{q}{m}N(x)(m - N(x)) \]  

(3.18)

where,

- \( N(x) \) is the cumulative number of adopters due to the value of the product \( x \);
- \( n(x) \) is the number of adopters due to the value of the product \( x \);
- \( m \) is the initial market size;
- \( p \) and \( q \) are the innovation and imitation coefficients respectively;
- \( r_s \) is the revenue generated per unit for the products.

With the initial condition \( x(0, 0) = 0 \) and \( N(0, 0) = 0 \), on solving Eq. (3.18) we have

\[ N(t, r_s) = m \left( \frac{1 - e^{-(p+q)t^{r_s}r_s^{-1-a}}}{1 + \frac{q}{p}e^{-(p+q)t^{r_s}r_s^{-1-a}}} \right) \]  

(3.19)

After discussing the formulation of the two dimensional model we focus on profit maximization that uses revenue, warranty length and production function as simultaneous decision variables. We again make use of the Cobb–Douglas functional form of production functions which has been extensively used to characterize the rapport of an output to inputs. It was proposed by Knut Wicksell in 1906, and tested against statistical evidence by Charles Cobb and Paul Douglas in 1928. The factors like fixed cost, production cost and inventory cost are also considered in the problem of maximizing profit. The current section takes into consideration two different scenarios in which the management of an organization firstly decides to sacrifice their initial profit by providing longer warranty length to ensure marketability and later decides to shorten the warranty length to increase overall profitability.

### 3.2.1 Warranty From Management’s Perspective

Consumerism in the last couple of decades has substantially redefined the relationship of buyer and seller in the marketplace. Consumer activism, legislation, and heightened
expectations have placed more responsibility on the manufacturer for the performance of his goods, with a corresponding increase in the "rights" of the consumer. Direct complaints to manufacturers from the better-educated, increasingly affluent and aware consumer have burgeoned in this new climate. Although business appears to be doing a much better job in dealing with irate consumers, expectations have apparently outstripped this improvement. Organizations thrive for better and efficient means to market their products and services in the market to yield maximum benefits.

Increasing profit is the prime goal of any organization and an effective way to achieve this goal is by promoting sales and goodwill of the manufacturer. In order to stimulate purchase willingness, manufacturers must convince consumers of the product quality. In such an increasingly intense global competition, a good warranty policy assists in leveraging the image of a high-quality product.

A warranty is a formal commitment by a manufacturer to provide certain responsibilities for product quality after the sale of the product. Through warranties, customers are provided guarantees for failure free, acceptable service for a period of time following the purchase of a product. In general, buyers believe that the length of the warranty period majorly coincides with higher quality and more reliability of a product. For manufacturers, a warranty program is an important tool in marketing products. It not only serves as a sales weapon to increase the sales volume, but also brings considerable profits. However, if the product quality is low, it can be expensive. Customers while purchasing any product carefully examines the warranty policy alongside its duration. Thus, a proper warranty plays an increasingly important role in commercial transactions. In literature, warranty has also been defined as one time decision for the manufacturer wherein the producer generally incorporates the component of warranty cost to its overall profit, as and when a customer raises a concern and the company decides to replace the faulty product (Free Replacement Warranty Policy). By doing so, producer not only restores faith within the consumer regarding the product of its future usage but also provides them with a chance to expect increase in sales during the later part of the product life cycle. The amount of goodwill a producer earns by providing warranty during the initial life cycle of the product ensures its carry over effect over the next stages of the product life cycle.
Many different warranty policies are available, and a summary of these may be found in Elsayed (1996) and references therein. Warranties offered by producers constitute one of the major characteristics of the quality of products. An examination of the current literature on the subject reveals that most publications deal with the evaluation of the economic cost of various types of warranties and the best warranty policy for a given warranty period. None of these investigations has coped with the problem of determining the optimal warranty period with maximizing overall profit. The closest treatment of optimal warranty was by Chun and Tang (1995), but they have dealt with the optimal warranty price as opposed to optimal warranty period.

According to a study by Ladany & Shore (2003, 2007) in which the manufacturer's lower specification limit (LSL) coincides with the offered warranty period, it was assumed therein that a longer warranty period increases the sale-price of the product, and the optimal warranty period, i.e., the optimal lower specification limit, was determined as to maximize the expected revenue per item. Further, they have extended their study and have addressed the problem of determining the optimal warranty period. In the present framework like Ladney & Shore (2003, 2007), we have also considered LSL to be the period upto which the manufactures provides warranty to the consumer and similarly upper specification limit (USL) to be the period by which the manufacturer expects his product to fail.

The current proposal takes into consideration two different scenarios befitting the focus of the study, i.e., the overall maximization of profit for an organization in presence of warranty as a key component

(I) how a producer sacrifices his initial profit and provides longer warranty length to ensures better marketability, and

(II) how during the later part of the product life cycle, a producer earns more profit by shortening the warranty length.

3.2.2 Proposed Profit Model

During the initial stages of the product life cycle when the sole motive of the organization is to increase sales, the manufacturer thrive to make their presence felt and for that they are ready to provide a longer warranty period and earn lower revenue. But
once their stand in the market becomes stable, the manufactures may go on to decide about earning higher revenue by decreasing the warranty length. In the present work, we propose to study the impact of warranty length in attaining higher profits for producer. In the first framework i.e. (M1) we have incorporated the effect of providing warranty on sales-price in periods to the consumer for the initial stages during the beginning of the sales cycle. As discussed earlier, it is the time when the manufacturer is ready to earn lesser revenue and creates his position in the market. This leads to reduced revenue earned in each period of the product life cycle, i.e.

- revenue loss due to product replacement within warranty period,
- revenue loss within Lower Specification Limit to Upper Specification Limit period (revenue loss accounted here is because of loss of goodwill)
- post Upper Specification Limit period, revenue loss due to non-failure of the product and the producer losing in terms of repeat sales.

Companies ensure extended product life cycle by providing longer warranty period and thereby increasing sales price of their products. However, the organization needs to focus primarily on increasing their overall profit rather than expected revenue per item. Customers intend to buy products with longer warranty length as providing longer warranty is considered as a symbol of manufacturer’s reliability and indemnity over the product.

In all the three cases discussed above, producer is aware of the lower revenue earned per period while being more inclined towards generating higher sales driven by longer warranty commitment.

With manufacturers product now established in the market, looks to earn higher profit by gaining an increment in revenue and by decreasing the warranty length during the later part of the product life cycle. Therefore, in the second framework i.e. (M2) we have incorporated the effect of providing warranty on revenue in periods to the consumer for the time when product is there in the market and organization aims to generate higher revenue. The behavior of the above described three cases in the new frame will be as follows:
• revenue loss due to product replacement within warranty period,
• no revenue loss within Lower Specification Limit to Upper Specification Limit period as organization now starts to think in terms of increasing their profit.
• post Upper Specification Limit period, revenue loss due to non-failure of the product and the producer losing in terms of repeat sales.

We will now have a look at the methodology and modeling of the proposed profit (warranty) function.

3.2.2.1 Profit Model Based on First Market Scenario (M1)

Let us assume that the manufacturer offers a warranty policy for an initial period upto Lower Specification Limit (LSL), during which he expects the product to perform to the best of its capabilities and expects the product to last before its Upper Specification Limit (USL). Let $r_s$ denotes revenue generated per unit in the absence of warranty. As discussed earlier, regarding the lower revenue earned during the product’s initial stages in the market, we assume:

• Let $(r_L)r_s, (r_L < 1)$, denotes the lower revenue (contribution to profit) per unit earned by the manufacturer during the warranty period as some part of the revenue, i.e. $(1 - r_L)$ the fractional (relative) loss per unit for items with life-time below the LSL acts as a warranty cost for the units failed.

• Let $(r_K)r_s, (r_L < r_K < r_U)$, denotes the lower revenue (contribution to profit) per unit earned by the manufacturer during LSL to USL as part of the revenue, i.e. $(1 - r_K)$ the fractional (relative) loss per unit for items with life-time between LSL & USL acts as a loss of goodwill of the manufacturer.

• Let $(r_U)r_s, (r_U < 1)$, denotes the lower revenue (contribution to profit) per unit earned by the manufacturer for products with life-time that exceeds the USL, as some part of the revenue, i.e. $(1 - r_U)$ the fractional (relative) loss per unit for these items, acts as a loss to the manufacturer due to lesser sales.

Let $f(x)$ denotes the lifetime density function of a product and the expected revenue earned per item is given by $R$: 

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\[ R = r_s \cdot LSL \cdot \int_{0}^{x} f(x)dx + r_u \cdot USL \cdot \int_{x}^{\infty} f(x)dx \] (3.20)

This can be further rewritten in terms of the cumulative probability function F as:

\[ R = r_s \cdot [r_L F(LSL) + r_K (F(USL) - F(LSL)) + r_U (1 - F(USL))] \] (3.21)

We also consider that the sales-price of a product linearly increases with the warranty period, so we have

\[ r_s = a + b \cdot LSL \] (3.22)

Let Q denotes the quantity produced by the manufacturer and is dependent on the demand of the product (expressed by number of units sold, \( N(t, r_s) \)), and on the product’s sale price \( (r_s) \). The relationship between these factors is expressed by the Cobb-Douglas type production function,

\[ Q(N, r_s) = \theta \cdot N^{\eta} \cdot r_s^{1-\eta} \] (3.23)

where \( \theta \) and \( \eta \) are positive parameters.

Let \( C_0, C_1, \) and \( C_2 \) denotes the set up cost, per unit production cost and per unit inventory carrying cost, respectively. The total cost comprising of fixed and the variable component at any time t and sales-price \( r_s \) will be

\[ [C_0 + C_1 \{Q(N, r_s)\} + C_2 \{Q(N, r_s) - N(t, r_s)\}] \] (3.24)

The Net Profit \( \psi \) from manufacturing and selling of the product at any time t and sales-price \( r_s \) will be

\[ \psi(t, r_s) = r_s \cdot [r_L F(L) + r_K (F(U) - F(L)) + r_U (1 - F(U))) \cdot N(t, r_s) \]
\[ - [C_0 + C_1 \{Q(N, r_s)\} + C_2 \{Q(N, r_s) - N(t, r_s)\}] \] (3.25)

for convenience we replace \((LSL, USL)\) by \((L, U)\). Given the parameters in the above profit equation and also the distribution of the life-time of the product, \( \psi \) becomes a function of \( t \) and \( r_s \). Our aim is to find optimal L that we extract by maximizing \( \psi \) and finding optimal \( r_s \), which when substituted in equation (3.22) yields optimal L.
3.2.2.2 Profit Model Based on Second Market Scenario (M2)

Once the manufacturer begins to realize that the marketability of his products has reached its desired level, he decides to alter his approach of sacrificing profit by providing longer warranty length and moves towards profitability. In order to ensure this, let us assume that the manufacturer offers a warranty policy for an initial period up to Lower Specification Limit (LSL), during which he expects the product to perform to the best of its capabilities and expects the product to last before its Upper Specification Limit (USL). Let \( r_s \) denotes revenue generated per conforming unit (those units whose lifetime lies in between LSL and USL). As discussed earlier, regarding the lower revenue earned during the product’s initial stages in the market, we continue to assume:

- **Let** \( (r_s)_L, (r_L < 1) \), denotes the lower revenue (contribution to profit) per unit earned by the manufacturer during the warranty period as some part of the revenue, i.e. \((1 - r_L)\) the fractional (relative) loss per unit for items with life-time below the LSL, acts as a warranty cost for the units failed.

- **Let** \( (r_s)_U, (r_U < 1) \), denotes the lower revenue (contribution to profit) per unit earned by the manufacturer for products with life-time that exceeds the USL, as some part of the revenue, i.e. \((1 - r_U)\) the fractional (relative) loss per unit for these items, acts as a loss to the manufacturer due to lesser sales.

Let \( f(x) \) denotes the lifetime density function of a product and the expected revenue earned per item is given by \( R \):

\[
R = r_s \int_0^{LSL} f(x)dx + r_s \int_{LSL}^{USL} f(x)dx + r_s r_u \int_{USL}^{\infty} f(x)dx
\]

(3.26)

It’s important to note that the revenue earned during the period LSL to USL is different from the lower revenue earned as proposed in equation (3.20). The full revenue earned during this period is due to the better market positioning of the manufacturer as a result of providing longer warranty during the initial stages of the product life cycle, which results in creating goodwill in the market and hence higher revenue.
This can be further rewritten in terms of the cumulative probability function F as

\[ R = r_s \cdot [r_L F(LSL) + (F(USL) - F(LSL)) + r_U (1 - F(USL))] \]  

(3.27)

The Net Profit \( \psi' \) in the current scenario from manufacturing and selling of the product at any time \( t \) and sales price \( r_s \) after incorporating the set up, production and inventory cost will be

\[
\psi'(t, r_s) = r_s \cdot [r_L F(L) + (F(U) - F(L)) + r_U (1 - F(U))] \cdot N(t, r_s) \\
- [C_0 + C_1 \{Q(N, r_s)\} + C_2 \{Q(N, r_s) - N(t, r_s)\}] 
\]  

(3.28)

where again for convenience we replace (LSL, USL) by (L, U). Once again our aim is to find optimal L that we extract by maximizing \( \psi' \) and finding optimal \( r_s \), which when substituted in equation (3.22) yields optimal L.

### 3.2.3 Numerical Illustration

In order to validate our model, we tested it on real data sets for DRAM computers and used it for estimation of the model parameters. The results of the two-dimensional diffusion innovation model parameter estimation are given in the Table 3.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>315.679</td>
</tr>
<tr>
<td>( p )</td>
<td>0.003</td>
</tr>
<tr>
<td>( q )</td>
<td>0.994</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 3.4

Parameter Estimates
The exponential distribution is obviously a more realistic representation of the distribution of life-time. Suppose that \( x \) is exponentially distributed with parameter \( L \). Hence,

\[
F(L) = 1 - e^{-\lambda L} = 1 - e^{-\lambda \left( \frac{r_s - a}{b} \right)} = F(a + b r_s)
\]  

(3.29)

Thus all expressions in the two models (equations 3.25 and 3.28) would be explicitly expressed in terms of \( r_s \) and \( t \). Now in order to solve the models proposed earlier, we have made use of the Maple software and the following numerical data:

\( a=3, b=0.4, \lambda=0.04, U=24, r_L=0.2, r_U=0.9, \eta=0.603, \theta=4 \).

Further, we assume that \( L_0 = \frac{r_L + r_U}{2} = 0.55 \), for computational tractability.

\[
F(L) = 1 - \exp(-\lambda L) = 1 - \exp(-\lambda \left( \frac{r_s - a}{b} \right))
\]  

(3.30)

\[
=1-\exp^{100000000\times r_s + 300000000}
\]  

(3.31)

\[
F(U) = 1 - \exp(-\lambda U) = 0.6171071140
\]  

(3.32)

Further, the behavior of the profit functions \( M_1 \) and \( M_2 \) is shown in Fig 1 and 2 respectively. Table 3.5 represents the optimal results.

**Table 3.5: Optimal Results for Price, Warranty Length And Overall Profit**

<table>
<thead>
<tr>
<th>M1 Profit Function Optimal Results</th>
<th>M2 Profit Function Optimal Results</th>
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<tbody>
<tr>
<td>( r_s )</td>
<td>( r_s )</td>
</tr>
<tr>
<td>10.89</td>
<td>9.13</td>
</tr>
<tr>
<td>( L )</td>
<td>( L )</td>
</tr>
<tr>
<td>19.73</td>
<td>15.33</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>( \varphi' )</td>
</tr>
<tr>
<td>809.27</td>
<td>1026.20</td>
</tr>
</tbody>
</table>
Fig: 3.2: Profit Behavior for M1

Fig: 3.3: Profit Behavior for M2