Chapter-4

Innovation Diffusion & Launch Time of Successive Generational Technologies

An important problem in marketing research is the diffusion of new products or of products in new markets. The diffusion process begins when the product is introduced into the market and progresses through a series of adoption events. This chapter is focused on studying the extensions of the very famous Bass Model. In the first part we have relaxed the assumption of market size to be fixed. Since, in reality the market size tends to change with time. The concept of change point has also been incorporated. In the later part of the chapter, we study another aspect which Bass initially did not put the light on, i.e. successive generations. Due to demand of highly qualitative and technically improved products the developers are forced to come up with new features in their products. The later section of the chapter is focused on this survival mantra that firms are using to survive in this intensified competitive environment i.e. bringing out their newer generation product frequently. In the earlier chapters of the thesis we have discussed some of the important and well known IDMs for multi-generations (Section 1.6.5, Chapter 1). Moving from single generation to multi-generation framework increases the complexity in the modeling framework. We are living in an era of globalization where cost is an important component amongst 4 C’s of marketing that

This chapter is based on the following papers:


determines the success of a product in the market. Clients/Marketers in the industries are now getting aware and are equally concerned about systematizing, strategizing, and optimizing their available resources. Furthermore, almost all industries aim at improving their existing products and so they have to come up with the newer versions of their product quickly in order to sustain themselves into the market. Therefore, one of their targets is set to focus on finding out optimal time of launch of their newer version product in the market and to spend as less as possible. Here, using some of the concepts from earlier studies, we have developed a mathematical model to find out the time of introduction of next generation when the overall cost is minimized. We have discussed the problem with the help of numerical illustration for a case of two generational products.

4.1 A DYNAMIC POTENTIAL ADOPTER DIFFUSION MODEL INCORPORATING CHANGE IN THE ADOPTION RATE

4.1.1 Notations

\( N(t) \) : Number of adoptions during time \( t \).

\( m(t) \) : Total population

\( m \) : Constant, representing the initial number of adoptions when the process starts.

\( p \) : Coefficient of innovation.

\( q \) : co-efficient of imitation.

\( \alpha \) : The rate at which the market is expanding.

\( b_i(t) \) : Time dependent rate of adoption per remaining number of adopters before and after the change point (\( i=1,2 \)).

\( \beta_i \) : Learning Parameter before and after the change point (\( i=1,2 \)).

\( b \) : Proportionality constant for adoption rate per remaining number of adopters.

4.1.2 Framework Used for the Proposed Model

The proliferation of newly introduced information, entertainment, and communication products and services and the development of market trends such as globalization and increased competition have resulted in diffusion processes that go beyond the classical
scenario of a single market monopoly of durable goods in a homogenous, fully connected social system. The diffusion modeling literature since 1990 has attempted to extend the Bass framework to reflect the increasing complexity of new product growth. The topic of innovation diffusion has been studied in depth by researchers from different disciplines, including sociology, economics, and marketing. The diffusion process in marketing is related to penetration of a market by a new product. Individuals who constitute the market may differ by the manner in which information about the innovation reaches them with respect to their responses to information. Innovators are those who purchase the product through external influence. During early stages of the process innovators alone are involved in the purchase decisions. Later through word-of-mouth from adopters, imitators adopt the product. Wide variety of diffusion processes have been studied in the literature. They can be classified into three categories viz. pure innovative, pure imitative and mixed influence. (Refer Section 1.5, Chapter 1 for detail).

Bass model has been widely used to model the diffusion flow in the market. To avoid mathematical complexity many simplifying assumptions have been taken while developing the Bass model. One such assumption is that the number of potential adopters \((m)\) is constant over the product life cycle. Modifications in the model have been proposed for increasing market size, where \((m)\) of equation (1.8), chapter 1 is substituted as \(m(t)\). The nature of \(m(t)\) depends on various sociological and economic factors. Hence no single functional form can describe the growth in market size for different products. This necessitated a modeling approach that can be modified without unnecessarily increasing the complexity of the resultant model. In 2004, Kapur et al gave a modeling approach that was achieved by changing the rate function. Equation (1.8) could be restated as

\[
\frac{dN(t)}{dt} = b(t)(m(t) - N(t))
\]

(4.1)

It is an accepted fact that an individual cannot be permanently classified as an innovator or an imitator. One who is an innovator for one product may be an imitator for another (Schiffman and kanu, 1995), again an imitator may also be exposed to media through which a product is promoted. Moreover, an individual who qualifies otherwise to be
called as an innovator may be by chance or by choice obtain an opinion about the product from a purchaser. Therefore, it was concluded that, though knowledge about the new product grows over time, attributing it to the distinct category as defined by Bass may be avoided.

Before the spread of the Bass model, a number of models were proposed, that linked the rate of adoption to the potential adopters. But Bass model due to its inherent flexibility could describe many of them. Hence, it was imperative to capture this flexibility and this article shows how it was done by proposing a logistic time dependent form for $b(t)$, i.e

$$b(t) = \frac{b}{1 + \beta e^{-bt}}$$

(4.2)

Consequently, the diffusion model takes the form:

$$\frac{dN(t)}{dt} = \frac{b}{1 + \beta e^{-bt}} (m(t) - N(t))$$

(4.3)

The solution for equation (4.3) obtained was:

$$N(t) = m \left( \frac{1 - e^{-bt}}{1 + \beta e^{-bt}} \right)$$

(4.4)

The equivalency of the two frameworks has been already discussed in Chapter 1.

### 4.1.3 A Dynamic Potential Adopter Diffusion Model

The Bass model assumes that the number of potential adopters remains constant over time. But with the increase in population due to factors like action of marketing mix variables and cross country effects, the market size tends to increase over time. Sharif and Ramanathan (1982) through their study of diffusion of oral contraceptives in Thailand illustrated the application of dynamic potential adopter diffusion models. They proposed a mathematical form for $m(t)$ for growth in potential adopter diffusion models. This form was substituted in external, internal and mixed diffusion models. A significant improvement in the fit for the resultant model was reported. Yet these models are complex and only one form could be tried by the authors on mixed influence diffusion model (Bass model).

In this article, we will discuss about the exponential form of growing pattern and so the equation (4.3) can be written as:
\[
\frac{dN(t)}{dt} = \frac{b}{1+\beta e^{-\beta t}} (m(t) - N(t)) \tag{4.5}
\]

We assume that the market size is increasing with an exponential rate and keep 
\[m(t) = me^{\alpha t}.\]

Consider \[\frac{dN(t)}{dt} = \frac{b}{1+\beta e^{(\alpha - \beta) t}} (me^{\alpha t} - N(t)) \tag{4.6}\]

the solution for equation (4.6) is:

\[
N(t) = \frac{mb}{\alpha + b} \left[ e^{\alpha t} - e^{\beta t} \right] \tag{4.7}
\]

where \(\alpha = \) rate of increase in population and the nature of \(m(t)\) is dependent upon the social system.

### 4.1.4 Dynamic Potential Adopter Diffusion Model with Change Point

We have already discussed about the concept of change point that can take place in marketing context. The rate of innovation per remaining potential adopters can be changed because of changes in marketing strategies. An innovation’s rate is also affected by the impact of change agents like promotional efforts. There are various factors which can cause change points in sales data that may include change in packaging, change in advertising strategy, increasing outlets, and availability of pack sizes, combination offers and discounts. The rates of innovation and imitation will also be changed (not in the same proportion), so the values of \(\beta\) i.e. \(\beta = \frac{q}{p}\) will also change.

Hence we will assume two values of \(b(t)\).

\[
b(t) = \begin{cases} 
  \frac{b_1}{1 + \beta_1 e^{-\beta_1 t}} & t \leq \tau \\
  \frac{b_2}{1 + \beta_2 e^{-\beta_2 t}} & t > \tau 
\end{cases}
\]

where \(\tau\) is the change point

Under the above assumptions, the following differential equations arise:
Case 1:
when \(0 < t \leq \tau\)

\[
\frac{dN(t)}{dt} = \frac{b_1}{1 + \beta e^{-(b_1) t}} (me^{\alpha t} - N(t))
\]

(4.8)

Solving and assuming \(N(t) = 0; \) at \(t=0\) we get:

\[
N(t) = \frac{mb_1}{\alpha + b_1} \left[ \frac{e^{\alpha t} - e^{-(b_1) t}}{1 + \beta e^{-(b_1) t}} \right]
\]

(4.9)

Case 2:
when \(t > \tau\)

\[
\frac{dN(t)}{dt} = \frac{b_2}{1 + \beta e^{-(b_2) t}} (me^{\alpha t} - N(t))
\]

(4.10)

Solving and assuming at

\( t = \tau; N(t) = N(\tau) \)

We get:

\[
N(t) = \frac{mb_2}{\alpha + b_2} \left[ \frac{e^{\alpha t} - e^{-(b_2) t}}{1 + \beta e^{-(b_2) t}} \right] + \frac{mb_1}{\alpha + b_1} \left[ \frac{e^{\alpha \tau} - e^{-(b_1) \tau}}{1 + \beta e^{-(b_1) \tau}} \right] \left[ \frac{1 + \beta e^{-(b_1) \tau}}{1 + \beta e^{-(b_2) \tau}} \right] e^{-(b_2) (t-\tau)}
\]

(4.11)

4.1.5. Description of Data

The proposed model has been validated on the sales data sets of three products namely room air conditioners, color television receivers and telephone answering machines (Bass, Krishnan and Jain 1994). We consider the parameter as given, since the marketing strategy and promotional effort allocation can be tracked all the time during the life cycle of a product.

Consider Table 4.1 for the location of change point of DS-I, DS-II and DS-III.
4.1.6 Parameter Estimation and Comparison Criteria

We have estimated the parameters of the proposed model using SPSS tool based on the non-linear least squares method. The estimated parameter values of the proposed model are given in Table 4.2. The $R^2$ (coefficient of determination) values in Table 4.2 are close to 1, signifying a good fit of the proposed model.

4.1.7. The Predictive Validity of the Change Point Model

Predictive validity is defined as the validity of the model to determine the future behavior of the sales curve from present and past data. Assume that data is available for n time periods and that the corresponding sales figure is $S_n$. The data up to $s_i (\leq S_n)$ is used to estimate the parameters of $N(t)$. The sales figure for $S_n$ can be then predicted from $\tilde{N}(S_n)$, which can be then compared with the actually observed value $N(S_n)$. This process is repeated for various values of $S_i$. The normalized error

$$\frac{\tilde{N}(S_n) - N(S_n)}{N(S_n)}$$

called relative predictive error for different sample sizes is given in Table 4.3. A low error value indicates better predictive validity of the models.

Table 4.1: Location of change point

<table>
<thead>
<tr>
<th>S. No.</th>
<th>DS</th>
<th>Change point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Room air conditioners (I)</td>
<td>at 7th</td>
</tr>
<tr>
<td>2</td>
<td>Color television receivers (II)</td>
<td>at 6th</td>
</tr>
<tr>
<td>3</td>
<td>Telephone answering machines (III)</td>
<td>At 8th</td>
</tr>
</tbody>
</table>
### Table 4.2: Estimation Results

<table>
<thead>
<tr>
<th>Model under comparison</th>
<th>DS-I</th>
<th>DS-II</th>
<th>DS-III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dynamic Diffusion Model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>15150</td>
<td>36590</td>
<td>23350</td>
</tr>
<tr>
<td>$b$</td>
<td>.429</td>
<td>.534</td>
<td>.332</td>
</tr>
<tr>
<td>$\beta$</td>
<td>49.498</td>
<td>65.062</td>
<td>36.244</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.011</td>
<td>.022</td>
<td>.104</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.99868</td>
<td>.99960</td>
<td>.99942</td>
</tr>
<tr>
<td>M.S.E</td>
<td>29001.03</td>
<td>495047.7</td>
<td>29125.58</td>
</tr>
<tr>
<td><strong>Proposed Model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>15440</td>
<td>36640</td>
<td>32570</td>
</tr>
<tr>
<td>$b_1$</td>
<td>.001</td>
<td>.001</td>
<td>.021</td>
</tr>
<tr>
<td>$b_2$</td>
<td>.192</td>
<td>.054</td>
<td>.019</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>6.198</td>
<td>.652</td>
<td>1.060</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>169.754</td>
<td>272.891</td>
<td>4.702</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.053</td>
<td>.193</td>
<td>.313</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.99915</td>
<td>.99959.</td>
<td>.99962</td>
</tr>
<tr>
<td>M.S.E</td>
<td>19504.7</td>
<td>20451.42</td>
<td>16198.7</td>
</tr>
</tbody>
</table>

### Table 4.3: Predictive Validity

<table>
<thead>
<tr>
<th>Years</th>
<th>DS-I dynamic</th>
<th>Proposed</th>
<th>DS-II Dynamic</th>
<th>Proposed</th>
<th>DS-III dynamic</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>6</td>
<td>---</td>
<td>---</td>
<td>-1241.77</td>
<td>7389.404164</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>7</td>
<td>4401.735</td>
<td>4700.493261</td>
<td>-58.3874</td>
<td>10926.66693</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>8</td>
<td>6016.08</td>
<td>6071.077453</td>
<td>589.4263</td>
<td>15220.53235</td>
<td>7706.317</td>
<td>7662.807346</td>
</tr>
<tr>
<td>9</td>
<td>7824.701</td>
<td>7583.125993</td>
<td>633.4042</td>
<td>20443.52937</td>
<td>11162.83</td>
<td>10943.76437</td>
</tr>
<tr>
<td>10</td>
<td>9684.31</td>
<td>9229.050699</td>
<td>552.6451</td>
<td>26806.83092</td>
<td>15715.37</td>
<td>15465.2287</td>
</tr>
<tr>
<td>11</td>
<td>11440.78</td>
<td>10997.75198</td>
<td>-269.528</td>
<td>34568.78355</td>
<td>21491.82</td>
<td>21697.02138</td>
</tr>
<tr>
<td>12</td>
<td>12978.9</td>
<td>12875.66626</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>13</td>
<td>14247.3</td>
<td>14848.0718</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
Figures 4.1, 4.2 and 4.3 represent the goodness of fit curves for the three data sets respectively. It can be seen that the proposed model fits all the three data sets reasonably well. This can be further justified by the value of $R^2$ which are .99915, .99959 and .99962 for room air conditioners, color television receivers and telephone answering machines respectively. Also, from the table 4.2 it can be seen that the value of MSE is lower as compared to the value of MSE for the IDM in dynamic environment. Therefore, it can be concluded that the concept of change point covers the realistic situation quite well.

![Figure 4.1: Goodness of Fit for DS-I](image)

**Figure 4.1: Goodness of Fit for DS-I**
Chapter 4  Innovation Diffusion & Launch Time of Successive Generational Technologies

Figure 4.2: Goodness of Fit for DS-II

Figure 4.3: Goodness of Fit for DS-III
4.2 MANAGING LAUNCH TIME OF SUCCESSIVE INDUSTRIAL TECHNOLOGIES

Since the competition is at its peak level, the industries are rapidly coming up with the newer versions of their products. As new projects are launched, a strategy to meet the challenges from the existing players, or future competitors is needed. Hence, a carefully researched, planned and calculated move is necessary in every field. Further in the development of products, cost associated in development and their promotions for launching the newer product are also an important factor to be considered. Therefore, the role of successful planning for the firm to keep its product high in the market becomes more critical.

Today’s business world is client oriented, one who satisfy customer requirements can sustain in the market effectively & profitably for long. In context of diffusion of a product, adoption and cost are two major concerns of client satisfaction. Both the constraints being measures of business require their quantification. Adoption on one hand is of great importance to quantify the performance the product in the market. The price associated with the product on the other hand plays a vital role in describing the quality of the product and indirectly its adoption capacity. But things have changed from past drastically, the mind set is now shifting; Cost to satisfy (vs. Price). It is to note that price is one part of the cost to satisfy. If you sell hamburgers, for example, you have to consider the cost of driving to your restaurant, the cost of conscience of eating meat, etc. One of the most difficult places to be in the business world is the retailer selling at the lowest price. If you rely strictly on price to compete you are vulnerable to competition in the long term.

Now-a-days the major concern for the firms is to plan the launch of the next generation product. Owing to the prevailing paradox between the newer generation’s launch time and resources limitation for the developers; an imperative decision problem, which arises is to determine when to stop spending on the product and release it into the market. Such problems are called Optimal Launch Time Problems. Product users crave for faster deliveries; cheaper as well as quality products whereas developers desire to minimize their development cost, maximizes the profit margins and meet the competitive requirements. Refer Fig 4.4; if the firm enters the market too soon it might have the space to itself, well at least for a little while, or share it with only a few
incumbents but it will also bear the brunt of promoting that space and be a pioneer in finding out if that space is a good place to be profit-wise.

Figure 4.4: Innovation life cycle

On the flip side, if the firm wait too long to enter the market it might lose ground to competitors, and it risks the window of opportunity, but one might also gain valuable insights from observing the incumbents as well as benefiting from the marketing efforts of others who are growing the market in that space. In context of successive generations of the products we can say that if the launch of the successive generation is overly delayed, the manufacturer (developer) may undergo thrashing by means of penalties and revenue loss, while a premature release of new version may cost heavily in terms of product failure and this might also consequently harm manufacturer’s reputation. Thus, a tradeoff between conflicting objectives is required. In this paper we formulate a cost optimization model in order to find out optimal launch time of the product under the constraint of having a desired proportion of adoptions from the n\textsuperscript{th} generation.

Study of technology-market structure analysis involves describing how product generations compete against each other. New innovation in the market doesn’t immediately replace the previous one that it intends to substitute, but starts to compete with it. This creates a sequence of parallel diffusions of the existing generational
products in the market. The need for introduction of a new technology generation can be explained with the help of S-curve or sigmoid diffusion curve (fig 4.4). During the initial period of the product life cycle the adoption may be slow with promoters trying for acceleration (intensity of diffusion). Later the rise in the curve is steep especially for successful products. But sales (profitability) of a technology product can be improved only till it attains its natural performance limit. When a technology starts approaching its natural limit of performance, a new technology needs to take over and begin its life cycle (fig 4.5). Not all technology substitution becomes successful, making new products risky but necessary ventures for companies. Quite naturally the new technology which is replacing the older one has a natural limit of performance that exceeds the market saturation point of the old technology. Technological progress in a particular field is a result of sequence of growth curves. When these families of S-shaped curves are taken together, they depict the progress of a particular technology.

The present work focuses on deciding the introduction timing of the newer generation product keeping the cost associated with the product’s development, maintenance and promotion minimum. The following section talks about the modeling framework used to develop the optimizing problem in hand. The model beautifully discusses the concept of substitution. It is interesting in the sense that it incorporates the repeat-purchase behavior as well.

**Fig 4.5: Different launching times**

The present work focuses on deciding the introduction timing of the newer generation product keeping the cost associated with the product’s development, maintenance and promotion minimum. The following section talks about the modeling framework used to develop the optimizing problem in hand. The model beautifully discusses the concept of substitution. It is interesting in the sense that it incorporates the repeat-purchase behavior as well.
4.2.1 Diffusion of Products: From Single to Successive Generations

As already discussed in Chapter 1, the Bass diffusion model was developed by Frank Bass and describes the process of how new products get adopted as an interaction between users and potential users. It has been described as one of the most famous empirical generalizations in marketing. The model is widely used in forecasting, especially product forecasting and technology forecasting. It is considered as to be the base model for many diffusion theories that have been developed since its origin in 1960s. As discussed earlier the classical Bass model doesn’t recognize the relationships between different product categories. Thus the model holds that the adoption of an innovation doesn’t complement, substitute, eliminate or enhance the adoption of another product (or vice versa).

Dante and Weil argued that innovation in the market can’t remain in isolated and suggested that another innovation or existing product can either positively or negatively influence its diffusion process. The market success of a given innovation may even be aided by another product (multi-product interactions) or product generation (successive generations). Therefore the models of technology generations should be distinct and should identify the potential market for each generation, the growth rate of consumer preference towards a generation and repeat buying behaviors. Norton and Bass provided one of the earliest diffusion models that can describe the sales growth phenomenon for multiple generations competing in the market. Norton and Bass model is a classic example of multiple generation models, which is again built upon the Bass model. In the model it is assumed that the coefficients of innovation and imitation remain unchanged from generation to generation of technology. But many authors have argued against this assumption. Islam and Meade have tested the hypothesis of coefficient constancy across generation of Norton–Bass (1987) model. Their empirical work relaxed the assumption of constant coefficient. They proposed that the coefficients of later generation technology are constant increment/decrement over the coefficients of the first generation. Mahajan and Muller (1996) proposed a model, which is again an extension of Bass model to capture simultaneously both the substitution and diffusion patterns for each successive generation of technological
products. Speece and MacLachlan (1995), Danaher (2001) et al., developed a model in a different way by incorporating price as an explanatory variable. In this article, we propose a more general model that captures both diffusion and substitution process for each generational technology product. The latest in the fame is a model developed by Jiang (2010), which described the substituters and switchers of generations which many other authors did not put the light on. The optimization model developed here is dependent on the modeling framework by Kapur et al (2010) which is described in the following section.

4.2.2 A Diffusion Model with Reverting Possibility (Kapur et al (2010)):
The Framework for Proposed Model

Technology advances significantly speed up new product development, resulting in several generations of the same product coexist in the consumer market. The most familiar examples include Apple's iPod and iPhone family of products and Microsoft Windows and Offices lines of products, Samsung memory cards and so on. Newer models and versions are continuing to emerge (Refer Fig 4.6). As the time interval between new product releases decreases, it is important to understand the impact of new technologies on earlier ones.

Fig 4.6: Successive Generations of memory cards
The new generation technology gives consumers a better product that they value more. When new technologies become available, it provides an opportunity for potential adopters of earlier technologies to substitute the more recent technologies for earlier ones. In anticipation of price reductions, consumers may choose to wait for the sale. This is termed as inter-temporal substitution by customers, which diminishes the potential sales of the earlier technologies.

Their model is based on the following assumptions:

- Once adopters adopt a new technology, she/he may revert to earlier generation technology.
- Each adopter (New Purchasers) can purchase exactly one product unit and she/he makes no further purchases of the product generations that they have adopted. Also, each adopter after having made the first purchase may make a repeat purchase of exactly one unit in each successive generation or they can skip a generation and wait for more advanced one.
- A consumer's choice in each time period can be independent of her/his choice in previous periods and depends on the utility of the new product.
- Like the first time purchasers, repeat purchasers (i.e. buyers who have also purchased from previous generations) can also be influenced by the word-of-mouth effect and as well as the marketing-effort made by the firm. Both the first time purchasers and the repeat purchasers can be modeled according to the Bass model.

The authors here assumed a monopoly market situation and that each adopter can adopt only one unit from each generation. In general we do expect the potential market size to increase monotonically after $\tau_1$ (introduction time of second generation product), but it may not hold for all generations. This model shows continuous flow dynamic character. The model for different market situations has been build up as follows:

**Case 1:** When only first generation product exists in the market:
When there is only first generation product in the market, the cumulative sales pattern of that generation can be given by the Bass model.

\[
N_1(t) = m_1 \left( \frac{1 - e^{-(p+q)t}}{1 + \left( \frac{q}{p} \right) e^{-(p+q)t}} \right)
\]  

(4.12)

**Case 2:** When first two generations are in the market:

In the innovation diffusion model discussed above the Bass model the potential adopter population for the first generation is assumed to be constant. When the second generation is introduced, there would be a sizable portion of this population who are yet to purchase. So, when they have the option to choose they will go with the product having maximum utility. Satisfied adopters can influence others, with positive word-of-mouth. As a result a fraction of the potential adopters of first generation (say, \( \alpha_2 \)) switches to the second generation technology and the remaining \([1 - \alpha_2]\) will adopt the first generation technology; and at the same time a fraction of the potential population from second generation \( \alpha_1 \) (say) may find the first generation more appealing. As a result they may skip the second generation for the first generation product. \( \alpha_j \) can be defined as the skipping parameter (to the jth generation). Mahajan and Muller have considered the parameter as constant over time. Apart from it, due to the influence of promotional effort and word-of-mouth some buyers of the existing generation technology may go for repeat purchase of the other generation technology and eventually become the potential repeat purchasers of the either of the generations. Thus in two generation market scenario cumulative number of purchasers for two generation can be given as:

\[
N_2(t) = m_2F_2(t - \tau_1) + B_2(t - \tau_1) - B_1(t - \tau_1) + R_2(t) \quad t \geq \tau_1
\]  

(4.13)

\[
N_1(t) = m_1F_1(t) - B_2(t - \tau_1) + B_1(t - \tau_1) + R_1(t) \quad t \geq \tau_1
\]  

(4.14)

In the above equation \( B_2(t - \tau_1) \) are those first time purchasers who have skipped the first generation product and instead purchased the second generation product.
\( B_1(t - \tau_1) \) are those first time purchasers who have skipped the second generation product and instead purchased the first generation product.

\( R_j(t) \) are the repeat purchasers of the \( j^{th} \) generation product.

Many authors have suggested different mathematical forms for the skipping parameter (Danaher et al. 2003). In this approach, the authors assumed that throughout the sales horizon skipping takes place at the constant rate. The mathematical form can be given as follows:

\[
B_2(t - \tau_1) = \begin{cases} 
0 & \text{when } t < \tau_1 \\
\alpha_2 m_1 (F_1(t) - F_1(\tau_1)) & \text{when } t \geq \tau_1
\end{cases} \quad (4.15)
\]

\[
B_1(t - \tau_1) = \alpha_1 m_2 F_2(t) \quad \text{when } t \geq \tau_1 \quad (4.16)
\]

Where \( F_j(t) \) is the cumulative adoption function with parameters \( p_j \) and \( q_j \) respectively.

Buyers from any of the generation product can become the potential purchasers of the other technology, thus the total number of potential repeat purchasers of any one generation is all the prior purchasers of other generation. Thus all the prior first time purchasers of second generation product can become the potential repeat purchasers of first generation product and vice versa. Thus \( R_1(t) \) and \( R_2(t) \) can be defined as:

\[
R_1(t) = \left( m_1 F_2(t - \tau_1) + B_2(t - \tau_1) - B_1(t - \tau_1) \right) F_1(t) \quad t \geq \tau_1 \quad (4.17)
\]

\[
R_2(t) = \left( m_1 F_1(t) - B_2(t - \tau_1) + B_1(t - \tau_1) \right) F_2(t - \tau_1) \quad t \geq \tau_1 \quad (4.18)
\]

The present study is based on this model in determining the number of adopters for the next generations. Successful product planning requires creating a long-term plan for generational changes and advances to maintain market share and customer loyalty. From electronic to software products, the long-term trajectory of products requires
changes to keep products fresh and appealing to customers. The most effective multi
generational product planning generally includes defensive measures to keep your
product competitive. Frequent new product release and technological uncertainty about
the release time pose significant challenges for firms to manage successive generation
of products. On the one hand, strategic consumers may delay their purchase decision
and substitute the earlier generation with the newer generation product. On the other
hand, the firm must fully anticipate consumer reactions and take into account the effect
of their strategic behavior on product pricing and successive generation product
diffusion.

4.2.3. The Optimal Launch Planning

A major concern for the industries is to plan the launch of the successive generational
product. Many problems have been studied in the literature focusing on the specific
context of durable goods only. However, in this section, we describe the cost modeling
framework that has been generalized to a broader range of marketing settings where
consumers make decisions about related products that are sequentially released. The
research is based on the monopolistic assumption, where a firm introduces an advance
generation product at time $\tau$. Each consumer needs at most one unit in that product
category. We are interested in understanding the optimal time of offering the advance
generation product in the market place under the cost constraint. In the planning of
launch decision for the product that is to be brought into the market with new versions;
the firm has to take into consideration two things:

(i) Adopters of the newer generation.
(ii) The remaining number of adopters from the previous generation.

In determining the optimal launch policy for a product with no generation, only (i)
prevails. On the other hand, if (ii) is not taken into consideration for the optimal
planning of multi-generational products, then the notion of coming up with various
versions is lost. This is because the decision of launching the product lays its
foundation on the total adopters in the market. In multi-generational case, the adopters
from the previous version also become the potential adopters for the newer ones.
Therefore, to formulate an optimal launch plan for multi-generational product we require diffusion model incorporating the concept of multi-generations.

4.2.3.1 The Cost Model

Suppose the firm has to launch the $n^{th}$ version of the product into the market. Then the cost function will include the production costs for $n^{th}$ generational product and the promotional cost.

Let,

- $\tau_{n-1}$ be the time of launch of the $n^{th}$ generation product.
- $C_{n1}$ be the cost of production of the $n^{th}$ generation product till the $(n+1)^{th}$ generation product is introduced in the market (i.e. $t \leq \tau_n$).
- $C_{n2}$ be the cost of production of the $n^{th}$ generation product after the introduction of the $(n+1)^{th}$ generation product in the market (i.e. $t > \tau_n$).
- $C_{n3}$ is the promotional cost per unit time for the $n^{th}$ generation product (i.e. $t \leq \tau_n$).
- $N_n(t)$ is the cumulative number of adopters of the $n^{th}$ generation product till time $\tau_n$.
- $C_n(t)$ is the total expenditure made by the firm on the $n^{th}$ generation product at time $\tau_n$.

Throughout the analysis we have assumed that $C_{n2} > C_{n1}$. The assumption is quite intuitive in nature. Till time ‘$\tau_{n-1}$’ (i.e. before introduction of the $n^{th}$ generation product) the sales of earlier generation will be on the higher side as a result the cost per unit production decreases. But as the $n^{th}$ generation product is introduced at time ‘$\tau_{n-1}$’, it will reduces the rate of adoption of earlier generation product considerably and that increases the per unit production cost drastically.

Under the model assumption and notation, the cost function $C_n(t)$ can be defined as:

$$C_n(t) = C_{n1}N_n(t) + C_{n2}\left[m_n + m_{n-1}F_{n-1}(t) - N_n(t)\right] + C_{n3}t \quad (4.19)$$

It is to note that here $\{m_{n-1}F_{n-1}(t)\}$, is actually combination of two components; $m_{n-1}(F_{n-1}(t) - F_{n-1}(\tau_{n-1}))$ that represents the remaining number of adopters of earlier
generation who would have bought the older generation if new generation would not have been there in the market. As discussed, we term these adopters to be SUBSTITUTERS. The second component \( m_{n-1} F_{n-1}(\tau_{n-1}) \) that describes adopters who have purchased the older generation and yet they plan to buy the newer generation again, so, represents the number of adopters who actually switch. We call this behavior as SWITCHING. Here, \( m_n \) represents the potential number of adopters for \( n^{th} \) generation specifically. Therefore we can say that together with \( m_n \) they (substituters and switchers of the earlier generation) become the potential buyers for the newer generation.

To determine the optimal time \( t^* \), the objective function can be written as

\[
\min C_n (t) = C_n t N_n (t) + C_{n2} \left[ \{m_n + m_{n-1} F_{n-1}(t)\} - N_n (t) \right] + C_{n3} t
\]

subject to

\[
N_n (t) \geq \rho \{m_n + m_{n-1} F_{n-1}(t)\}
\]

where,

\( \rho = \text{desired proportion of adoption} \)

\( F_n(t) \) represents the fraction of adoption for \( n^{th} \) generation as defined by equation 7.

4.2.4. Data

We assume the firm sells two generations of the product. The second generation is introduced at \( \tau_r > 0 \). The model setup is motivated by Colour TV release in the market. Television in India has been in existence for about four decades. For the first 17 years, it spread haltingly and transmission was usually in black and white. In the initial years the Indian Television Industry expanded very slowly. Many factors were responsible for the lukewarm response from the consumers that include the disastrous government policies along with the popular belief that television was a luxurious element that Indians could do without. Sales of TV sets, as reflected by licences issued to buyers were just 676,615 until 1977. Post liberalization the industry has been witnessing robust demand; fuelled by revival in economy, increase in individual disposable income and liberal incentive schemes by banks and financial institutions. The demand for CTV grew at 15% during 1985-89 but witnessed a slump from 1990-
With the entry of MNCs and thereby aggressive marketing, the period between 1995-1996 to 1999-2000 saw a surge in growth rate to 29%. In between this period though the market share of B&W television declined rapidly but it was never been out of the market. Colour television was introduced in Indian market in 1981. Though post liberalization colour television to large extent substitute the market share of B&W television but it couldn't cannibalize the market completely.

4.2.5. Model Validation

The parameters present in equations (4.13 and 4.14) were estimated using nonlinear least squares (NLLS) by software package SAS. To validate the predictive performance of the model we have considered data from television industry (1970-1992). Estimated value of diffusion parameters of each of the two technologies are given in Table 4.4. Figure 4 provides the goodness of fit curve.

Table 4.4: Parameter Estimates; (*) in millions

( j = 1, 2; for B&W and Color TV)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(m_j)</th>
<th>(p_j)</th>
<th>(q_j)</th>
<th>(\alpha_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&amp;W</td>
<td>284.94</td>
<td>0.00015</td>
<td>0.18684</td>
<td>0.01442</td>
</tr>
<tr>
<td>Color</td>
<td>20.983</td>
<td>0.00256</td>
<td>0.73477</td>
<td>0.67706</td>
</tr>
</tbody>
</table>
For demonstrating the formulation and solution of the optimal launch plan (OLP) as proposed in sec 4.2.3.1, we choose the above data. Here, B&W Television is already there in the market. And color Televisions have been launched 10 years after their arrival in the market. The problem formulated in equation (4.20) determines when to launch the next version (i.e. the Color television) such that the cost of development/production and promotion is minimized.

Therefore the problem developed in section 4.2.3.1 and equation (4.20) can be written as:

\[
\begin{align*}
\text{Min. } C_2(t) &= C_1 N_2(t) + C_2 \left[ (m_2 + m_1)F_1(t) - N_2(t) \right] + C_3 t \\
\text{subject to } N_2(t) &\geq \rho \{m_2 + m_1, F_1(t)\}
\end{align*}
\]

(4.21)
Now as can be seen from table 4.4, the worldwide estimated initial market size of B&W Television can be given as 284.94 (in millions) and for color televisions 20.983 (in millions).

The other base values considered in the model are:

\[ C_1 = 5 \text{ (in lakhs)} \quad C_2 = 18 \text{ (in lakhs)} \quad C_3 = 3 \text{ (in lakhs)} \quad C_B = 500 \text{ (lakhs)} \quad \rho = 67\% \]

i.e. before introduction of the second generation product the firm wants to capture the 67% market share of the first generation product. Thus, assuming all these values the objective function stated in equation (4.21) can be written as:

\[
\begin{align*}
\text{Min. } C_2(t) &= 5N_2(t) + 18 \left[ \frac{1}{2}[287.94 + 20.983.F_1(t)] - N_2(t) \right] + 3t \\
\text{subject to} \quad N_2(t) &\geq 0.67[287.94 + 20.983.F_1(t)] \\
\end{align*}
\]

The objective here is to find the optimal value of \( t \) (i.e. the optimal time of introduction of color TV sets) under the minimum cost criterion. The above function is maximized by usage of Maple package and the minimum cost comes out to be 253.21 (lakhs). Since we are considering the cost criteria, we should launch the product at this time i.e. the time when cost involved is minimum. The time to launch is \( t^* = 20.4 \). It is note that if the above problem is optimized without the constraint that is if we are not worried about desired proportion of adoption for the newer generation, the minimum cost comes out to be 248.32 (lakhs) and time to launch is \( t = 21.3 \).
Figure 4.8 represents the behavior of the cost function that has been modeled. As can be seen from the figure also, the cost is high in the beginning. Later on, it starts to decline and it reaches to its minimum value at time t=20.4. Therefore, satisfying the cost objective, this is optimal time to launch the product (color tv) in the market.