CHAPTER 5

2D FINITE ELEMENT MODELLING OF BURIED PIPE TESTS

5.1 INTRODUCTION

Finite element method has been proved to be very useful tool in the analysis of buried structures. The method allows for different boundary conditions to be applied in such a way that an acceptable global approximate solution to the physical problem can be achieved.

‘PLAXIS’ version 8 used in this thesis is a finite element code specifically developed for the analysis of geotechnical problems such as the settlement of foundations, the stability of slopes, or the deformation of buried structures. PLAXIS stands for PLANE strain and AXISymmetric analysis, two idealizations commonly used in geotechnical engineering. The plane strain idealization is used for structures having a constant cross section over a significant length. This idealization simplifies the problem because it is assumed that the displacements perpendicular to the cross-section are zero. The axisymmetric model is used for problems that are symmetrically relative to the central axis. This idealization simplifies the problem by assuming the deformation and the stress state identical in any radial direction. Geotechnical problems involve nonlinear and time-dependent behaviour of soils that can only be handled by advanced constitutive models. PLAXIS integrates some well known constitutive models that can deal with such complex analysis, as
well as pore pressure in multi-phase materials. In addition, PLAXIS can model the interaction between a structure and soil.

This chapter deals with the modeling of the buried pipe tests conducted in this study as a plane strain problem i.e. in only two dimensions. The tests with and without geogrid were modeled. Parametric studies have been conducted by varying the trench width, plate dimensions, properties of the pipe and backfill. It is revealed that overall two dimensional simulations were reasonable. A brief summary of all the features of plaxis is given below.

5.2 REVIEW OF THE FEATURES OF PLAXIS

5.2.1 Graphical Input of Geometry

The version 8.2 of PLAXIS includes a computer assisted drawing program. This drawing program is convenient and allows the possibility to model a wide variety of geometry. Applied loads, prescribed displacements and structural members can be directly added to the geometry in the drawing area.

5.2.2 Automatic Mesh Generation

Precious time is saved by the automatic mesh generation of finite elements. This feature allows the automatic generation of a random mesh of triangular elements. There is an option available to refine the mesh where the displacement and stress are concentrated.

5.2.3 High Order Continuum Elements

Two types of triangular elements are available in PLAXIS. They are the quadratic 6-noded element and the cubic 15-node elements. These elements are chosen depending on the problem and the accuracy required. It
might be advantageous to use the 6-noded element in problems with very large number of elements, since this element type is less time consuming. A smooth distribution of stresses can be obtained if 15 noded elements are used.

5.2.4 Beam Elements

A beam in PLAXIS is "a structural object used to model slender structures in the ground with a significant flexural rigidity and normal stiffness. The beam element can include three or five nodes depending on the type of elements previously chosen. Each node has three degrees of freedom. There are two degrees of freedom related to the displacement, and one related to the rotation. It is also possible to simulate the development of a plastic hinge when the maximum bending moment or maximum axial force is reached. The weight of the beam can be taken into account. In the formulation of PLAXIS, beams are superimposed as a continuum and therefore "overlap" the soil. It is then necessary to subtract the unit weight of the soil from the unit weight of the beam.

5.2.5 Plates

Special beam elements are used to model the bending of retaining walls, tunnel linings, shells, and other slender structures. The behaviour of these elements is defined using a flexural rigidity, a normal stiffness and an ultimate bending moment. A plastic hinge may develop for elastoplastic plates as soon as the ultimate moment is mobilized. Plates with interfaces may be used to perform realistic analysis of geotechnical structures.

5.2.6 Geogrids

Geogrids (or geotextiles) are often used in practice for the construction of reinforced embankments or soil retaining structures. These
elements can be simulated in PLAXIS by the use of special tension elements. It is often convenient to combine these elements with interfaces to model the interactions with the surrounding soil.

5.2.7 Tunnels

The PLAXIS program offers a convenient option to create circular and non-circular tunnels using arcs and lines. Plates and interfaces may be used to model the tunnel lining and the interaction with the surrounding soil. Fully isoparametric elements are used to model the curved boundaries within the mesh. Various methods have been implemented to analyze the deformations that occur as a result of various methods of tunnel construction.

5.2.8 Steady State Pore Pressure

Complex pore pressure distribution may be generated on the basis of a combination of phreatic levels or direct input of water pressures. As an alternative, a steady state groundwater flow calculation can be performed to calculate the pore pressure distribution in problems that involve steady flow or seepage.

5.2.9 Excess Pore Pressures

PLAXIS distinguishes between drained and undrained soils to model permeable sands as well as nearly impermeable clays. Excess pore pressures are computed during plastic calculations when undrained soil layers are subjected to loads. Undrained loading situations are often decisive for the stability of geotechnical structures.
5.2.10 Interface

The formulation of interface element in PLAXIS uses an elasto-plastic model. The model is governed by the Coulomb failure criterion, which differentiate between plastic and elastic behaviour. Elastic displacement occurs when the shear stress is lower than \( \sigma_n \tan \phi_i + c_i \), where \( \sigma_n \) is the normal stress acting on the interface, \( \phi_i \) is the fiction angle of the interface and \( c_i \) is the adhesion of the interface. The parameters \( \phi_i \) and \( c_i \), are determined from the properties of the soil. Usually, the strength of the interface is less than the strength of the soil. Therefore, a reduction factor is applied to the fiction angle and the cohesion of the soil resulting in the properties of the interface.

However, the reduction factor does not apply to the dilation angle \( \psi \), of the interface. The dilation angle of the interface is set to zero if the reduction factor is smaller than 1, otherwise, it is equal to the dilation angle of the soil.

5.2.11 Automatic Load Stepping

This feature allows optimizing the step size to get an efficient calculation process. Load increments that are too small would require many steps, and the computer-time could be excessive. Nevertheless, too large increments would require an excessive number of iteration to reach equilibrium, and the solution could even diverge.

5.2.12 Staged Construction

This procedure is important to get the most realistic results. The staged construction feature simulates the construction, excavation, and backfill processes by changing the properties of soil clusters and structural
elements during the calculation. For example, soil clusters, which are part of the geometry, are switched off to simulate the excavation. Structural elements can also be switched off.

5.2.13 Presentation of the Results

The results in PLAXIS are presented graphically in the following formats. First, a view of the deformed mesh is presented. Second, it is possible to visualize the total, incremental, horizontal and vertical deformations by vectors, contour lines, or shaded areas. Third, the effective and total stresses are shown in the form of principal stresses, mean contour lines, or mean shaded areas. It is possible to visualize deformations, bending moments, shear stresses and normal stresses for structural and interface element. Underground water flow and pore pressure outputs are also available.

In addition, PLAXIS includes a special curve program to visualize load or time versus displacement, and stress-strain diagrams. This information is particularly useful to analyse local behaviour of soils.

5.2.14 Stress Paths

A special tool is available for drawing load-displacement curves, stress and strain paths, stress-strain diagrams and time-settlement curves. The visualization of stress paths provides a valuable insight into local soil behaviour and allows a detailed analysis of the results of a PLAXIS calculation.
5.3 MODELLING OF SOIL

PLAXIS includes some advanced constitutive models, as well as some simple models. The simplest model is the well-known Mohr-Coulomb model. This model gives a good approximation of the ultimate load for simple problems. For more complex problems, involving time-dependent behaviour or unloading – reloading residual strain, more advanced models are necessary. The soft-soil model, which is based on the cam-clay model (Schofield and Wroth, 1968), is efficient to analyse the behaviour of normally consolidated soft soils. Secondary compression can also be modeled. For stiffer soils, the Hardening Soil model of Scahnz (PLAXIS, 2002) which is based on the Hyperbolic (Duncan and Chang, 1970) relation between stress and strain. A detailed presentation of the Mohr-Coulomb model is presented below.

5.3.1 The Mohr-Coulomb Model

Plasticity is associated with the development of irreversible strains. In order to evaluate whether or not plasticity occurs in a calculation, a yield function $f$ is introduced as a function of stress and strain. A yield function can often be presented as a surface in principal stress space. A perfectly-plastic model is constitutive model with a fixed yield surface, i.e. a yield surface that is fully defined by model parameters and not affected by (plastic) straining. For stress states represented by points within the yield surface, the behaviour is purely elastic and all strains are reversible.

5.3.1.1 Elastic perfectly – plastic behaviour

The basic principle of elastoplasticity is that strains and strain rates are decomposed into an elastic part and a plastic part. Hooke’s law is used to relate the stress rates to the elastic strain rates. According to the classical theory of plasticity, plastic strain rates are proportional to the derivative of
the yield function with respect to the stresses. This means that the plastic strain can be represented as vectors perpendicular to the yield surface. This classical form of the theory is referred to as associated plasticity. However, for Mohr-Coulomb type yield function, the theory of associated plasticity leads to an over prediction of dilatancy. Therefore in addition to the yield function, a plastic potential function is introduced.

5.4 BASIC PARAMETERS OF THE MOHR-COULOMB MODEL

The Mohr-Coulomb model requires a total of five parameters, which are generally familiar to most geotechnical engineers which can be obtained from basic tests on soil samples. These parameters with their standard units are listed below.

\[ E: \text{Young’s Modulus (kN/m}^2) \]
\[ \nu: \text{Poisson’s ratio} \]
\[ \phi: \text{Friction angle (°)} \]
\[ C: \text{Cohesion (kN/m}^2) \]
\[ \psi: \text{Dilatancy angle (°)} \]

5.4.1 Young’s Modulus (E)

PLAXIS uses the young’s modulus as the basic stiffness modulus in the elastic model and the Mohr-Coulomb model, but some alternative stiffness moduli are displayed as well. In soil mechanics the initial slope is usually indicated as \( E_0 \) and the secant modulus at 50% strength is denoted as \( E_{50} \). For materials with a large linear elastic range it is realistic to use \( E_0 \), but for loading of soils, one generally uses \( E_{50} \). Considering unloading problems, as in the case of tunneling and excavations, one needs \( E_0 \) instead of \( E_{50} \).
5.4.2 Poisson’s Ratio (υ)

The selection of a Poisson’s ratio is particularly simple when the elastic model or Mohr-coulomb model is used for gravity loading. Poisson’s ratio is evaluated by matching $K_0$. In many cases the value of Poisson’s ratio ranges from 0.3 to 0.4. In general, such values can also be used for loading conditions other than one-dimensional compression. For unloading conditions, however it is more common to use values in the range between 0.15 and 0.25.

5.4.3 Cohesion (c)

The cohesive strength has the dimension of stress. PLAXIS can handle cohesionless sands ($c = 0$) but some options will not perform well. To avoid complications, non experienced users are advised to enter at least a small value. PLAXIS offers a special option for the input of layers in which the cohesion increases with depth.

5.4.4 Friction Angle (φ)

The friction angle is entered in degrees. High friction angles, as sometimes obtained for dense sands, will substantially increase plastic computational effort. The computing time increases more or less exponentially with the friction angle. Hence in high friction angles should be avoided when performing preliminary computations for a particular project. The friction angle largely determines the shear strength.

5.4.5 Dilatancy Angle (ψ)

The dilatancy angle, specified in degrees. Apart from heavily over consolidated layers, clay soils tend to show little dilatancy ($ψ = 0$). The dilatancy of sand depends on both the density and on the friction angle. For
quartz sands the order of magnitude is $\psi = \varphi - 30^\circ$. For $\varphi$ values of less than 30°, however the angle of dilatancy is mostly zero. A small negative value for $\psi$ is only realistic for extremely loose sands.

5.5 VALIDATION OF PLAXIS

In order to assess the capability of the plaxis software in solving certain idealised problem it is required to test the software by solving similar problem for which solution is available.

A PVC pipe of 630 mm diameter and SDR of 35 was modelled using plaxis FE code for different soil covers above the crown of the pipe. In this FE model the pipe is modelled as beam element of uniform thickness and the material of the pipe is idealised as elastic material. The soil medium in which the pipe buried is idealised as Mohr-Coulomb material to include elasto-plastic response of soil. The element used for the soil medium is 15 noded triangular element. Interface elements are also used between the soil and the pipe elements. The properties of soil and pipe material adopted in the analysis are presented in Table 5.1 which is as reported by Suleiman et al (2002).

Figure 5.1 presents the typical problem chosen and simulated FE model for the comparative study. In Plaxis automatic mesh generation was adopted after creating the geometric model (half boundary). Symmetric boundary condition was adopted along the centre line of the pipe and displacements at two other boundaries (both bottom and right vertical boundary) were arrested. The analysis was performed for various cover depths under gravity loading.
Table 5.1 Properties of Pipe and Soil used in the analysis (Suleiman et al 2002)

<table>
<thead>
<tr>
<th>Property</th>
<th>PVC (E (kPa))</th>
<th>PVC (υ)</th>
<th>PVC (t (mm))</th>
<th>PVC (Density (kN/m³))</th>
<th>PVC (ø (deg.))</th>
<th>PVC (Ψ (deg.))</th>
<th>PVC (C (kPa))</th>
<th>SOIL (E (kPa))</th>
<th>SOIL (υ)</th>
<th>SOIL (t (mm))</th>
<th>SOIL (Density (kN/m³))</th>
<th>SOIL (ø (deg.))</th>
<th>SOIL (Ψ (deg.))</th>
<th>SOIL (C (kPa))</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (kPa)</td>
<td>2756000</td>
<td>6890</td>
<td>18.0</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>υ</td>
<td>0.45</td>
<td>0.38</td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>t (mm)</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density (kN/m³)</td>
<td>-</td>
<td>18</td>
<td></td>
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<td>ø (deg.)</td>
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<tr>
<td>Ψ (deg.)</td>
<td>-</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>C (kPa)</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

a) Physical model  
b) FE model

Figure 5.1 Problem chosen for comparative study
From the results of the FE analysis conducted the vertical deflections of the crown of the pipe are determined for different embedment depths and are compared in Figure 5.2 with the deflections reported by Moser (1990) in his study conducted on a PVC pipe with SDR 35 at Buried Structural Laboratory, Utah state university.

The vertical deflection of pipe is increased with cover depth and the rate of increase in deflection is reduced as reported by Moser (1990) based on his experimental studies. Plaxis results compare well the experimental results till the soil cover of 3 m and the difference between them is increased with cover depth for the depth of cover more than 3. The maximum variation between the results of Plaxis and Moser is found to be 8% even for the soil cover of 9 m. The comparison throughout Plaxis FE code simulates behaviour of buried pipe reasonably well, hence Plaxis programme is used to study the performance of buried pipe subjected to surcharge load.

![Figure 5.2](image)

**Figure 5.2** Vertical deflection percent with respect to soil cover for PVC pipe compared with Moser (1990)
Finite element analysis program “PLAXIS” was used to study the interaction of PVC pipe with the backfill soil. The cross trench condition was analysed as plane strain condition. Six circular segment elements were used to represent the PVC pipe. Interface elements were used around the pipe to ensure a complete soil-structure interaction phenomenon. Fifteen noded plane strain triangular elements were used to model the backfill and the soil is assumed to follow Mohr-coulomb material model. A vertically free and horizontally fixed boundary was used for the soil box sidewalls. For the numerical analysis the vertical outer boundary was assumed to be rigid. The distance between this boundary and the pipe springing line ranged one pipe diameter. The influence of this boundary was investigated in the 2D parametric analysis.

The soil was zoned into two sections as bedding and backfill in order to represent normal method of installation. They have been assigned appropriate properties. The boundary between the bedding and the surround was approximately horizontal, the irregularity of the layer interface being a function of the method of generating the mesh and the number of elements used. Figure 5.3 shows the geometry of the model with the boundary condition.
Figure 5.3 Plot of geometry model with boundary conditions

The loading plate of width 0.2 m and length 0.6 m was discretised as a plate element with the engineering properties of steel. Loading was simulated by applying a uniform pressure to the width of the plate. For the finite element tests the vertical outer boundary of the soil was considered to be rigid.

5.7 PIPE PROPERTIES

The pipe is modeled as beam element and the material of the pipe is assumed to behave as perfectly elastic material. The properties adopted for the FE study is outlined in Table 5.2. The values of hoop stiffness $EA$ and the flexural stiffness $EI$ per m length of the pipe were input directly into the material data set for the beam column element used in the two dimensional analysis.
Table 5.2 Properties of Materials

<table>
<thead>
<tr>
<th>Properties</th>
<th>Loose Sand</th>
<th>Dense Sand</th>
<th>PVC</th>
<th>Steel Plate</th>
<th>GeoGrid (CE 121)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ (kN/m$^3$)</td>
<td>15</td>
<td>17</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$E$ (kN/m$^2$)</td>
<td>9000</td>
<td>19000</td>
<td>0.933E+06</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\upsilon$</td>
<td>0.30</td>
<td>0.30</td>
<td>0.31</td>
<td>0.30</td>
<td>-</td>
</tr>
<tr>
<td>$EA$ (kN/m)</td>
<td>-</td>
<td>4665.00</td>
<td>2.4E+06</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>$EI$ (kNm$^2$ /m)</td>
<td>-</td>
<td>-</td>
<td>9.7E-03</td>
<td>28.8</td>
<td>-</td>
</tr>
<tr>
<td>$\phi$ (deg.)</td>
<td>32</td>
<td>42</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\psi$ (deg.)</td>
<td>2</td>
<td>12</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Relative Density (%)</td>
<td>15</td>
<td>74</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5.8 PRESENTATION AND DISCUSSION OF RESULTS

The results of the laboratory tests and data from 2D finite element analysis for 200 mm diameter pipes embedded in sand are compared.

5.8.1 Effect of Cover Depth

The influence of cover height on the performance of the 200 mm diameter pipe predicted by FEA is provided in Figure 5.4 for loose sand backfill condition. Plots of diametric strains are given against applied pressure for FEA of tests for cover heights (embedment depths) of 200, 400 and 600 mm cover. It is evident that relatively small increases in cover height theoretically afford better protection for the pipe. Increasing the cover height from 200 mm to 400 mm, the prediction of vertical diametric strain at a surface pressure of 100 kPa was approximately halved. Lateral pipe strains although small as seen in model tests, were similarly reduced. The predictions over estimated the observed diametric strains by 7%.
**Figure 5.4** Influence of cover height on deflections of 200 mm diameter pipe as predicted by 2D FEA

**Table 5.3** Comparison of Vertical diametric strains (%) for different embedment ratios of the pipe in dense sand backfill

<table>
<thead>
<tr>
<th>Surface pressure (kN/m²)</th>
<th>H/D = 1</th>
<th>H/D = 2</th>
<th>H/D = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>FEM</td>
<td>Experiment</td>
</tr>
<tr>
<td>50</td>
<td>-0.745</td>
<td>-0.804</td>
<td>-0.415</td>
</tr>
<tr>
<td>100</td>
<td>-1.770</td>
<td>-1.911</td>
<td>-0.810</td>
</tr>
<tr>
<td>150</td>
<td>-2.725</td>
<td>-2.943</td>
<td>-1.165</td>
</tr>
</tbody>
</table>

The comparison of vertical diametric strain for embedment ratios of 1, 2 and 3 for the applied surface pressures of 50 kN/m², 100 kN/m² and 150 kN/m² is presented in Table 5.3. The FEM results show a good agreement with the experimental results for all the different levels of embedment of the
pipe. With the increase in surface pressure the FE results slightly overestimated the vertical diametric strains of the pipe by 8%. The horizontal diametric strains obtained from the FEA are compared in Table 5.4 and are found to overestimate the results. The maximum difference of 9% was observed when compared with the experimental values. This is due to the rigid boundary conditions generated in the FE model.

### Table 5.4 Comparison of Horizontal diametric strains (%) for different embedment ratios of pipe in dense sand backfill

<table>
<thead>
<tr>
<th>Surface pressure (kN/m²)</th>
<th>H/D = 1</th>
<th>H/D = 2</th>
<th>H/D = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>FEM</td>
<td>Experiment</td>
</tr>
<tr>
<td>50</td>
<td>0.320</td>
<td>0.352</td>
<td>0.255</td>
</tr>
<tr>
<td>100</td>
<td>0.725</td>
<td>0.783</td>
<td>0.505</td>
</tr>
<tr>
<td>150</td>
<td>1.08</td>
<td>1.188</td>
<td>0.740</td>
</tr>
</tbody>
</table>

#### 5.8.2 Effect of Cover Density

The density indices of the backfill and the surround soil for each FEA are provided in the legend. It is obvious from the Figure 5.5 that the pressure – diametric strain behaviours were essentially the same. It appeared that the soil density variations had marginal influence on the lateral pipe deflections with loading as observed in model test. The changes in the backfill and surround soil properties seemed to have little influence on the development of horizontal pipe diametric strain with loading and again, the pipe-soil system was predicted to behave linearly.
Figure 5.5  Influence of soil density on deflections of 200 mm diameter pipe with 400 mm of cover, from 2D FEA

5.8.3 Comparison of FEM and Experimental Results without Geogrid Reinforcement

The match with the experimental data was satisfactory as evidenced by the plots of diametric strain against applied pressure in Figure 5.6. The ratio of FEA to observed vertical diametric strains at an applied pressure of 150 kPa was around 8%.

The correspondence of the FEA output with the horizontal pipe strain was also satisfactory. Observed pipe diametric strains were almost identical between the horizontal and vertical directions in the experimental tests conducted for various cases but the 2D FEA could not exactly simulate this behaviour owing to the assumption of rigid side wall boundaries. A typical plot of the vertical deflection of the pipe under a surface pressure of 150 kPa in loose sand backfill is shown in the Figure 5.7.
Figure 5.6  Plot of Diametric strain against applied pressure 2D analysis, 200 mm pipe diameter at 400 mm cover without geogrid reinforcement in loose sand.

Figure 5.7  Vertical deflection in metres for a 200 mm diameter pipe, 600 mm cover 2D FEA at a surface pressure of 150 kPa in loose sand.
(a) Deviatoric stress

(b) Vertical stress at the level of crown of pipe

(c) Horizontal stress on a plane passing at springline of pipe

Figure 5.8  Stresses in kpa for a 200 mm diameter pipe, 600 mm cover 2D FEA at a surface pressure of 150 kPa in loose sand
From the Figure 5.8 it is clear that the major principal stress from the FEA was highest below the edge of the loading plate and adjacent to the center of the loading plate. The stress concentration from the edge of the loading plate headed towards the quarter point of the pipe. The plots of deviator stress show extensive, but similar zone of high stresses, which reaches the quarter point of the pipe. Such a pattern of stress development could be expected of granular soil which is conducive to arching. The crown of the pipe, having deflected significantly has shed load to neighboring pipe sections. The vertical stresses at the level of crown of pipe are found to be -29.27 kN/m² and the horizontal stresses at the springline of the pipe is found to be -25.91 kN/m².

![Image](image.png)

**Figure 5.9  Development of plasticity for a pressure of 150 kPa in loose sand**

Typical plot of the development of plasticity in loose sand backfill is presented in Figure 5.9. The Figure is relevant to an applied surface pressure of 150 kPa. By the time the surface loading had reached 150 kPa, a
zone of plasticity had formed below the soil surface. Plasticity had developed along the unloaded surface adjacent to the loading plate and was heading downwards along the wall and towards the shoulder of the pipe. More significantly, a tongue like area of plasticity extended below the edge of the loaded area. The depth of this zone extended for half the backfill cover height. The development of this zone as loading continued resulted in the instability of the finite element analysis more often in the case of loose sand backfill. This observation is in good agreement with the earlier investigation carried out by Cameron (2005) using Finite element code ‘AFENA’

Figure 5.10  Plot of diametric strain against applied surface pressure for 200 mm diameter pipe with 600 mm cover in dense sand, against 2D analysis

A further example is provided in Figure 5.10 in which the applied pressure and corresponding diametric strain of the pipe are compared for the pipe tests with 600 mm of cover. The test observations indicated a slight difference in both the horizontal and vertical pipe responses with loading. The
match between the FE predictions and the experimental results was appreciable upto 50 kpa and with the increase in surface pressure the FEA predictions slightly overestimated the test deflection data.

It is also apparent in the plots that the two-dimensional FEA predictions of pipe–soil system behaviour indicated an almost linear development of pipe deflection with applied pressure.

Figure 5.11 provides further data from the FEA of the test with the dense sand backfill. In this Figure, plots are provided for vertical deflection contour, principal stress and deviatoric stress, which corresponded to the average applied surface pressure of 100 kPa.

(a) Vertical deflection

Figure 5.11 (Continued)
The major principal stress was highest below the edge of the loading plate and the stress concentration headed towards the pipe. Deviatoric stresses show more extensive but similar zone of stresses which reaches the quarter point of the pipe. The crown of the pipe having deflected significantly has shed load to the neighbouring pipe sections. The development of plasticity
in the analysis is indicated in the Figure 5.12. The corresponding average surface pressure is 100 kPa. It is predicted that plasticity will quickly develop at shallow depth below the unloaded surface of the backfill. With loading, plasticity progresses downwards beside the unloaded surface of the backfill.

![Figure 5.12](image)

**Figure 5.12** Development of Plasticity with displacement of the loading plate, 2D FEA of test, 600 mm of cover in dense sand

With loading, plasticity progresses downwards, beside the first two elements from the axis of symmetry that is beside the directly loaded area. Further loading not only deepens the plastic zone but also induces plasticity immediately below the edges of the plate. The depth of the zone almost reaches half the cover height.

### 5.8.4 Comparison of FEM and Experimental Results with Geogrid Reinforcement

The effect of single layer of geogrid reinforcement provided at 100 mm (0.5D) and 200 mm (D) above the pipe crown on vertical and horizontal deflections are compared between finite element and experimental results and are presented in the Figure 5.13. The provision of geogrid reinforcement
reduced the vertical deflections on the crown of the pipe by 20%. However the FEM results slightly overestimated the experimental results on the average by 8% for the range of surface pressures studied. The horizontal strains in the pipe were satisfactory and the FEM results matched well with the experimental results.

(a) Geogrid reinforcement at 100 mm above the pipe crown in loose sand (Type I reinforcement)

(b) Geogrid at 200 mm above the pipe crown in loose sand (Type II reinforcement)

Figure 5.13 Plots of Diametric strain Vs Applied surface pressure for 400 mm cover with geogrid reinforcement as predicted in 2D FEA
(a) Single layer of Geogrid reinforcement at the springing line of the pipe in Loose sand (Type III reinforcement)

(b) Two layers of Geogrid reinforcement at the springing line of the pipe with 50mm loose sand packing in between the layers (Type IV reinforcement)

Figure 5.14 (Continued)
(c) Two layers of Geogrid reinforcement at the springing line of the pipe with 50 mm dense sand packing in between the layers (Type V reinforcement)

Figure 5.14 Plot of Diametric strain Vs Applied surface pressure for 200 mm diameter pipe with 400 mm cover for Type III, Type IV and Type V reinforcements as predicted in 2D FEA

The Figure 5.14 shows the comparison of finite element results with the experimental results when geogrid reinforcement was placed in a single layer along the springline and when two layers geogrid are placed along the springline of the pipe with 50 mm of loose or dense sand packing in-between the layers. It was observed that the presence of geogrid reinforcement along the springline of the pipe reduced the lateral deflection of the pipe significantly by 38%. However these results were found to be 9% higher than the experimental results even though there was reasonable agreement with the horizontal pipe strain. In the case of Type IV reinforcement the FE results closely matched with the experimental results for the horizontal diametric strain of the pipe. The FE results were found to be 5% higher than the experimental observations for an applied surface pressure
of 150 kN/m². A similar observation was made with two layers of geogrid placed along the springline with 50 mm dense sand packing in-between the layers. The reduction in vertical and horizontal deflections observed in FE results were 17% and 65% respectively. But these slightly overestimated the experimental results on the average by 9%.

Table 5.5 Comparison of Horizontal diametric strains (%) for H/D = 3 in loose and dense sand

<table>
<thead>
<tr>
<th>Surface pressure (kN/m²)</th>
<th>Reinforcement</th>
<th>Loose sand</th>
<th>Dense sand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Experiment</td>
<td>FEM</td>
</tr>
<tr>
<td>50</td>
<td>Type III</td>
<td>0.024</td>
<td>0.025</td>
</tr>
<tr>
<td>100</td>
<td>Type III</td>
<td>0.045</td>
<td>0.048</td>
</tr>
<tr>
<td>150</td>
<td>Type III</td>
<td>0.064</td>
<td>0.069</td>
</tr>
<tr>
<td>50</td>
<td>Type IV</td>
<td>0.036</td>
<td>0.038</td>
</tr>
<tr>
<td>100</td>
<td>Type IV</td>
<td>0.050</td>
<td>0.054</td>
</tr>
<tr>
<td>150</td>
<td>Type IV</td>
<td>0.070</td>
<td>0.075</td>
</tr>
<tr>
<td>50</td>
<td>Type V</td>
<td>0.014</td>
<td>0.015</td>
</tr>
<tr>
<td>100</td>
<td>Type V</td>
<td>0.024</td>
<td>0.026</td>
</tr>
<tr>
<td>150</td>
<td>Type V</td>
<td>0.030</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Typical results of horizontal diametric strain (%) obtained for an embedment ratio of 3 (i.e. 600 mm from the surface of the backfill) corresponding to 50 kN/m², 100 kN/m² and 150 kN/m² are presented in Table 5.5. The FE results showed a slight overestimation in the range of 7.5 to 8% in the observed horizontal strains for the surface pressures considered. This observation was evident for the pipe buried in both loose and dense sand backfills.
The hoop stresses obtained at the crown of the pipe with Type I and Type II reinforcement for the pipe embedded at a depth of 400 mm from the surface of the loose sand backfill is presented in Table 5.6. The FEM shows higher values for both the types of reinforcements. The stress values obtained from the experimental studies are found to be 9% lesser than the finite element results for the surface pressures considered.

The FE predictions of the hoop stresses at the springline of the pipe embedded at 400 mm from the surface of the backfill with geogrid reinforcement is compared with the experimental results obtained from laboratory model tests in Figure 5.15. From the FE predictions it is clear that the Type V reinforcement reduces the hoop stresses by 40% at a surface pressure of 100 kPa. But the FE predictions tend to be higher than the experimental results by 7.5% on the average for the surface pressures considered.
Figure 5.15  Comparison of Hoop stresses at the springline of the pipe with geogrid reinforcement for H/D = 2 in loose sand

Typical contour plots obtained for the pipe embedded at a depth of 400 mm from the surface of the dense sand backfill with Type II reinforcement (i.e. provision of geogrid at 200 mm above the crown of the pipe) for an applied surface pressure of 150 kPa is shown in Figure 5.16. The plots give a clear understanding of the mechanism of the buried pipe. Figure 5.16(a) shows the FE model with the provision of geogrid reinforcement at 200 mm above the crown of the pipe. The deformed mesh obtained at an applied surface pressure of 150 kN/m$^2$ shown in Figure 5.16(b) indicates that the deformations of buried pipe need not be exactly elliptical. This is in agreement with the conclusions reported by Sivakumar et al (2006) in his finite element studies on buried steel pipes subjected to gravity loading.

The displacement contour is shown in Figure 5.16(c) and the maximum vertical displacement as predicted by FEM is found to be -3.44 mm. The stresses developed in the principal directions are shown in Figure 5.16(d) and the extreme principal stress predicted by FE analysis is -198.99 kN/m$^2$. The mean deviatoric stress reaches a maximum value of 118.30 kN/m$^2$ and the stress contour is illustrated in Figure 5.16(e). The finite element studies predicted a maximum shear strain of 9.51%. The contour
lines shown in Figure 5.16(f) indicate the shear strains developed at the shoulder, haunch of the pipe and the edge of the loading plate.

(a) FE model with geogrid reinforcement at 200 mm above the pipe crown
(Type II reinforcement)

(b) Deformed mesh

Figure 5.16 (Continued)
(c) Vertical displacements in metres, 400 mm cover 2D FEA

(d) Principal stress

Figure 5.16 (Continued)
(e) Deviatoric stress in kPa for 400 mm cover of 200 mm diameter pipe, 2D FEA

(f) Shear strain (%) for 400 mm cover of 200 mm diameter pipe, 2D FEA

Figure 5.16 Contour plots of 2D FEA for 200 mm diameter pipe, 400 mm cover in dense sand at 150 kPa
The variation of vertical stress on a horizontal plane at the level of the crown of the pipe is found to be \(-81.22\) kN/m\(^2\) which is less than the effective overburden pressure. Similarly the horizontal stress at the springline is determined and its value is \(-59.68\) kN/m\(^2\). The stress variations are illustrated in Figure 5.17.

Figure 5.17 Stresses at the crown and springline of the pipe

Figure 5.18 shows the compatibility of FE and experimental results as observed in the dense sand with Geogrid reinforcement placed at 200 mm above the pipe crown. The pipe is embedded at a depth of 600 mm from the surface. The results are satisfactory with the reduction in the vertical crown deflection observed as 40% in 2D analysis. But the FE results have overestimated the vertical deflection by 10% and the lateral deflection by 9% when compared with the experimental results.
Figure 5.18  Plot of Diametric strain Vs Applied surface pressure for 200 mm diameter pipe, 600 mm cover in dense sand with geogrid reinforcement at 200 mm above pipe crown as predicted in 2D FEA

Figure 5.19  Comparison of Experimental strain results with 2D finite element analysis for 400 mm with and without geogrid reinforcement in Dense sand
From the Figure 5.19 it is evident that the use of geogrid reinforcement reduces the strain on the crown of the pipe thereby offering better protection to the pipe. The finite element results agree reasonably well with the experimental results. Lateral strains on the pipe were similarly reduced.

5.9 PARAMETRIC STUDIES

A series of parametric studies to gain further insights into the soil-pipe interaction problem was conducted using Finite element method, PLAXIS. The input parameters for the material and the soil were chosen as given in Table 5.2

In the first case of parametric study the trench width was increased by one pipe diameter (i.e. 200 mm) on either side and was designated as the full model analysis. The soil on either side was idealized as natural soil with modulus of elasticity 7000 kN/m² and density 20 kN/m³. In the second study the thickness of the pipe was varied with the commercially available thicknesses such as 5 mm, 6.8 mm and 7.9 mm for 200 mm diameter PVC pipe. In the third case the trench width was varied to study the effect of distributed load on the pipe deflections. In the last case of this parametric analysis the influence of interface reduction factor on the pipe deflections and stresses were studied. Results of the studies are summarized and the comments are made while examining the plots.

5.9.1 Influence of Trench Width

Comparisons of the typical results obtained from the analysis of the experimental model and the full model are provided in Figure 5.20 although the vertical deflections appeared to be similar above the pipe crown the deflections were laterally restricted within the backfill of the full soil model.
The perfectly rough interface between the sand in the trench and the natural soil at the side appeared to have restricted the development of vertical deflection. Lateral deflection although small, whereas expected more pronounced for the full model.

It was evident from Fig 5.20 that the pipe between the spring line and the crown was able to expand more freely in the full model as the crown deflected. As the trench width decreases, the proportionate sharing of the soil load between the pipe and the side fill is increased due to the weakening of soil arch resulting from the limited soil fill at the springing. A flexible pipe buried with similar boundary conditions will be subjected to even higher displacement than those buried in wider trench. The distribution of soil load due to soil arching causes the shoulder of the pipe to carry a higher soil load and the pipe is supported more at the haunch than at the pipe invert.

In flexible buried pipes a higher vertical stress in the soil is developed at the level of the pipe springing, where soil in this zone act as vertical earth column supporting the soil arch. In the case of narrow trench width this zone is in close proximity to the trench boundary but in wider trench this zone separates from trench boundary.
Figure 5.20 Deflection contours for 2D FEA of 200 mm pipe test
(Experimental model is above the predicted full model)
(a) Deviatoric stress

Figure 5.21 (Continued)
A comparison of stress development is provided in the Figure 5.21. The solutions corresponded to average applied surface pressures of 50 kpa. Deviatoric stress contours were similar although the stress contours were more intense between the loading plate and the pipe crown in the experimental model. In the full model moderate deviatoric stress was evident across the boundary between the sand backfill and the natural soil near the springline of the pipe.

A tongue of plasticity had reached from the elements immediately adjacent to the loading plate in both the cases. Further plasticity was associated with the tensile zone of the unloaded soil near the haunch of the pipe as observed from Figure 5.21(b).

5.9.2 Influence of Stiffness of Pipe

The influence of stiffness of the pipe on the vertical crown deflection has been compared with the commercially available thicknesses of 5 mm, 6.8 mm and 7.9 mm for 200 mm diameter PVC pipe and presented.
Figure 5.22 Variation of vertical deflection with applied surface pressure for 200 mm pipe with varying thickness for H/D ratio of 1

From the Figure 5.22 it is observed that the variation in deflection with respect to decrease in wall thickness is not significant and the deflection obtained is considerably lower than the allowable limit of 5% of the diameter of the pipe under static loading conditions. This typical behaviour was observed in the pipe embedded at depths of 400 mm and 600 mm from the surface of the loose and dense sand backfills. Stress increases with the decrease in the thickness of the pipe as evident from the contour plots shown in Figure 5.23.
Figure 5.23 (Continued)
(c) 7.9 mm

**Figure 5.23  Development of deviatoric stresses in 200 mm diameter pipe with Varying thickness, 2D analysis at 200 mm cover**

It is expected that an increase in pipe stiffness would result in an increase in imposed deviatomic stress on the pipe, the degree of load shedding (Positive arching) being controlled by the relative stiffness of the pipe and the sidefill reducing as the pipe stiffens and thus the ratio increasing. These studies are useful to estimate the deflection of the PVC pipe buried in soil for a specified range of stiffness ratios, but only for simple boundary and loading conditions considered. The approach can be extended to consider the installation processes and the site conditions specific to regions or local bodies.

### 5.9.3 Effect of Loading Area

The width of the loading plate varied from 0.5D to 1.5D and subjected to 50 kpa of static loading in loose sand backfill. The vertical deflection and the stresses observed are presented and discussed.
Figure 5.24 (Continued)
Figure 5.24  Deflection contours for 200 mm diameter pipe with variable width of loading plate, 2D analysis

(a) 100 mm wide loading plate

Figure 5.25 (Continued)
Figure 5.25 Stress contours for 200 mm diameter pipe with variable width of loading plate, 2D analysis
The development of displacements and stresses in the 2D analysis of 200 mm diameter pipe with variable width of the loading area is indicated in the Figures 5.24 and 5.25 respectively. Three contour plots of deformation and stresses corresponding to the loading area for an applied surface pressure of 50 Kpa is shown. It is evident from the plots that the deflection and the stresses on the crown of the pipe increased with the increase in the width of the loading area. The extreme displacements obtained were 2.18 mm, 2.67 mm and 2.89 mm for 100, 200 and 300 mm width of the loading plate respectively. The corresponding extreme deviatoric stresses obtained were 37.13 kN/m$^2$, 37.51 kN/m$^2$ and 39.19 kN/m$^2$.

5.9.4 Influence of Interface Element

Interface elements allow the soil element to slip or separate from the pipe surface when the shear stresses and the normal stresses in the interface reach a threshold. The absence of interface element leads to the direct connection between the soil elements and the structural elements thereby leading to higher values.

In the light of the numerical results it can be stated that the interface elements have a significant influence on the deformations of the pipe and the stresses generated in the pipe. It is observed that the first step of the compaction simulation produced a smaller vertical elongation and horizontal deflection in the case without interface. This is related to the interface elements attached to the beam elements and generating tension limiting vertical elongations and horizontal deflection. The absence of interface elements generated an increase of earth pressure at the pipe’s haunch leading to higher vertical elongation and horizontal deflection.
The arching effect described by Marston does not take into consideration the effect of friction of the backfill material on the pipe’s surface. Marston relates the arching effect to the relative movement of the backfill directly above the pipe with respect to the soil on both sides of the trench. The rigidity of the pipe and the density of the backfill have a considerable effect on the arching. For instance a loose soil placed above the pipe will settle more than the side fills. The friction between the sides fills and the loose soil will reduce the load applied on top of the pipe resulting in arching. However the numerical analysis with interface elements generated greater arching than the analysis without interface elements.

Figure 5.26  Deformed mesh for 200 mm diameter pipe, 2D analysis with interface reduction factor of 1.0
The deformed mesh shown in the Figure 5.26 indicates the effect of interface element on the pipe buried at 200 mm from the surface and subjected to a surface pressure of 50 kPa.

The influence of interface reduction factor on the deformation of the pipe was insignificant. The vertical elongation and the horizontal deflections were almost the same using the interface reduction factors of 0.5, 0.67, 0.85 and 1.0. The total deformations were higher in the case of weaker interface. The interface is particularly deformed at the haunch of the pipe which was predominant when the pipe was buried at 600 mm from the surface. The interface strength has an influence on the normal effective stresses applied on the buried PVC pipe. However, the deformation of the pipe did not reflect this increase of stress. The overburden pressure may be too low to generate loads that would have a considerable effect on the deformation of the pipe related to the interface strength. The normal effective stress at the pipe’s crown was about 28.07 kN/m² at the completion of the backfill with an interface reduction factor of 0.5. This stress was increased to 35.5 kN/m² with an interface reduction factor of 1.0. The analysis with higher interface reduction factors produced higher stresses. The relative earth movements along the top of the pipe relax the pressure at the pipe’s crown. This phenomenon is increased as the interface reduction factor, or the interface strength is decreased.

5.10 SUMMARY

The experimental tests conducted with and without geogrid reinforcement in loose and dense conditions of sand backfill were adequately modelled by the two dimensional FEA program ‘PLAXIS’. The 2D FEA could provide some guidance on the likely response and patterns of the behaviour of pipes subjected to applied surface pressures. The finite element predictions slightly overestimated the experimental results. This could have
been due to the assumption of rigid side boundary in FE analysis. However, the predictions clearly indicated that the 2D FEA could be used with some caution to provide a relative measure of the importance of backfill cover height and the provision of geogrid reinforcement at different levels and locations to offer better protection for flexible plastic pipes.