CHAPTER 4

DESIGN OF REDUCED ORDER SLIDING MODE CONTROLLER FOR PFC ĆUK CONVERTER

4.1 INTRODUCTION

This chapter discusses a simple reduced order sliding surface design approach for a PFC Ćuk converter, wherein a new and systematic technique for the selection of sliding surface co-efficients to implement ROSMC is attempted. Pade’s approximation technique is used to obtain the reduced order model of the PFC converter.

The converter switches are driven as a function of the instantaneous values of the state variables in a way that forces the system trajectory to stay on a suitable selected surface in the state space called the Sliding Surface (SS). This control method offers several advantages over the other linear control techniques: stability even for large line and load variations, robustness, good dynamic response and simple implementation.

4.2 REDUCED ORDER SLIDING MODE CONTROL SCHEME

This section presents the design of the ROSMC controller. The controller is comprised of an inner current loop which uses ROSMC for shaping the source current and an outer voltage control loop using PI controller to regulate the output voltage as shown in Figure 4.1. The input to the PI controller is the voltage error and the output sets the reference inductor
current amplitude for inner current loop. The inputs to the ROSMC are voltage error $e_1$ and the current error $e_2$. The output $u$ is the control signal, which in turn sets the new duty ratio of the switching pulse for triggering the switch.

\[ e_1 = w_1 - x_1 = V_{\text{ref}} - V_o \]  
\[ e_2 = w_2 - x_2 = i_{\text{ref}} - i_{L_i} \]

To derive the sliding surface $\sigma (e,t)$, the phase variable canonical form representation of the system is considered. The reduced order state model of the Cuk converter is given by

\[ W = [w_1 w_2]^T, \quad X = [x_1 x_2]^T, \quad e = [e_1 e_2]^T \]

Here the state variables considered are $x_1 = V_{\text{co}} = V_o$, $x_2 = i_{L_i}$, $w_1 = V_{\text{ref}}, w_2 = i_{\text{ref}}$, the values of errors $e_1$ and $e_2$ are given by

**Figure 4.1 Closed loop control using ROSMC**

4.2.1 Derivation of Reduced Order Sliding Mode Control Law

Let $W$ be the vector which contains the reference dynamic variables, $X$ be the actual state variables and $e$ be the error vector,

\[ W = [w_1 w_2]^T, \quad X = [x_1 x_2]^T, \quad e = [e_1 e_2]^T \]

Here the state variables considered are $x_1 = V_{\text{co}} = V_o$, $x_2 = i_{L_i}$, $w_1 = V_{\text{ref}}, w_2 = i_{\text{ref}}$, the values of errors $e_1$ and $e_2$ are given by

\[ e_1 = w_1 - x_1 = V_{\text{ref}} - V_o \]  
\[ e_2 = w_2 - x_2 = i_{\text{ref}} - i_{L_i} \]  

To derive the sliding surface $\sigma (e,t)$, the phase variable canonical form representation of the system is considered. The reduced order state model of the Cuk converter is given by
\[
\dot{X} = AX + BU
\]  

(4.3)

where

\[
A = \begin{bmatrix}
0 & 1 \\
-1.64 \times 10^5 & -49.34
\end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]  

(4.4)

Sliding surface \(\sigma(e,t)\) is defined as the linear function of tracking errors and is expressed as

\[
\sigma(e,t) = [G][e]
\]  

(4.5)

where

\[
G = [G_1 \quad G_2] \quad \text{and} \quad G_1 \quad G_2 > 0
\]

The objective of the tracking error problem is to keep the error vector \((e)\) on the surface \(\sigma(e,t)=0\), which implies that the error converges exponentially to zero and is given by \(\sigma(e,t)=[G][e]=0\)

\[
\sigma(e,t) = [G][\dot{e}] = 0
\]  

(4.6)

On the sliding surface, a second order system is reduced to first order system with stable linear differential equation. Also the system dynamics on the sliding surface is determined only by the co-efficient vector \(G\). Hence the control is insensitive to parameter variations. To determine the control law, the error state equation using the accessible states is derived as follows

\[
\ddot{e} = W - \dot{X}
\]  

(4.7)
\[ e = W - AX - Bu \]  

(4.8)

Substituting \( X = W - e \) in Equation (4.8), \( e \) is expressed as

\[ e = W - AW + Ae - Bu \]  

(4.9)

The Filippov's average equivalent switch control \( u_{eq} \) that guarantees \( \sigma^*(e,t) = 0 \) is obtained as,

\[ \dot{\sigma} = Ge = [G][W - AW + Ae - Bu_{eq}] = 0 \]  

(4.10)

This gives the value of control signal as,

\[ u_{eq} = [GB]^{-1}G[W - AW + Ae] \]  

(4.11)

Substitution of this control law into Equation (4.9), gives the error dynamics as

\[ e = W - AW + Ae - B(GB)^{-1}G[W - AW + Ae] \]  

(4.12)

\[ e = [I - B(GB)^{-1}G][W - AW + Ae] \]  

(4.13)

By applying invariance condition \( [W - AW = 0] \), the above equation is modified as:

\[ e = [I - B(GB)^{-1}G]Ae = A_{eq}e \]  

(4.14)

If \( (GB)^{-1} \) exists, the vector \( G \) is derived by selecting the eigenvalues of \( A_{eq} \) such that it guarantees the asymptotic convergence of error to
zero at the desired rate. The matrix $A_{eq}$ is chosen to satisfy (4.14) and is given by

$$A_{eq} = \begin{bmatrix} -4.79 & 0 \\ 0 & -0.21 \end{bmatrix} \quad (4.15)$$

The matrix $G$ is then obtained using Equation (4.14) as

$$G = \begin{bmatrix} G_1 & G_2 \end{bmatrix} = \begin{bmatrix} 1 & 5 \end{bmatrix} \quad (4.16)$$

Hence the sliding surface $\sigma$ is given by

$$\sigma = G_1 e_1 + G_2 e_2 \quad (4.17)$$

Equation (4.17) shows that if the Ćuk converter operates in sliding mode (when $\sigma = 0$, stability condition), the dynamics of errors $e_1$ and $e_2$ tend exponentially to zero with a time constant $G_2 / G_1$. Also the error state equation given in Equation (4.14) describes the error motion under SMC. Once the sliding surface $\sigma (e, t) = Ge$ is derived, then the control law for the hitting condition is defined as

$$u = M \text{sgn}(\sigma) x_1$$

$$= U x_1 \quad (4.18)$$

where $U = 1$ for $\sigma > \delta$

$U = 0$ for $\sigma < -\delta$

Here, $\delta$ is chosen as 0.0005. This value is selected by iterative procedure. Equation (4.18) is used to generate the gate pulse to switch, which in turn shapes the source current. Here $M$ is a constant equal to unity so that $\sigma \dot{\sigma} < 0$ (existence condition) is satisfied. The reaching condition ensures that
the tracking error trajectory is asymptotically attracted to $\sigma = 0$ (stability condition). It is observed that the control law $u$ in Equation (4.18) doesn’t depend on the operating conditions, converter parameters or bounded disturbances. This is achieved as long as the control input $u$ is large enough to maintain the plant subsystem in sliding mode. Therefore, it is said that the plant dynamics operating in sliding mode is robust against the disturbances as mentioned. The desired dynamics of the input current is determined only by the coefficient $G$ of the control law.

It is convenient to select the sliding surface as a linear combination of the errors since it results in equivalent control method to describe the system dynamics in the sliding mode, thus

$$\sigma(t) = e_1 + G_2 e_2 = G^T e = 0,$$  \hspace{1cm} (4.19)

where $G^T = [1, G_2]^T$ is the vector of sliding surface co-efficient which corresponds to $G$ in Equation (4.16)

$$\hat{\sigma}(e,t) = \begin{cases} \begin{aligned} & G^T A e + G^T B U + C^T D, \text{for} \sigma(t) > 0 \\ & G^T A e + G^T B U^- + C^T D, \text{for} \sigma(t) < 0 \end{aligned} \end{cases}$$  \hspace{1cm} (4.20)

After substituting the values of $A$, $B$, $C$ and $G$, Equation (4.20) becomes

$$\dot{\lambda}_1(t) = \left( G_1 - \frac{G_2}{R_i C_o} \right) e_2 - \frac{G_2}{L_o C_o} e_1$$  \hspace{1cm} (4.21)

$$\dot{\lambda}_2(t) = \left( G_1 - \frac{G_2}{R_i C_o} \right) e_2 - \frac{G_2}{L_o C_o} e_1 + G_2$$  \hspace{1cm} (4.22)

Equations $\lambda_1(t) = 0$ and $\lambda_2(t) = 0$ define two lines in the phase plane with the same slope passing through origin. These equations represent the sliding surface for switch ON and OFF conditions which are confined to a
single sliding surface as shown in Figure 4.2. Phase trajectory is drawn using the errors $e_1$ and $e_2$. The phase trajectory of ROSMC controlled PFC Ćuk converter for $G_1, G_2 > 0$ is shown in Figure 4.2.

![Phase trajectory of ROSMC controlled PFC Ćuk converter and the sliding surface on phase plane](image)

**Figure 4.2** Phase trajectory of ROSMC controlled PFC Ćuk converter and the sliding surface on phase plane

### 4.3 SIMULATION AND EXPERIMENTAL RESULTS

To observe the system performance, simulations have been performed on a digital computer using MATLAB /SIMULINK software. The following parameters are considered for simulation: the input AC voltage = 24 Vrms, desired output voltage $V_o = 24$ V, input inductance $L_i = 2$ mH, output inductance $L_o = 1$ mH, transfer capacitor $C_t = 25$ µF, output capacitor $C_o = 2000$ µF, switching frequency $f_s = 50$ kHz, load resistance $R_L = 10$ Ω. SIMULINK model implementing ROSMC is shown in Figure 4.3. In ROSMC scheme, the voltage error $e_1$ and current error $e_2$ are calculated and
the sum of the gain product of errors produces the control law $\sigma$. Based on the magnitude and the sign of $\sigma$, the control signal produces the gating pulse to the switch $S$, which in turn shapes the source current.

**Figure 4.3 SIMULINK Model of ROSMC**

The ROSMC algorithm is experimentally tested in the laboratory with the same design parameter values as used in the simulation. The hardware implementation of the experimental setup based on a digital PWM is shown in Figure 4.4. The DS1104 controller board of dSPACE specifically designed for the development of high-speed multivariable digital controllers is plugged into a PCI slot of the PC. The board also includes a slave-subsystem based on the TMS320F2407 DSP processor. The input inductor current, rectified input voltage and output voltage signals are sensed and given to the controller through the ADC channels of the DSP. The ROSMC controller is then designed in SIMULINK and downloaded to the DSP which generates the necessary switching signals to the driver circuit. In the power circuit, the input and output inductors are of ferrite core type and the
capacitors are of plain polyester type. Power MOSFET IRF540N is used as a switch and IN 4007 is used as a diode. IR2110 is used to drive the power MOSFET.

**Figure 4.4 Hardware implementation of ROSMC**

Figures 4.5(a) and 4.5(b) depict the closed loop simulation and experimental results of PFC Ćuk converter after employing ROSMC control technique. Figure 4.5(b) shows experimental waveforms of source current, input inductor current, output voltage and sliding surface of the PFC Ćuk converter controlled by ROSMC strategy using dSPACE 1104 digital controller. UPF operation at the line side and tight regulation of the DC output voltage are evident from the Figures 4.5(a) and 4.5(b). It is also observed that the sliding surface is oscillating around zero. The closed loop control of PFC Ćuk converter using ROSMC forces the source current to follow the source voltage more effectively.
Figure 4.5  Performance of PFC Ćuk converter after applying ROSMC
(a) From Simulation (b) From Experimentation
Figures 4.6(a) and 4.6(b) show the harmonic spectrum of source current obtained by simulation and experimentation and the % THD of the source current is 3.79% in simulation and 6% in experimentation.

Figure 4.6 Harmonic spectrum of source current after applying ROSMC (a) From Simulation (b) From Experimentation

The regulated voltage for step change in load obtained both by simulation and experimentation are shown in Figures 4.7(a) and 4.7(b).

Figure 4.7 Regulated output voltage after applying ROSMC for 15 % increase in load current (a) From Simulation (b) From Experimentation
The converter gets back the reference voltage with a settling time of 150 ms for 15% increase in load current. Table 4.1 presents the performance parameters of PFC Ćuk converter as a function of load current.

### Table 4.1 Performance parameters for load variation using ROSMC

<table>
<thead>
<tr>
<th>Load Current(A)</th>
<th>2.4</th>
<th>2.09</th>
<th>1.96</th>
<th>1.84</th>
<th>1.71</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>THD (%)</td>
<td>3.79</td>
<td>3.87</td>
<td>4.07</td>
<td>4.15</td>
<td>4.23</td>
<td>4.37</td>
</tr>
<tr>
<td>P.F</td>
<td>0.9989</td>
<td>0.9987</td>
<td>0.9982</td>
<td>0.9976</td>
<td>0.9968</td>
<td>0.9957</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>82.1</td>
<td>80.38</td>
<td>79.85</td>
<td>79.4</td>
<td>79.24</td>
<td>78.61</td>
</tr>
</tbody>
</table>

The graphical representation of data given in Table 4.1 is presented in Figure 4.8.

**Figure 4.8 Performance of the system with ROSMC for load variation**
It is observed that the $\% \eta$ is maintained above 75\% for all the loads and the maximum efficiency at rated load is found to be 82.1 \%. The power factor is also maintained close to unity irrespective of the load variation which shows the effectiveness of ROSMC control.

4.4 CONCLUSION

In this chapter, the derivation of control algorithm for ROSMC is reported and a systematic technique for the selection of sliding surface coefficients to implement ROSMC is attempted. The effectiveness of the controller is verified by both simulation and dSPACE 1104 based experimental studies. The output voltage is also regulated for step load variation which shows the robustness and effectiveness of ROSMC. It is concluded that ROSMC is a robust nonlinear controller for input current shaping and output voltage regulation. The proposed method can be used for SMPS applications.