CHAPTER 3

DESIGN OF HYSTERESIS, REDUCED ORDER LINEAR QUADRATIC REGULATOR CONTROLLERS FOR PFC ČUK CONVERTER

3.1 INTRODUCTION

This chapter focuses on the derivation of state model, reduced order model and design of HC and constant frequency ROLQR controllers for PFC Čuk converter. The reduced order model is obtained from the original higher order state-space averaged model of the DC-DC Čuk converter. The model reduction method used here is Pade’s approximation technique. Reduced order model makes the implementation of control technique easier and requires less computation time. A prototype of front end diode rectifier followed by DC-DC Čuk converter controlled by a dSPACE 1104 signal processor for implementing the HC and ROLQR algorithms is set up. Experimental results are presented to validate the simulation studies.

3.2 MODELING OF DC-DC ČUK CONVERTER

The buck and boost feature of the DC-DC Čuk converter is utilized in PFC circuits for instantaneous input current control along with output voltage regulation. To obtain the mathematical model of the controller, the averaged state model of DC-DC Čuk converter is derived by considering the equivalent circuits during switch ON and switch OFF conditions as shown in Figures 2.2 and 2.3. The state space averaged equations are given as follows:
\[
\frac{dx_1}{dt} = -\frac{(1-D)}{L_i} x_3 + \frac{V_s}{L_i} \tag{3.1}
\]
\[
\frac{dx_2}{dt} = \frac{D}{L_o} x_3 - \frac{1}{L_o} x_4 \tag{3.2}
\]
\[
\frac{dx_3}{dt} = \frac{(1-D)}{C_t} x_1 - \frac{D}{C_t} x_2 \tag{3.3}
\]
\[
\frac{dx_4}{dt} = \frac{1}{C_o} x_2 - \frac{1}{R_c C_o} x_4 \tag{3.4}
\]

where \(x_1, x_2, x_3, x_4\) are the current through the input inductor (\(L_i\)), current through the output inductor (\(L_o\)), voltage across the transfer capacitor (\(C_t\)), voltage across the output capacitor (\(C_o\)) and \(D\) represents the duty cycle.

From Equations (3.1)-(3.4), the averaged system matrices obtained are given below,

\[
A = \begin{bmatrix}
0 & 0 & \frac{-(1-D)}{L_i} & 0 \\
0 & 0 & \frac{D}{L_o} & -\frac{1}{L_o} \\
\frac{(1-D)}{C_t} & -\frac{D}{C_t} & 0 & 0 \\
0 & \frac{1}{C_o} & 0 & -\frac{1}{R_c C_o}
\end{bmatrix}, \quad B = \begin{bmatrix}
\frac{1}{L_i} \\
0 \\
0 \\
0
\end{bmatrix} \tag{3.5}
\]

The design values given in Equation (2.8) are substituted in Equation (3.5). After applying phase variable transformation, \(A\) and \(B\) matrices become,

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-6.3 \times 10^3 & -1.87 \times 10^6 & -3.8 \times 10^7 & -50
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix} \tag{3.6}
\]
3.3 DERIVATION OF REDUCED ORDER MODEL

The main objectives are to shape the source current and to regulate the output voltage of the DC – DC Ćuk converter. To meet the requirements, the converter requires sensing of four state variables, which is not preferred from the practical point of view. In order to reduce the complexity in controller design, the fourth order model of a Ćuk converter is reduced to a second order model. The model reduction technique used is Pade’s approximation, wherein the two dominant state variables of the system i.e. the input inductor current, $i_{Li}$ and the output capacitor voltage, $v_{Co}$ are retained and the effects of the transfer capacitor $C_t$, and the output inductor $L_o$ are neglected. Hence it becomes sufficient to regulate only two state variables $i_{Li}$, $v_{Co}$ using the ROLQR control strategy. The reduced order $A$ and $B$ matrices for the above system are given as

$$A = \begin{bmatrix} 0 & 1 \\ -1.64 \times 10^5 & -49.34 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$ (3.7)

3.4 CLOSED LOOP CONTROL OF PFC ĆUK CONVERTER

This section presents the design of the controller. The closed loop system is comprised of an outer voltage control loop to regulate the output voltage and an inner current loop for source current shaping. A PI controller is used for the voltage loop. HC and ROLQR controllers are proposed individually for the inner current loop and their performances are compared.

3.4.1 Principle of PI Controller

A PI compensator is selected for providing DC voltage regulation. The output voltage $V_o$ is sensed and compared with reference voltage $V_{ref}$ and the error is processed by the PI controller to keep the output voltage constant.
The values of $K_p$ and $K_i$ are found by Ziegler-Nichols tuning method. The tuning method provides sustained oscillations with ultimate gain $K_u=4$ and ultimate period $P_u=0.065s$. By this technique, the values of $K_p (=K_u/2)$ and $K_i$ are found to be $2$ and $18.46$ ($T_i=P_u/1.2$, $K_i=K_p/T_i$) respectively.

### 3.4.2 Principle of HC

The block diagram of hysteresis current controlled scheme is shown in Figure 3.1. It consists of an outer PI compensator and an inner HC. In this scheme, the output of PI controller is used to set the peak amplitude of input reference current and is then multiplied with a sine template extracted from the input voltage. This reference current is compared with the actual current and the error is fed to the hysteresis controller which generates ON –OFF signals for the switch $S$.

![Figure 3.1 Closed loop control using HC](image)

The hysteresis controller turns ON the switch when the inductor current goes below the lower reference $i_{Li} - h/2$ and turns OFF the switch when the inductor current goes above the upper reference $i_{Li} + h/2$, giving rise to a variable frequency control as shown in Figure 3.2. For the proposed system, hysteresis band ‘h’ considered is $+0.0005$. 

...
3.4.3 ROLQR Control Scheme

The theory of optimal control is concerned with operating a dynamic system at minimum cost. The time invariant linear quadratic regulator is used as tracking current controller. The Q matrix is chosen in such a way that maximum weightage is given to input inductor current, so that input current shaping is performed more effectively by ROLQR controller. Q and R should be positive semi-definite and positive definite respectively. They are selected such that the scalar quantity $x^T Q x$ is always positive or zero at each time $t$ for all functions $x(t)$, and the scalar quantity $u^T R u$ is always positive at each time $t$ for all values of $u(t)$. Also, the eigen values of $Q$ should be non-negative, while those of $R$ should be positive. The computed Q and R matrices are

$$Q = \begin{bmatrix} 1.16 \times 10^7 & 0 \\ 0 & 1 \end{bmatrix}; \quad R = [1]$$

For a continuous time system, the state-feedback control law $u = -K_p x$ minimizes the quadratic cost function

$$J(x(.), u(.)) = \frac{1}{2} \int_0^t (x^T Q x + u^T R u) dt$$
subject to the system dynamics

\[ \dot{x} = Ax + Bu \]  

(3.10)

For the proposed system, the quadratic cost function is obtained after substituting the values of \( x, Q, R \) in Equation (3.9)

\[ \int_{0}^{\infty} \left[ 1.16 \times 10^7 x_1^2 + x_2^2 + u^2 \right] dt \]  

(3.11)

The control law is found to be

\[ u = -R^{-1}B^T Kx = -K_F x \]  

(3.12)

where \( K_F \) is the feedback gain matrix and \( K \) is the return function matrix. The unknown coefficients of the return function matrix are found by solving the Riccati equation

\[ -Q - A^T K - KA + KBR^{-1}B^T K = 0 \]  

(3.13)

After substituting the values of \( A, B, Q, R, K \) in (3.13), the Riccati equation becomes

\[
\begin{bmatrix}
1.16 \times 10^7 & 0 \\
0 & 1
\end{bmatrix}
- \begin{bmatrix}
0 & -1.64 \times 10^5 \\
1 & -49.34
\end{bmatrix}
\begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
- \begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\begin{bmatrix}
0 & 1 \\
-1.64 \times 10^5 & -49.3
\end{bmatrix}
+ \begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix} = 0
\]  

(3.14)

On solving the Equation (3.14), the return function matrix \( K \) is found to be

\[
\begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
= \begin{bmatrix}
1.2 \times 10^5 & 40 \\
40 & 1
\end{bmatrix}
\]  

(3.15)
After substituting the value of K matrix in the Equation, \( K_F = R^{-1}B^T K \) the feed back gain matrix \( K_F \) is found to be \([35.362 \quad 0.7216]\).

Hence the control law becomes

\[
\begin{align*}
  u &= -35.362(x_{1ref} - x_1) - 0.7216(x_{2ref} - x_2) \\
  \text{(3.16)}
\end{align*}
\]

The Equation (3.16) can be rewritten as

\[
  u = -(K_1 e_1 + K_2 e_2) \\
  \text{(3.17)}
\]

where \( K_1 = 35.362 \) and \( K_2 = 0.7216 \)

### 3.4.3.1 Closed Loop Control Using ROLQR

The proposed ROLQR scheme for a PFC Ćuk converter is illustrated in Figure 3.3. Here PI and ROLQR act as outer voltage and inner current controllers respectively. The input to the PI controller is the voltage error and the output of the PI controller sets the amplitude of the reference inductor current for inner current loop. The inputs to the ROLQR are voltage error \( e_1 \) and the current error \( e_2 \). The resultant control signal \( u = -(K_1 e_1 + K_2 e_2) \) is compared with the saw tooth carrier signal of 50 kHz frequency. The output of the comparator is the PWM signal, which in turn is used for triggering the MOSFET switch. \( K_1 \) and \( K_2 \) as derived in section 3.4.3 are the state feedback gains and are found to be 35.362 and 0.7216 respectively.

![Figure 3.3 Block diagram of ROLQR control](image)

3.5 SIMULATION AND EXPERIMENTAL RESULTS

To study the system performance, simulations have been performed on a digital computer using MATLAB/SIMULINK software tool. The following parameters are considered for simulation: the input AC voltage = 24Vrms, desired output voltage $V_o = 24V$, input inductor $L_i = 2$ mH, output inductor $L_o = 1$ mH, transfer capacitor $C_t = 25 \mu F$, output capacitor $C_o = 2000 \mu F$, switching frequency $f_s = 50$ kHz and load resistance $R_L = 10 \Omega$. SIMULINK model implementing HC is shown in Figure 3.4. In HC scheme, the output of PI controller decides the amplitude of input reference current. This reference amplitude is then multiplied with the sine template extracted from the input voltage. The reference current is compared with the actual current and the error is fed to the HC. HC produces the gating pulse to the switch S, which in turn shapes the source current.

![SIMULINK model implementing HC](image)

**Figure 3.4** SIMULINK model implementing HC

SIMULINK model implementing ROLQR control is shown in Figure 3.5. In constant frequency ROLQR control scheme, the voltage error $e_1$
and current error $e_2$ are calculated and the sum of the gain product of errors produces the control law $u$ which is compared with the saw tooth carrier signal. The output of the comparator is the PWM signal, which in turn is used for triggering the MOSFET switch. A prototype model of the PFC ĉuk converter is developed and controlled with the proposed control strategies using dSPACE 1104 signal processor with the same design values as used in simulation.

![Diagram of DC-DC Ĉuk Converter](image)

**Figure 3.5 SIMULINK model implementing ROLQR Control**

The photograph and hardware implementation of the experimental setup based on a digital PWM are shown in Figures 3.6 and 3.7. The DS1104 controller board of dSPACE specifically designed for the development of high-speed multivariable digital controllers is plugged into a PCI slot of the PC. The board also includes a slave-subsystem based on the TMS320F2407 DSP processor.
The input and output inductors are of ferrite core type and the capacitors are of plain polyester type. Power MOSFET IRF540N is used as a switch and IN 4007 is used as a diode. IR 2110 is used to drive the power MOSFET. The input inductor current, rectified input voltage and output voltage signals are sensed and given to the controller through the ADC channels of the DSP. The 12 bit ADC unit in the dSPACE accepts a maximum analog input voltage of ±10V. During the closed loop operation, the converter output voltage and input inductor current are sensed and scaled down to less than ±10V using the signal conditioning circuit. The HC and ROLQR controllers are then designed in SIMULINK and downloaded to the DSP which generates the necessary switching signals to the driver circuit. The pulses from the dSPACE DS1104 signal processor are not capable of directly driving the MOSFET. In order to increase the pulse amplitude and to provide isolation between the power circuit and control circuit, opto coupler and driver circuits are provided.

Figure 3.6 Hardware Setup of PFC Ćuk converter
Figure 3.7 Hardware implementation of HC/ ROLQR control

Figures 3.8(a) and 3.8(b) present the simulation and experimental results of the closed loop operated PFC Ćuk converter after employing conventional HC. It is observed from Figures 3.8(a) and 3.8(b) that source current becomes sinusoidal and is in phase with the source voltage. Output voltage is also maintained at -22V with a steady state error of 8%.
Figure 3.8 Performance of PFC Ćuk converter after applying HC
(a) From Simulation (b) From Experimentation
The harmonic spectrum of source current after employing HC technique in simulation and experimentation are shown in Figures 3.9(a) and 3.9(b). The % THD of the source current is found to be 5.61% in simulation and 6.5% in experimentation.

![Figure 3.9](image1)

(a)

(b)

**Figure 3.9** Harmonic spectrum of source current after applying HC (a) From Simulation (b) From Experimentation

The output voltage for variation in load is shown in Figures 3.10(a) and 3.10(b). The converter gets back the reference voltage for 12% increase in load current with a settling time of 200ms.

![Figure 3.10](image2)

(a)

(b)

**Figure 3.10** Response of output voltage for 12% increase in load current after applying HC (a) From Simulation (b) From Experimentation
Table 3.1 presents the closed loop performance parameters of HC controlled PFC Ćuk converter as a function of load current.

**Table 3.1 Performance parameters for load variation using HC scheme**

<table>
<thead>
<tr>
<th>Load Current (A)</th>
<th>2.22</th>
<th>2.02</th>
<th>1.94</th>
<th>1.86</th>
<th>1.79</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>THD (%)</td>
<td>5.61</td>
<td>5.73</td>
<td>5.99</td>
<td>6.21</td>
<td>6.43</td>
<td>6.6</td>
</tr>
<tr>
<td>P.F</td>
<td>0.9867</td>
<td>0.9856</td>
<td>0.9843</td>
<td>0.9834</td>
<td>0.9822</td>
<td>0.9815</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>79.76</td>
<td>79</td>
<td>78.7</td>
<td>78.6</td>
<td>78.1</td>
<td>77.75</td>
</tr>
</tbody>
</table>

The graphical representation of data given in Table 3.1 is presented in Figure 3.11. It is inferred that, the $\% \eta$ is maintained above 75% and the maximum efficiency at rated load is found to be 79.76 % for HC.

![Figure 3.11 System performance for load variation using HC](image)

Figures 3.12(a) and 3.12(b) present the simulation and experimental closed loop results of PFC Ćuk converter after employing ROLQR control.
Figure 3.12  Performance of PFC Čuk converter after applying ROLQR Control (a) From Simulation (b) From Experimentation
It is observed from Figures 3.12(a) and 3.12(b) that source current becomes sinusoidal and is in phase with the source voltage. Output voltage is also maintained at the reference value. The harmonic spectrum of source current after employing ROLQR technique in simulation and experimentation are shown in Figures 3.13(a) and 3.13(b). The % THD of the source current is found to be 3.98% in simulation and 6.3% in experimentation.

![Figure 3.13 Harmonic spectrum of source current after applying ROLQR control (a) From Simulation (b) From Experimentation](image)

The regulated output voltage for variation in load with ROLQR control strategy is shown in Figures 3.14(a) and 3.14(b). The converter gets back the reference voltage 16% increase in load current with a settling time of 165ms for this control strategy. Table 3.2 presents the variation of performance parameters of PFC Ćuk converter as a function of load current for ROLQR control.
Figure 3.14  Regulated Voltage for 16% increase in load current after applying ROLQR Control (a) From Simulation (b) From Experimentation

The graphical representation of data given in Table 3.2 is presented in Figure 3.15.

Table 3.2 Performance parameters for load variation using ROLQR control

<table>
<thead>
<tr>
<th>Load Current(A)</th>
<th>2.38</th>
<th>2.19</th>
<th>2</th>
<th>1.83</th>
<th>1.76</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>THD (%)</td>
<td>3.98</td>
<td>4.16</td>
<td>4.23</td>
<td>4.54</td>
<td>4.76</td>
<td>5.12</td>
</tr>
<tr>
<td>P.F</td>
<td>0.9974</td>
<td>0.9967</td>
<td>0.9962</td>
<td>0.9956</td>
<td>0.9948</td>
<td>0.9937</td>
</tr>
<tr>
<td>Efficiency η (%)</td>
<td>80.26</td>
<td>78.3</td>
<td>77.9</td>
<td>77.8</td>
<td>77.5</td>
<td>76.4</td>
</tr>
</tbody>
</table>

It is observed that the %η is maintained above 75% for all the loads and the maximum efficiency at rated load is found to be 80.26 % for ROLQR control. In ROLQR control, the power factor is almost unity for load variations which shows the effectiveness of ROLQR control over HC.
Figure 3.15 System performances for load variation using ROLQR control

3.6 CONCLUSION

The derivation of control algorithms for fixed frequency ROLQR and HC is presented in this chapter. The effectiveness of the controller is verified by both simulation and dSPACE 1104 based experimental studies. The variation of performance parameters of PFC Ćuk converter with change in load proves the effectiveness of ROLQR control over HC. In commercial applications, a fixed frequency is preferred because of low power loss and EMI issues. Hence, it is concluded that fixed frequency ROLQR control is a better choice for input current shaping and output voltage regulation for the considered system.