CHAPTER 4
THEORETICAL INVESTIGATION ON LOW TEMPERATURE FLASH VAPORIZATION

4.1 INTRODUCTION

This chapter describes the theoretical investigation of low temperature flash vaporization process for desalination of saline water. An energy balance model and vapour diffusion model developed are used for calculating the rate of vaporization. Also the vapour diffusion model gives the droplet vaporization time.

The rate of vaporization at the low pressures is particularly significant when the difference between the temperature of the liquid and the saturation temperature i.e., the superheat is high. By reducing the ambient pressure to an extent such that the saturation temperature corresponding to the low pressure is below the saline water temperature, the water can evaporate so long as its temperature remains higher than the saturation temperature. The superheat comes from the reduced pressure and the vapor, so generated at the low pressures, can be condensed in a low-pressure condenser. The temperatures of vaporization and condensation can be tailored to correspond to the upper and lower strata values obtainable in the ocean.
4.2 ENERGY BALANCE MODEL FOR RATE OF VAPORIZATION

The feasibility of low-pressures (sub-atmospheric pressures) vaporization of warm saline water is demonstrated through a simple Energy Balance model. The warm saline water is assumed to be dispersed as a cloud of droplets in a vaporizer column (i.e., flash chamber) maintained at low-pressures. The choice of the level of vacuum pressures to give sufficient vaporization and the requirements of the critical parameters to achieve certain targets of performance are determined from the model. Vaporization from a single representative droplet is considered and the energy balance at the surface of the droplet is modeled.

The following assumptions are made to reduce the number of unknowns. The assumptions used to develop the model include: System operates under steady-state conditions, which is the industry standard, where steady-state conditions are adopted during summer/winter or for high/low production capacity.

a) In the model, saline water is assumed to be distributed in the vaporizer as fine droplets of average diameter D and the drops constituting the spray are spherical and non-deformable.

b) The droplet is a single component pure liquid with zero solubility for gases i.e., and it does not contain impurity.

c) The droplet is at uniform temperature \( T_e \) before flashing.

d) The pressure in the vaporizer is reduced to \( P_{\text{vac}} \) at the time \( t=0 \) and after that it is maintained at the constant pressure.
e) The heat loss or gain from the vaporizer wall, pipelines and pumping system is neglected. Also the rise in the temperature of feed water as it passes through the pump is negligible.

f) The droplet evaporates in a quiescent, infinite medium i.e., the evaporation process quasi-steady. This means that any instant in time the process can be described as if it were in steady state (i.e., the evaporation rate is assumed to be steady vapour flow away from the droplet surface). This assumption eliminates the need to deal with partial differential equations.

g) Vaporization process is not affected by the salt content in the water droplet, because the pressure in the vaporizer is very small.

h) The temperature outside the boundary layer is assumed to be constant.

i) The rate of cooling of the liquid surface by heat conduction in the vapour phase is assumed to be negligible compared with the heat removed by the phase change.

j) The vapour is in equilibrium with the residual brine leaving the vaporizer.

k) Effect of non-condensable gases released from water droplet during vaporization is neglected.

The saline water corresponding to the warm ocean surface temperatures is assumed to be injected at different flow rates into a low-pressure vaporizer. If the average mass of a water droplet formed in the spray is m kg, the heat balance at the droplet-vapor interface is given by Kuo (1986):
\[
\frac{m_\text{c}}{C} \frac{dT_\text{d}}{dt} = \frac{dm}{dt} h_\text{fg} + h_\text{c} \pi D^2 (T_\text{g} - T_\text{d}) + \pi D^2 k \varepsilon_\text{d} (T^4_\text{g} - T^4_\text{d})
\]

(4.1)

Here \( h_\text{c} \) is the heat transfer coefficient at the droplet surface in kW/m\(^2\)K, \( h_\text{fg} \) is the enthalpy of vaporization in kJ/kg, \( D \) is the droplet diameter in m, \( T_\text{g} \) is the ambient gas temperature in \(^\circ\)C, \( T_\text{d} \) is the temperature of droplet in \(^\circ\)C, \( C \) is the specific heat of water in kJ/kgK, \( k \) is the Stefan–Boltzmann’s constant (=5.67 \times 10^{-5} \text{ kW/m}^2\text{K}^4) \) and \( \varepsilon_\text{d} \) is the emissivity of the droplet surface. The vapour flow away from the droplet surface i.e. Vaporization at the droplet surface is shown in Figure 4.1.

In the present investigation water is not significantly heated over the ambient temperature and the main contributing factor for the variation of droplet temperature with time in equation (4.1) is the vaporization at the surface. Hence a lumped mass assumption is used since the Biot modulus \(< 0.1 \), and convective heat transfer is negligible. The radiation heat transfer term in equation (4.1) is neglected, because at equilibrium condition \( T_\text{g} \) is equal to \( T_\text{d} \).

![Figure 4.1 Vaporization at the droplet surface](image)
The initial value of droplet temperature ($T_d$) is same as the feed water temperature ($T_{feed}$), the vaporization results from superheat of the liquid due to the low pressure ambient i.e., due to ambient pressure being lower than the saturation pressure at the given temperature of the saline water droplet, the vaporization can be considered to be driven by the pressure difference between the saturation pressure($P_{sat}$) at the surface temperature of the droplet and the ambient low pressure($P_{vac}$) (Muthunayagam et al 2005a). The latent heat of vaporization of water and specific heat is significant and is reasonably constant at about 2450 kJ/kg and 4.19 kJ/kgK respectively, in the temperature range between 11°C and 32°C. The main contributing factor for the variation of droplet temperature with time in equation (4.1) is the evaporation at the surface. Therefore equation (4.1) can be simplified as;

$$m_d C \frac{dT_d}{dt} = \frac{dm}{dt} h_{fg} \quad (4.2)$$

$$\frac{\Delta m}{m} = \frac{C \Delta T_d}{h_{fg}} \quad (4.3)$$

Here $\Delta m$ is the mass of water vaporized when the temperature of the droplet of mass $m$ falls by $\Delta T_d$ (i.e., $T_d - T_{sat}$). Since the convective transport at the surface is neglected in equation, (4.1), the fractional mass evaporated ($\Delta m/m$) is seen to be independent of diameter. On substituting the values of $h_{fg}$ and $C$ in equation (4.3), therefore it is simplified as;

$$\Delta m/m = 1.71 \times 10^3 \times \Delta T_d \quad (4.4)$$

Vaporization is possible only when the saturation vapour pressure corresponding to the temperature of droplet ($P_{sat} T_d$) exceeds the ambient vacuum pressure in the vaporizer ($P_{vac}$), stated differently; for a given value of
vaporizer pressure, the vaporization of the water droplets would be possible only when the temperature of the droplet exceeds the value of the saturation temperature \( T_{\text{sat}} \) corresponding to the pressure in the vaporizer. For temperature of feed water varying between 26°C and 32°C and pressure in the vaporizer between 10 mm of Hg and 18 mm of Hg (abs) the maximum temperature drop of droplet \( \Delta T_d \) (i.e., Liquid superheat) up to which vaporization continues is given in the Table 4.1, the properties of water and vapour are taken from the steam table (Ballaney 2003).

**Table 4.1 Liquid superheat (\( \Delta T_d \)) in °C**

<table>
<thead>
<tr>
<th>Pressure in the vaporizer ( [P_{\text{vac}}] ) (mm of Hg)</th>
<th>( T_{\text{sat}} ) (°C)</th>
<th>Temperature of feed water ( [T_{\text{feed}}] ) (°C)</th>
<th>26</th>
<th>28</th>
<th>30</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11.25</td>
<td>14.75</td>
<td>16.75</td>
<td>18.75</td>
<td>20.75</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>16.42</td>
<td>09.58</td>
<td>11.58</td>
<td>13.58</td>
<td>15.58</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>20.43</td>
<td>05.57</td>
<td>07.57</td>
<td>09.57</td>
<td>11.57</td>
<td></td>
</tr>
</tbody>
</table>

The Liquid Superheat (\( \Delta T_d \)) for different levels of pressure in the vaporizer \( P_{\text{vac}} \) and temperature of droplet \( T_d \) is given in Figure 4.2.
Figure 4.2  Liquid superheat as a function of temperature of droplet for different pressures in the vaporizer

It is seen that the temperature difference progressively increases as the pressure in the vaporizer decreases and temperature of droplet increases. Also it is seen that the temperature drop of droplet ($\Delta T_d$) is very sensitive to the level of vacuum in the vaporizer. The liquid superheat rapidly increases as the pressure in the vaporizer decreases. Using equation (4.4) the percentage mass of injected water vaporized ($\Delta m/m$) is estimated for different temperatures of droplet and pressures in the vaporizer and are given in Table 4.2.
Table 4.2 Percentage mass of water droplet vaporized (Δm/m)

<table>
<thead>
<tr>
<th>Pressure in the vaporizer ( P_{\text{vac}} ) (mm of Hg)</th>
<th>Temperature of feed water ( T_{\text{feed}} ) (°C)</th>
<th>26</th>
<th>28</th>
<th>30</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td>2.52</td>
<td>2.86</td>
<td>3.21</td>
<td>3.55</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>1.64</td>
<td>1.98</td>
<td>2.32</td>
<td>2.66</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>0.95</td>
<td>1.29</td>
<td>1.64</td>
<td>1.98</td>
</tr>
</tbody>
</table>

The percentage mass of water vaporized as a function of temperature of droplet for different pressures in the vaporizer is shown in Figure 4.3A.

Figure 4.3A Percentage mass of water droplet vaporized as a function of temperature of droplet for different pressures in the vaporizer
It is seen that the rate of vaporization increases as the temperature of the droplet increases, the rate of vaporization increases for a given pressure in the vaporizer; however, in the range of the temperature of droplet between 26°C and 32°C, the increase in pressure from 10 mm of Hg to 18 mm of Hg leads to a very rapid deterioration of the fractional vaporization of water. The percentage mass of water vaporized as a function of pressure in the vaporizer for different temperatures of droplet is shown in Figure 4.3B.

![Graph showing the relationship between fraction of mass vaporized and pressure in the vaporizer for different temperatures of droplet.](image)

**Figure 4.3B** Percentage mass of water vaporized as a function of pressure in the vaporizer for different temperatures of droplet

It is seen that the rate of vaporization is very sensitive to the level of vacuum in the vaporizer. The evaporation rapidly falls as the pressure in the vaporizer increases especially at low temperature of droplet. Also as the initial temperature of droplet increases, the percentage of vaporization increases for
a given pressure in the vaporizer, same trend is also seen in pool flash evaporation (Osamu Miyatake 1973). The desirability to operate the system at higher temperature of feed water and lower pressure in the vaporizer is seen from the plots. The values shown in the Tables 4.1 and 4.2 are calculated by neglecting the effect of salt content in the water, since the pressure in the vaporizer is very small (Lange 1969).

4.3 VAPOUR DIFFUSION MODEL FOR RATE OF VAPORIZATION

The saline water is assumed in the model to be distributed in the vaporizer as fine droplets of average diameter D. The vaporization from droplets in a spray involves simultaneous heat and mass transfer process. The overall rate of vaporization depends on the temperature of the droplets, pressure in the vaporizer and diameter of the droplets in the spray. The liquid leaves the injector (nozzle) in the form of liquid sheets, which rapidly disintegrate into drops of varying size. The assumptions made in the energy balance model are also considered for this model.

The evaporation of droplets has been investigated extensively using equations of diffusion and continuity (Faeth 1983 and Kuo 1986). Here the transport processes within the droplet is neglected and the flow around the droplet in the vapour phase is taken to be quasi-steady. As the vaporization takes place from the droplet surface, its temperature and mass goes down depending on the ambient gas conditions and the initial water temperature at the same time lose part of their mass by vaporization and diffuses into the surrounding air. The term steady state is perhaps a misnomer when applied to droplet vaporization because a water droplet may not attain steady state vaporization during its lifetime.
The problem solved here is the determination of the mass flow rate of water vapour from the surface of the water droplet at any instant in time, knowledge of this will helps to calculate the droplet radius/diameter as a function of time and temperature and also to find out the droplet lifetime (i.e., droplet evaporation time), corresponding to different ambient pressures.

4.3.1 Rate of variation of droplet diameter

The present problem is eventually the diffusion of water vapour from the surface of the water droplet. The problem of vaporization from a single liquid droplet in a quiescent environment is the Stefan’s problem (Yunus A. Cengel 2003) for a spherically symmetric coordinate system. Figure 4.4 gives a spherically symmetric co-ordinate system for a droplet, the radius $r$ is the only co-ordinate variable. It has its origin at the centre of the droplet, the droplet radius at the liquid - vapour interface is denoted by $r_s$ and vary far away from the droplet surface is $r \rightarrow \infty$.

![Spherically symmetric co-ordinate systems for a droplet](image)

**Figure 4.4 Spherically symmetric co-ordinate systems for a droplet**
The Mass concentration of droplet vapour corresponding to \( r_s \) at the surface of the droplet is \( C_{v,s} \), this would change with time as the temperature at the droplet surface changes. The mass fraction of droplet vapour corresponding vary far away from the droplet surface \((r \rightarrow \infty)\) is \( C_{v,\infty} \). Mass concentration of water vapour at any radius \( r \) between \( r_s \) and \( \infty \) is \( C_{v,r} \) (i.e., \( r_s < r < \infty \)). Here the driving force for mass transfer is the difference of the water vapour concentration at the droplet surface and its environment.

Here the problem can be modeled as “mass transfer due to simultaneous action of convection and diffusion” i.e., Water vapour is transported by two means, first by the bulk motion of the fluid (by convection) and then by the diffusion of water vapour superimposed on the bulk flow (i.e., by molecular collision with air). By Fick`s law of diffusion (Cussler 1998):

\[
\begin{align*}
\{ \text{Mass flux of } & \} = \{ \text{mass flux of water vapour } \} + \{ \text{mass flux of vapour due to molecular diffusion } \} \\
\dot{m}_v = C_v [\dot{m}_f + \dot{m}_a] + \left[ -\rho_v \Gamma_{v,a} \frac{dC_v}{dr} \right]
\end{align*}
\]

(4.5)

In the above expression, \( \dot{m}_v \) is the mass flux of water vapour in \( \text{kg/s.m}^2 \), \( \dot{m}_a \) is the mass flux of air in \( \text{kg/s.m}^2 \), \( \Gamma_{v,a} \) is the diffusion coefficient of water vapor in the low pressure ambient (i.e., air) in \( \text{m}^2/\text{s} \), \( \rho_v \) is the density of water vapor in \( \text{m}^3/\text{kg} \) and \( C_v \) is the mass fraction (mass concentration) of water vapor. In equation (4.5) the convection of water vapour considered, but not the convection of air, therefore \( \dot{m}_a = 0 \).

\[
\dot{m}_v = C_v [\dot{m}_f] + \left[ -\rho_v \Gamma_{v,a} \frac{dC_v}{dr} \right]
\]

(4.6)
\[ [1 - C_v] \dot{m}_v = \left[-\rho_v \Gamma_{v,a} \frac{dC_v}{dr}\right] \]  

(4.7)

But, \( \dot{m}_v = \frac{\dot{m}_v}{A} \), where \( \dot{m}_v \) is the mass flow rate of water vapour in kg/s.

Therefore the above equation (4.7) becomes;

\[ [1 - C_v] \left[ \frac{\dot{m}_v}{4\pi r^2} \right] = \left[-\rho_v \Gamma_{v,a} \frac{dC_v}{dr}\right] \]  

(4.8)

\[ \left[ \frac{\dot{m}_v}{4\pi \rho_v \Gamma_{v,a}} \right] \left[ \frac{dr}{r^2} \right] = \left[-\frac{dC_v}{[1 - C_v]} \right] \]  

(4.9)

\[ \left[ \frac{\dot{m}_v}{4\pi \rho_v \Gamma_{v,a}} \right] \left[ \frac{dr}{r^2} \right] = \left[-\frac{dC_v}{[1 - C_v]} \right] \]  

(4.10)

Integrating the above equation from the surface of the droplet to any radius \( r \) from the centre of the droplet; \( C_v = C_{v,s} \) when \( r = r_s \) and \( C_v = C_{v,\infty} \) when \( r \rightarrow \infty \), the above equation becomes:

\[ \left[ \frac{\dot{m}_v}{4\pi \rho_v \Gamma_{v,a}} \right] \int_{r_s}^{r} \frac{dr}{r^2} = \left[\frac{C_v - C_{v,\infty}}{[1 - C_v]} \right] \]  

(4.11)

\[ \left[ \frac{\dot{m}_v}{4\pi \rho_v \Gamma_{v,a}} \right] \left[ -\frac{1}{r} \right]_{r_s}^{r = \infty} \left[\ln (1 - C_v) \right]_{C_s = C_{v,s}}^{C_v = C_{v,\infty}} \]  

(4.12)

\[ \left[ \frac{\dot{m}_v}{4\pi \rho_v \Gamma_{v,a}} \right] \left[ -\frac{1}{r} - \frac{1}{r_s} \right] = \left[\ln (1 - C_v,\infty) \right]_{C_s = C_{v,s}} - \ln (1 - C_{v,s}) \]  

(4.13)
In the above equation (4.13), the mass fraction $C_{v,s}$ at the surface would change with time as the temperature at the droplet surface changes. If the saline water droplets in the spray are assumed to be well dispersed without interacting with each other, the concentration $C_{v,\infty}$ would be zero at the initial times whereas for larger times $C_{v,\infty} \ll C_{v,s}$, therefore $C_{v,\infty} \approx 0$. For these conditions, equation (4.13) can be simplified and results in:

$$\left[ \frac{\dot{m}_v}{4\pi\rho_v r_{v,a}} \right] \left[ \frac{1}{r_s} \right] = -\ln(1 - C_{v,s}) \quad (4.14)$$

Substituting $D (= 2r_s)$ in the above equation results in:

$$\dot{m}_v = -2\pi\rho_v r_{v,a} D \ln(1 - C_{v,s}) \quad (4.15)$$

Depending on the amount by which the saturation vapor pressure at the droplet surface exceeds the ambient pressure, flash vaporization would occur. The process of such vaporization is not strictly by diffusion of water vapor away from the liquid surface under quasi-steady conditions for which equation (4.6) is applicable. However, diffusion is important in the transport of the vapour and as an approximation; the vaporization can be modeled to be driven by diffusion from the gradient in the vapor mass fraction at the droplet surface. The mass fraction of the water vapor at the droplet surface $C_{v,s}$ in equation (4.15) can be approximated for the vapor diffusion model as:

$$C_{v,s} = \frac{M_v}{M_{v+a}} \times Y_{v,s} \quad (4.16)$$

where $Y_{v,s}$ is the mole fraction of vapour at the droplet surface, $M_v$ denote the molecular weight of water vapour ($= 18.01527$ kg/kmol), $M_a$ denote the molecular weight of air ($= 28.9645$ kg/kmol) and $M_{v+a}$ denote the molecular weight of mixture of water vapour and air, which is given by;
\[ M_{v+a} = Y_{v,s} M_v + Y_a M_a \]  

(4.17)

where \( Y_a \) and \( Y_{v,s} \) is the mole fraction of air (or) pressure fraction of air \((= P_a / P)\) and mole fraction of vapour at the droplet surface \((= P_{sat} / P)\), also \( P \) is the total pressure of vapour and air in mm of Hg, \( P_a \) is the Partial pressure of air in mm of Hg and \( P_{sat} \) is the partial pressure of water vapour at the droplet surface in mm of Hg.

Assuming equilibrium exists between the liquid and vapour phase of water vapour and the assumption of ideal gases, the partial pressure of water vapour on the gas side of the interface must equal to the saturation vapour pressure associated with the temperature of the liquid i.e., \( P_{sat} \). By Dalton’s law of additive Pressure (Yunus A. Cengel 2003); \( P = P_{sat} + P_a \). In low pressure region \( P_a \) can be represented as \( P_{vac} \) where \( P_{vac} \) is the pressure in the vaporizer in mm of Hg. Therefore \( Y_a \) and \( Y_{v,s} \) can be written as;

\[ Y_{v,s} = \frac{P_{sat}}{P_{sat} + P_{vac}} \]  

(4.18)

\[ Y_a = \frac{P_{vac}}{P_{sat} + P_{vac}} \]  

(4.19)

On substituting the above values of \( Y_{v,s} \) and \( Y_a \) in equation (4.17), which results in;

\[ M_{v+a} = \frac{P_{sat}}{P_{sat} + P_{vac}} \times 18.015 + \frac{P_{vac}}{P_{sat} + P_{vac}} \times 28.97 \]  

(4.20)

\[ M_{v+a} = \frac{18.015P_{sat} + 28.97P_{vac}}{P_{sat} + P_{vac}} \]  

(4.21)
Put the value of \( Y_{v,s} \), \( M_v \) and equation (4.21) in equation (4.16);

\[
C_{v,s} = \frac{18.015P_{sat}}{18.015P_{sat} + 28.97P_{vac}} \quad (4.22)
\]

\[
C_{v,s} = \frac{P_{sat}}{P_{sat} + 1.6P_{vac}} \quad (4.23)
\]

Assuming ideal-gas behavior for vapour; which indicates that, the diffusion co-efficient for dilute gases at ordinary pressure is essentially independent of mixture composition the pressure and temperature dependence of binary diffusion co-efficient can be estimated using (Robert C. Reid et al 1989); \( \Gamma_{v,a} T^{1.5} \). It is seen that \( \Gamma_{v,a} \) tends to increase with temperature while decreasing with pressure. The value of \( \Gamma_{v,a} \) at \( T = 0^\circ C \) (273.15 K) and at 1 atmospheric pressure is \( 2.2 \times 10^{-5} \) m\(^2\)/s (Robert H. Perry and Don W. Green 1997) . Therefore;

\[
\left[ \Gamma_{v,a} \right]_{vac} = \left[ \Gamma_{v,a} \right]_{atm} \left( \begin{array}{c}
\frac{T^{1.5}}{P}_{vac} \\
\frac{T^{1.5}}{P}_{atm}
\end{array} \right) \quad (4.24)
\]

\[
\left[ \Gamma_{v,a} \right]_{vac} = \frac{3.707 \times 10^{-6} \times T^{1.5}}{P_{sat} + P_{vac}} \quad (4.25)
\]

In the above equation \( P_{sat}, P_{vac} \) are in mm of Hg and temperature in Kelvin. This equation (4.25) shows that the values of \( \Gamma_{v,a} \) increases as the temperature of the feed water, \( T \) increases and reduces as the pressure in the vaporizer, \( P_{vac} \) increases.
The droplet radius/diameter history can be obtained by writing droplet mass conservation (mass balance), which states that the rate at which the mass of the droplet decreases is equal to the rate at which the liquid is vaporized (Stephen R. Turns 1996):

\[ m_d = -m_v \]  \hspace{1cm} (4.26)

where
\[ m_v = \frac{d m_v}{d t} = \text{Rate of change in mass of vapour with respect to time in kg/s} \]
\[ m_d = \frac{d m_d}{d t} = \text{Rate of change in mass of droplet with respect to time in kg/s} \]

\[ m_d = \text{mass of water droplet in kg (}= \rho_d \times V_d \) \]
\[ \rho_d = \text{density of water droplet (}= 1000 \text{ kg/m}^3 \) \]
\[ V_d = \text{Volume of spherical water droplet of radius, } \frac{r}{6} \left( = \frac{\pi D^3}{6} \right) \]

Therefore,

\[ m_d = \rho_d \frac{\pi D^3}{6} \]  \hspace{1cm} (4.27)

On differentiating equation (4.27) with respect to

\[ m_d = \frac{d m_d}{d t} = \frac{\pi}{2} \rho_d D^2 \frac{d D}{d t} \]  \hspace{1cm} (4.28)
But, \( \frac{dD^2}{dt} = 2D \frac{dD}{dt} \); Therefore substituting this in the above equation results in:

\[
m_a = \frac{\pi}{4} \rho_d D \frac{dD^2}{dt}
\]

(4.29)

On substituting equations (4.29) and (4.15) in (4.26), which results in;

\[
\frac{\pi}{4} \rho_d D \frac{dD^2}{dt} = -\left\{ -2\pi \rho_v \Gamma_{v,a} D \ln (1 - C_{v,s}) \right\}
\]

\[
\frac{dD^2}{dt} = \frac{8\rho_v \Gamma_{v,a} \ln (1 - C_{v,s})}{\rho_d}
\]

(4.30)

The variation of diffusion coefficient \( \Gamma_{v,ar} \), the fractional concentration at the surface \( C_{v,s} \) and the vapor density \( \rho_v \) needs to be determined as a function of time in order to solve the above equation for rate of droplet diameter variations. All the three parameters \( \Gamma_{v,ar} \), \( C_{v,s} \) and \( \rho_v \) are strongly dependent on the temperature and since the droplet temperature changes with time, these parameters would vary significantly.

### 4.3.2 Temperature variation at the droplet surface with time

Kozyrev and Sitnikov (2001) pointed out the oversimplification of evaporation process in the above method since the non-equilibrium phenomenon is neglected. In the present model, the equilibrium assumption is not relaxed; however, a global droplet temperature variation is built into the diffusion model by determining its changes from the heat balance at the droplet-vapor interface given below.
\[
m_d C \frac{dT_d}{dt} = h_g \pi D^2 (T_g - T_d) + \frac{dm_d}{dt} h_{fg}
\]

(4.31)

The temperature \( T_d \) of the droplet changes as soon as evaporation occurs at its surface. A lumped mass assumption is used which is justifiable from the small convective heat transfer at the surface of the droplet. Therefore the above equation becomes;

\[
m_d C \frac{dT_d}{dt} = \dot{m}_d h_{fg}
\]

(4.32)

Substitute the equation (4.27) in (4.29)

\[
\rho_d \frac{\pi}{6} D^3 C \frac{dT_d}{dt} = \frac{\pi}{4} \rho_d D \frac{dD^2}{dt} h_{fg}
\]

(4.33)

\[
\frac{dT_d}{dt} = \frac{3h_{fg}}{2D^2} \frac{dD^2}{dt}
\]

(4.34)

The latent heat of vaporization of water \( (h_{fg}) \) and specific heat \( (C) \) is significant and is reasonably constant at about 2450 kJ/kg and 4.19 kJ/kgK respectively, in the temperature range between 11\(^\circ\)C and 32\(^\circ\)C. Substituting the values of \( h_{fg} \) and \( C \) in the above equation results in;

\[
\frac{dT_d}{dt} = \frac{878 \, dD^2}{D^2 \, dt}
\]

(4.35)

Instead of solving equations (4.30) and (4.35) simultaneously as an initial value problem, it is preferable to examine equation (4.14) for the explicit dependence of the droplet diameter variations at different values of temperature and vacuum pressures. This exercise will lead to an insight of the acceptable regimes of droplet temperatures and vacuum pressures over which vaporization takes place. Figure 4.5 shows the variation of the square of
droplet diameter with time at different specified values of droplet temperature as determined from equation (4.30). If the values of the surface concentration and binary diffusion coefficient $C_{v,s}$ and $\Gamma_{v,a}$ are determined at the different values of vacuum pressure, liquid droplet temperature and the corresponding values of the saturation pressure and the density of vapour adjacent to the droplet is obtained from the specific volume given in saturation vapour pressure tables (Ballaney 2003) and the density of water is taken as 1000 kg/m$^3$. The above values are substituted in equation (4.30) the value of $\frac{dD^2}{dt}$ can be determined, plotted and shown in Figure 4.5.

![Figure 4.5](image)

**Figure 4.5  Rate of droplet diameter variations with temperature and pressure**

It is seen from the Figure 4.5, higher values of droplet diameter variations are obtained at lower pressures in the vaporizer and higher
temperatures of droplet and this trend is to be anticipated. It is also seen from the Figure 3.5 that the variation of \( \frac{dD^2}{dt} \) is reasonable linear over a range of temperatures. The following linear relation is seen to fit the variation of \( \frac{dD^2}{dt} \) with temperature at the given specified values of pressure:

a) Pressure 10mm of Hg

\[
\frac{dD^2}{dt} = [18.14 - 0.0640 T] \times 10^{-7} ; 32^\circ C \geq T \leq 22^\circ C
\]  \hspace{1cm} (4.36A)

\[
\frac{dD^2}{dt} = [09.96 - 0.0363 T] \times 10^{-7} ; 22^\circ C \geq T \leq 11.25^\circ C
\]  \hspace{1cm} (4.36B)

b) Pressure 14mm of Hg

\[
\frac{dD^2}{dt} = [14.23 - 0.0500 T] \times 10^{-7} ; 32^\circ C \geq T \leq 24^\circ C
\]  \hspace{1cm} (4.37A)

\[
\frac{dD^2}{dt} = [09.96 - 0.0356 T] \times 10^{-7} ; 24^\circ C \geq T \leq 16.42^\circ C
\]  \hspace{1cm} (4.37B)

c) Pressure 18 mm of Hg

\[
\frac{dD^2}{dt} = [10.76 - 0.0378 T] \times 10^{-7} ; 32^\circ C \geq T \leq 20.43^\circ C
\]  \hspace{1cm} (4.38)

The value of temperature in the above set of equations is in Kelvin. The above set of equations for rate of droplet size variations gives a generalized correlation of the following form:

\[
\frac{dD^2}{dt} = a - bT
\] \hspace{1cm} (4.39)

The values of the constants \( a \) and \( b \) have units of \( m^2/s \) and \( m^2/s\ K \) respectively. The pressure in the vaporizer allows the vaporization to take place
till the temperature at the droplet surface reduces to the value of the saturation
temperature corresponding to the pressure in the vaporizer. The rate of droplet
diameter variation is therefore indicated in Figure 4.5 only for the temperatures
exceeding the saturation values corresponding to the vaporizer pressure. The
value of \( \frac{dD^2}{dt} \) is; however, not seen to be zero in Figure 4.5 as it should be at
the saturation temperature and a small but finite amount of vaporization is
observed due to errors in the input data.

### 4.3.3 Droplet vaporization time

Droplet vaporization time (or) Droplet Lifetime (or) Flashing time is
defined as the time taken for the droplet to reach saturation temperature
(corresponding to the pressure in the vaporizer \( (T_{sat}, P_{vap}) \)) from its initial
temperature \( (T_d) \), during the evaporation of droplet. Substituting the linear
variation observed for \( \frac{dD^2}{dt} \) with temperature (i.e., equation (4.17)) in equation
(4.16), the following expression is obtained:

\[
\frac{dT}{dt} = \frac{878}{D^2} (a - bT)
\]  

(4.40)

Equation (4.39) gives the variation of temperature as the droplet vaporizes. Vaporization will take place as long as the saturation vapor pressure at the droplet temperature exceeds the ambient vacuum pressure. The surface tension induced pressure at the droplet is \( 2\sigma/r \) where \( \sigma \) is the surface tension coefficient and \( r \) is the radius of the droplet. For a droplet of 1 mm in
diameter, this pressure is about 2.2 mm of Hg, which is very much smaller
than the lowest vacuum pressure of the experiments. The effect of surface
tension is therefore neglected when determining the phase equilibrium at the
liquid surface. The equation (4.39) can be further simplified to;
\[
\frac{dT}{(a - bT)} = \frac{878}{D^2} \, dt \tag{4.41}
\]

The amount of vaporization is generally small and as an approximation the variations in droplet diameter may be neglected while calculating the temperature variation of droplet with time. This is justified subsequently. Equation (4.41) can be integrated for the near-constant diameter of the droplet to give the droplet vaporization time \(t_v\) as:

\[
\int_{T=T_i}^{T=T_f} \frac{dT}{(a - bT)} = \frac{878}{D^2} \int_{t=t_i}^{t=t_f} dt \tag{4.42}
\]

\[
\ln\left(\frac{a - bT_f}{a - bT_i}\right) - \ln\left(\frac{a - bT_i}{a - bT_f}\right) = \frac{878}{D^2} [t_f - t_i] \tag{4.43}
\]

\[
\frac{1}{-b} \left[ \ln\left(\frac{a - bT_f}{a - bT_i}\right) \right] = \frac{878}{D^2} \left( t_e \right) \tag{4.44}
\]

\[
t_e = \frac{D^2}{878b} \left[ \ln\left(\frac{a - bT_i}{a - bT_f}\right) \right] \tag{4.45}
\]

Here \(T_f\) is the temperature at which evaporation ceases and is equal to the saturation temperature corresponding to the ambient pressure in the vaporizer and \(T_i\) is the initial droplet temperature which is same as the temperature of feed water into the vaporizer and \(t_e\) is the time for droplet vaporization. At lower value of vacuum pressure, two sets of evaporation constants \(a\) and \(b\) were obtained over the range of temperatures over which vaporization takes place (Figure 4.5). Denoting the temperature at which the constants change as \(T_c\) and denoting these constants as \(a_1\) and \(b_1\) in the range \(T_i\) to \(T_c\) and as \(a_2\) and \(b_2\) in range \(T_c\) and \(T_f\), the time of evaporation is:
\[ t_e = \left\{ \frac{D^2}{878 b_1} \ln \left( \frac{a_1 - b_1 T_i}{a_1 - b_1 T_c} \right) \right\} + \frac{D^2}{878 b_2} \ln \left( \frac{a_2 - b_2 T_c}{a_2 - b_2 T_r} \right) \]  

(4.47)

The time calculated from equations (4.46) and (4.47) are shown in Figure 4.6 for a droplet of initial diameter of 1 mm.

![Graph showing time for evaporation vs. droplet temperature for different pressures.](image)

**Figure 4.6** Droplet vaporization time for a 1 mm diameter of droplet for different pressures in the vaporizer

It is seen that the time is typically a few hundred millisecond. The time is higher for larger values of initial temperatures of the droplet and lower values of pressures in the vaporizer. The time varies directly as the square of the droplet diameter based on equations (4.46) and (4.47). Flashing time decreases with an increase of equilibrium temperature \( [T_{sat}]_{P_{sat}} \) and with the decrease of superheat. The vaporization time for a vacuum level is proportional to the square of the diameter of the particle and increases with the feed temperature...
due to larger temperature difference between the feed temperature and the final temperature at which vaporization ceases.

4.3.4 Diameter of the droplet after vaporization

Once the time of evaporation \( t_e \) is determined, the decrease in the diameter of the droplet can be determined from the observed linear variation of \( D^2 \) with time given by equation (4.39), this can be written as:

\[
\frac{D_i^2 - D_f^2}{t_e} = a - bT
\]  

(4.48)

An average value of temperature \( T \) in the linear range of the variation of \( dD^2/dt \) between the initial droplet temperature and the final temperature corresponding to the saturation temperature values corresponding to the pressure in the vaporizer is used, i.e., \( T_{av} = (T_i + T_f)/2 \), therefore:

\[
D_f = \sqrt{t_e \left(a - bT_{av}\right) + D_i^2}
\]  

(4.49)

A two-step variation, with corresponding average temperatures for each linear zone are used when the constants \( a \) and \( b \) change over the temperature interval, i.e., \( T_{av1} = (T_i + T_c)/2 \) and \( T_{av2} = (T_c + T_f)/2 \), therefore:

\[
D_f = \sqrt{[t_{e1}(a_1 - b_1T_{av1}) + t_{e2}(a_2 - b_2T_{av2})] + D_i^2}
\]  

(4.50)

Here \( t_{e1} \) and \( t_{e2} \) are evaporation times of the droplet, when the droplet temperature drops from \( T_i \) to \( T_c \) and \( T_c \) to \( T_f \) respectively. The final diameter of the droplet \( (D_f) \), when evaporation ceases, for an initial droplet diameter of 1mm is given in Table 4.3.
Table 4.3 Diameter of the 1mm diameter droplet after vaporization in mm

<table>
<thead>
<tr>
<th>$T_d$ (°C)</th>
<th>Pressure in the vaporizer $[P_{vac}]$ (mm of Hg)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>14</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.9876</td>
<td>0.9909</td>
<td>0.9930</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.9888</td>
<td>0.9921</td>
<td>0.9943</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0.9901</td>
<td>0.9932</td>
<td>0.9955</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>0.9912</td>
<td>0.9944</td>
<td>0.9968</td>
<td></td>
</tr>
</tbody>
</table>

The final diameter of the droplet ($D_t$), when evaporation ceases, for an initial droplet diameter of 1 mm is at the different levels of pressure in the vaporizer ($P_{vac}$) and temperature of droplet ($T_d$) is given in Figure 4.7.

Figure 4.7 Diameter of the 1mm droplet after vaporization as a function of $T_d$ for different pressure in the vaporizer
The final diameters are seen to be very near to the initial value of 1 mm indicating the assumption of constant D while integrating equation (4.41) to be reasonable.

4.3.5 Prediction for percentage of vaporization

The mass of droplet \( m_d \) is directly proportional to cube of diameter of droplet (i.e., \( m_d \propto D^3 \)). The initial mass of droplet \( (m_{d,i}) \) is directly proportional to cube of initial diameter of droplet \( (D_i) \) and the final mass of droplet \( (m_{d,f}) \) is also directly proportional to cube of final diameter of droplet \( (D_f) \). The fractional mass evaporated from the droplet can be found out by the following equations:

\[
\text{Fractional mass of water evaporated} = \frac{m_{d,i} - m_{d,f}}{m_{d,i}} = \frac{D_i^3 - D_f^3}{D_i^3}
\]  

(4.51)

\[
\text{Fractional mass of water evaporated} = 1 - \left[ \frac{D_f}{D_i} \right]^3
\]  

(4.52)

The fractional mass evaporated from a 1 mm droplet as a function of temperature of droplet is plotted in Figure 4.8, for the different values of vacuum pressures and injection temperatures of the saline water. An approximate estimate for percentage vaporization from the energy balance model using equation (4.4) is calculated for the above value of \( T_d \) and \( P_{vac} \), and this is shown as a dotted line in Figure 4.8.
**Figure 4.8** The fractional mass vaporized from a 1 mm droplet as a function of temperature of droplet

It is seen that the evaporation is very sensitive to the level of vacuum in the vaporizer. The evaporation rapidly decreases as the pressure in the vaporizer increases. As the temperature of the water droplet increases, the percent evaporation also increases; however, in the range of the water temperature between 26°C and 32°C, the increase in pressure from 10 mm of Hg to 14 mm of Hg in the vaporizer leads to a more rapid deterioration of the fractional vaporization of droplet. The very important role of the vacuum in the vaporizer is brought out through these calculations. The need to maintain the vacuum pressures between about 10 mm of Hg and 18 mm of Hg is seen to be essential to get a yield exceeding about 3% when the feed water temperatures are around 30°C. These parameters are adopted for the experiments.
The values are almost identical to those obtained from equation (4.52) when the pressure in the vaporizer is between 14 mm of Hg and 18 mm of Hg. At very low pressure vaporizer of 10 mm of Hg, the deviations become significant with the diffusion model predictions being much higher. Errors in diffusion model are expected to be significant at these small values of pressure since surface tension-induced pressures, though smaller, are about the same order of magnitude and effectively bring down the values of saturation temperature and pressure. Also it is seen that, when the pressure in the vaporizer is 18 mm of Hg, the deviation of energy balance model from the vapour diffusion model is high compared with $P_{\text{vac}}$ at 14 mm of Hg, this is because a single-step variation in $dD^2/dt$ is considered when the pressure in the vaporizer is 18 mm of Hg.

The energy balance and vapour diffusion models therefore suggests that if condensation of the evaporated water can be ensured, a good yield of fresh water is feasible from the low-pressure vaporization. However, the model does not give inputs on whether the warm saline water should be dispersed as coarse droplets or fine droplets in the low-pressure vaporizer to obtain sufficient vaporization. The diffusion of the vapor from water droplets of different sizes, the residence time of the droplets in the low-pressure vaporizer and the evaporation time of the vaporization process need to be considered. In general, dispersion of the warm water as finer droplets would present larger surface area for the vaporization process. The use of a low-pressure drop atomizer for generating small water droplets in the vaporizer seems desirable.