Chapter 3

Linear Discriminant Analysis for 3D Face Recognition System

3.1 Introduction

Face recognition and verification have been at the top of the research agenda of the computer vision community in recent times. Due to its applications envisaged in physical and logical access control, security, man-machine interfaces and low bitrate communication [Smeets et al., 2010]. To date, most of the research efforts, as well as commercial developments, have focused on two dimensional (2D) approaches. To overcome the limitation of 2D intensity images [Sandbach et al., 2012], 3D images are being used, such as 3D meshes and range images. The majority of the 3D face recognition studies have focused on developing holistic statistical techniques based on the appearance of face range images or on techniques that employ 3D surface matching. In this chapter, the objective is to propose discriminant analysis method for 3D face recognition based on Radon Transformation, principal component analysis (PCA) and linear discriminant analysis (LDA), which are applied on 3D facial range images. The experimentation is done

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A part of this chapter is published in the


using three publicly available datasets, namely, Bhosphorus, Texas and CASIA 3D face databases, and the results are analyzed.

3.2 Proposed method

3.2.1 Linear discriminant analysis

Linear discriminant analysis (LDA) and the related Fisher’s linear discriminant are the methods used in statistics, pattern recognition and machine learning to find a linear combination of features which characterizes or separates two or more classes of objects or events. The resulting combination may be used as a linear classifier, or, more commonly, for dimensionality reduction before later classification.

The principal component analysis (PCA) is a standard technique used to approximate the original data with lower dimensional feature vectors. The basic approach is to compute the eigenvectors of the covariance matrix, and approximate the original data by a linear combination of the leading eigenvectors. The mean square error (MSE) in reconstruction is equal to the sum of the remaining eigenvalues. The coefficients of projection of an arbitrary data vector along the principal components(eigenvectors) form the feature vector. In PCA, since no class membership information is used, the data vectors of the same class and of different classes are treated similarly. In the linear discriminant analysis (LDA), the class membership information is used to emphasize the variation of data vectors belonging to different classes and to deemphasize the variations of data vectors within a class [Zhao et al., 1998]. The LDA produces an optimal linear discriminant function \( f(x) = W^T x \) which maps the input into the classification space in which the class identification of this sample is decided based on a metric, e.g., Euclidean distance. A typical LDA implementation is carried out via scatter matrix analysis [Martinez & Kak, 2001] [Lu et al., 2003]. The within and between-class scatter matrices \( S_w \) and \( S_b \), respectively, are defined as follows:

\[
S_w = \frac{1}{M} \sum_{i=1}^{M} \Pr(C_i) \sum_i \\
S_b = \frac{1}{M} \sum_{i=1}^{M} \Pr(C_i)(m_i - m)(m_i - m)^T
\]  

(3.1)
Chapter 3

Here $S_w$ is the within-class scatter matrix showing the average scatter $\sum_i$ of the sample vectors $x$ of different class $C_i$ around their respective mean $m_i$:

$$\sum_i = E[(x - m_i)(x - m_i)^T | C = C_i]$$

Similarly, $S_b$ is the between-class scatter matrix, representing the deviation of the conditional mean vectors $m_i$’s from the overall mean vector $m$. Various measures are available for quantifying the discriminatory power, the commonly used one being,

$$J(W) = \frac{\|W^T S_w W\|}{\|W^T S_b W\|}$$

(3.2)

where $W$ is the optimal discrimination projection and can be obtained via solving the generalized eigenvalues problem $S_b W = \lambda S_w W$. The distance measure used in the matching could be a simple Euclidian distance. Thus, the fundamental difference between the PCA and LDA approaches is that, while PCA performs eigenvalues analysis on covariance matrix, the LDA does it on scatter matrices.

3.2.2 Proposed methodology

The Radon transform is applied to an input facial range image $I_1$ in steps of $h$ from $0^\circ$ to $180^\circ$ orientations, where $h$ may be $1^\circ$, $2^\circ$, $3^\circ$ or any convenient value. It yields a binary image $I_2$ with facial area being segmented. After superposing $I_2$ and $I_1$, the cropped facial range image $I_3$ is obtained. Next, the principal component analysis (PCA) technique is applied to the complete set of such cropped facial range images corresponding to the face images in the face database. It yields the set of eigenfaces. Then linear discriminant analysis (LDA) is performed to these eigenfaces which are used for face recognition in a given test face range image. The Figures 3.1 and 3.2 shows the overview of proposed framework and intermediate results of the Radon transformation of an input face image, respectively. The algorithms of the training phase and the testing phase of the proposed method are given below:
Algorithm 1: Training Phase

Step 1 : Input the range image $I_1$ from the training set containing $M$ images (Figure 3.2 (a)).

Step 2 : Apply Radon transform, from $0^\circ$ to $180^\circ$ orientations (in steps of $h$), to the input range image $I_1$ yielding a binary image $I_2$ (Figure 3.2 (c)).

Step 3 : Superpose the binary image $I_2$ obtained in the Step 2 on the input range image $I_1$ to obtain the cropped facial range image $I_3$ (Figure 3.2 (d)).

Step 4 : Repeat the Steps 1 to 3 for all the $M$ facial range images in the training set.
Step 5: Apply PCA to the set of cropped facial range images obtained in the Step 4 and obtain $M$ eigenfaces.

Step 6: Compute the weights $w_1,w_2,...,w_p$ for each training face image, where $p < M$ is the dimension of eigen subspace on which the training face image is projected.

Step 7: Store the weights $w_1,w_2,...,w_p$ for each training image as its facial features in the PCA feature library of the face database.

Step 8: Perform LDA on the feature subspace (i.e., weight vectors).

Step 9: Store the LDA components (feature vectors) in the LDA feature library of the face database.

**Algorithm 2: Testing Phase**

Step 1: Input the test range image $Z_1$.

Step 2: Apply Radon transform, from $0^\circ$ to $180^\circ$ orientations (in steps of $h$), to the input range image $Z_1$ yielding a binary image $Z_2$.

Step 3: Superimpose the binary image $Z_2$ on $Z_1$ to obtain the cropped facial image $Z_3$.

Step 4: Compute the weights $w_i^{test}, i = 1,2,...,p$, for the test image $Z_1$ by projecting the test image on the LDA feature subspace of dimension $p$.

Step 5: Compute the Euclidean distance $D$ between the feature vector $w_i^{test}$ and the feature vectors stored in the LDA feature library.

Step 6: The face image in the face database, corresponding to the minimum distance $D$ computed in the Step 5, is the recognized face.

Step 7: Output the texture face image corresponding to the recognized facial range image of the Step 6.
3.3 Experimental results

For experimentation, the three publicly available databases, namely, Bhosphorus, Texas and CASIA 3D face databases, are considered. The proposed method is implemented using Intel Core 2 Quad processor @ 2.66 GHz machine and MATLAB 2012b. In the training phase, 2 frontal face images with neutral expression of each 100 subjects are selected from the Texas 3D face database as training data set. In the testing phase, randomly chosen 200 face images of the Texas 3D face database with variations in facial expressions are used. The sample training images which are used for the experimentation are shown in the Figure 3.3, and their corresponding texture images are shown in the Figure 3.4. The eigenfaces and mean facial range image computed for PCA during the training phase are shown in the Figure 3.5 and Figure 3.6, respectively.

Figure 3.3: Sample 3D face range images of the training set.

Figure 3.4: The facial texture images corresponding to the training range images of the Figure 3.3.
The comparison of recognition rates obtained by the proposed (RT+PCA+LDA) approach, PCA (alone) and RT+PCA approaches are presented in the Table 3.1. The projection orientation of Radon transform is in steps of $1^\circ$, $2^\circ$ and $5^\circ$. It is observed that the proposed method, namely, RT (with steps of $1^\circ$ orientation) with PCA and LDA, yields better results as compared to the PCA (alone) and RT+PCA method discussed in the Chapter 2.

The graph of recognition rate versus the number of eigenfaces is shown in the Figure 3.7 for the proposed method (RT+PCA+LDA) along with other PCA and RT+PCA methods. It is observed that the recognition rate improves as the number of eigenfaces is increased. It is 99.20% for 40 eigenfaces in case of proposed method using SVM classifier. Further, the proposed method based on RT, PCA and LDA outperforms the PCA and RT+PCA methods. It entails an additional recognition time of 1 sec approximately.

The rank-one recognition rates of the proposed method are compared to that of the state-of-the-art 3D face recognition methods, namely, 3D face recognition using RT+PCA, LDA and sparse representation in the Table 3.2. The proposed method is found to be superior to the other two methods. Also, the experiments
Table 3.1: The comparison of recognition rates obtained by the proposed (RT+PCA+LDA) approach, PCA (alone) and RT+PCA approach

<table>
<thead>
<tr>
<th>No. of Eigen Spaces</th>
<th>Recognition Accuracy (in %)</th>
<th>Average recognition time for SVM classifier (in Sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PCA (Chapter 2)</td>
<td>RT+PCA (Chapter 2)</td>
</tr>
<tr>
<td>5</td>
<td>60.05</td>
<td>61.00</td>
</tr>
<tr>
<td>10</td>
<td>69.05</td>
<td>78.00</td>
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<tr>
<td>15</td>
<td>73.15</td>
<td>85.00</td>
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<td>20</td>
<td>82.55</td>
<td>91.10</td>
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<tr>
<td>25</td>
<td>88.65</td>
<td>94.50</td>
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<td>30</td>
<td>89.16</td>
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<td>35</td>
<td>90.05</td>
<td>95.60</td>
</tr>
<tr>
<td>40</td>
<td>91.02</td>
<td>96.00</td>
</tr>
</tbody>
</table>

Figure 3.7: The recognition accuracy (%) versus the number of eigenfaces of the proposed method and the other methods.
are performed using the other two face databases, namely, Bosphorus and CASIA 3D face databases, and the recognition performance is compared in the form of ROC curves as shown in the Figure 3.8 which demonstrates the dependence of recognition performance on the face database used for implementation.

![Figure 3.8: Receiver operating characteristic (ROC) curve for the proposed method RT+PCA+LDA, using SVM classifier and three different face databases.](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>Recognition Accuracy</th>
<th>Dataset used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method (RT+PCA+LDA)</td>
<td>99.20%</td>
<td>Texas 3D face database</td>
</tr>
<tr>
<td>3D face recognition using RT+PCA (Chapter 2)</td>
<td>96.00%</td>
<td>Texas 3D face database</td>
</tr>
<tr>
<td>LDA (Gabor features around facial fiducial points) [Gupta et al., 2010c]</td>
<td>96.80%</td>
<td>Texas 3D face database</td>
</tr>
<tr>
<td>Sparse Representation, (Triangular area/normal, Geodesic distance) [Hengliand Tang &amp; Ge, 2011]</td>
<td>95.30%</td>
<td>BJUT-3D and FRGC v2</td>
</tr>
</tbody>
</table>
3.4 Conclusion

The linear discriminant analysis (LDA) allows objective evaluation of the significance of visual information in different parts (features) of the face for identifying the human subject. The LDA of face images also provides a small set of features that carry the most relevant discriminative information for classification purposes, which is obtained through eigenvector analysis of scatter matrices with the objective of maximizing between-class variations and minimizing within-class variations. The result is an efficient projection-based feature-extraction and classification scheme. Each projection creates a decision axis with a certain level of discrimination power or reliability. Soft decisions made based on each of the projections are combined, and probabilistic or evidential approaches to multisource data analysis are used to provide more reliable recognition results.

In this chapter, a novel hybrid method is proposed for 3D face recognition using Radon transform with PCA and LDA on face range images for feature extraction. In this method, the LDA based feature computation is done and hence recognition can be done at high speeds. The proposed method yields 99.20% recognition performance using SVM classifier. Further it has low complexity due to a small number of LDA features, which compares well with other state-of-the-art methods. The experimental results demonstrate the efficacy and the robustness of the method to illumination and pose variations. The recognition accuracy can be further improved by considering an enriched feature set which captures facial variabilities with a better representation, such as symbolic data analysis.