CHAPTER 4

FOURIER TRANSFORM AND WAVELET TRANSFORM

4.1 INTRODUCTION

Most of the signals in practice are time domain signals in their raw format. In an AUSC system, the collected signals are presented in the format of an A-Scan, which is a plot of signal voltage (amplitude) versus time for one transducer position. Even though some defect information can be obtained from the amplitude of the reflected signal, this time amplitude presentation is not always the best representation of the signal (Lynn et al 1994 and Rao1985). For many signals, the signal’s frequency content is important and most distinguished information is hidden in the frequency components. In order to find the frequency content of a signal, the Fourier Transform (FT) is used for transforming mathematically our view of the signal from time-based to frequency-based. A raw time domain signal is broken down into constituent sinusoids of different frequencies by FT. Consequently the frequency-amplitude representation obtained by FT presents a frequency component for each frequency that exists in the signal (DeFatta et al 1988 and Kuc 1988).

4.2 FOURIER TRANSFORM

For many signals, Fourier analysis is extremely useful because the signal's frequency content is of great importance. However, Fourier analysis has a serious drawback, namely, the time information is lost in transforming
to frequency information. In other words, when looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place.

If the signal properties do not change much over time (that is, if it is what is called a stationary signal) this drawback is not very important. However, most practical ultrasonic NDT signals contain reflections from discontinuities, which result in time varying spectral characteristics of the signal.

Consequently, the conventional Fourier analysis technique is not quite appropriate to analyze the behavior of these signals. In an effort to correct this deficiency, time-frequency representation of a signal is suggested to analyze time-localized signals with time-varying spectra, where conventional Fourier transform analysis methods are proven to be inadequate.

The Short-Time Fourier Transform (STFT) is one of the time-frequency representations, and involves a technique called windowing of the signal by which the Short-Time Fourier transform is adapted to analyze only a small section of the signal. The STFT provides some information about both when and at what frequencies a signal event occurs, but we can only obtain this information with limited precision which is determined by the size of the window. While it is useful for the STFT to compromise between time and frequency information, the drawback is that once a particular size for the time window is chosen, that window size is the same for all frequencies. That is, narrow window sizes give good time resolution but poor frequency resolution while wide window sizes give good frequency resolution but poor time resolution. Most signals require a more flexible approach where we can vary the window size to determine more accurately either time or frequency.
Figure 4.1 illustrates the common formats for displaying a signal and contrasts the WT view with the time-based view, the frequency-based view and the STFT view.

![Diagram showing four different views of a signal: Time Domain, Frequency Domain (Fourier), STFT, and Wavelet Analysis]

**Figure 4.1** Four different views of a signal depending on different domains: time-domain, frequency-domain, STFT and Wavelet analysis

In order to overcome the dilemma of the resolution problem of the STFT, the Wavelet Transform (WT) was developed and applied in the area of signal processing. WT adapts a windowing technique with variable-sized regions, in which the use of long time intervals is allowed where more precise low-frequency information is required and shorter regions are used where high-frequency information is required. Hence wavelet analysis can be adapted for processing the ultrasonic flaws signals.
More precisely, while Fourier analysis consists of breaking up a signal into sine waves of various frequencies, wavelet analysis decomposes a signal into shifted and scaled versions of the original (or mother) wavelet. Here a wavelet is a waveform of effectively limited duration that has an average value of zero, while sinusoids, which are the basis of Fourier analysis, do not have limited duration and extend from minus to plus infinity. Due to this theory, wavelets tend to be irregular and asymmetric while sinusoids are smooth and predictable.

4.3 WAVELETS

Wavelets can be defined as a special kind of function that exhibits oscillatory behavior for a short period of time and then die out with mean approximately zero. The difference between a wave and a wavelet can be easily identified by the Figure 4.2.

![Figure 4.2 Wave and Wavelet](image)

Generally, wavelets are purposefully crafted to have specific properties that make them useful for signal processing. More technically, a wavelet is a mathematical function used to divide a given function into different scale components. Usually one can assign a frequency range to each scale component. Each scale component can then be studied with a resolution that matches its scale.
Manipulation of wavelet is in two ways

- Translation
- Shifting

Translation is shifting the central portion of wavelet along the time axis; this is done to extract the time information of a signal which is shown in Figure 4.3. In scaling the amplitude and time duration of the wavelet functions are changed to obtain frequency information which is shown in Figure 4.4. Thus because of the translation and scaling, the wavelet is localized in both time and frequency domain simultaneously.

![Figure 4.3 Translation of wavelets](image)

**Figure 4.3 Translation of wavelets**

![Figure 4.4 Change in scale of wavelets](image)

**Figure 4.4 Change in scale of wavelets**

Figure 4.5 shows a schematic of the wavelet transform which basically quantifies the local matching of the wavelet with the signal. If the wavelet matches the shape of the signal well at a specific scale and location, then a large transform value is obtained. If they don’t correlate well, a low transform value is obtained. The transformed value is plotted in two dimension transform plane which is shown in Figure 4.5.
Figure 4.5 Schematic diagram of Wavelet transform

Based on the characteristics of the wavelet functions (communities), they are grouped in to different families. Each member of a family has certain common characteristics that distinguish each member of a family. The different families are

- Wavelets for continuous wavelet transform (Gaussian, Morlet, Mexican hat)
- Daubechies Maxflat wavelets
- Symlets
- Coiflets
- Bi-orthogonal wavelets
- Complex wavelets

4.4 WAVELET TRANSFORM

Ultrasonic signals contain numerous non-stationary or transitory characteristics. These characteristics are often the most important part of the signal and Fourier analysis is not suitable to detect them since it can be processed only in frequency domain. Wavelet Transform is developed
especially to overcome these deficiencies. The signal analysis using Wavelet transform is faster than the Fourier transform analysis.

The Wavelet Transform (WT) is the most recent technique for processing non stationary (transient) signals simultaneously in time and frequency domains (Graps 1995 and Burrus et al 1998). It is a windowing technique with variable-sized regions, which allows the use of long time intervals to obtain more precise low frequency information and shorter regions where high frequency information is needed (Zafer Iscan et al 2006). Besides, Wavelet Transform is capable of decomposing a signal into shifted and scaled versions of the original (or mother) wavelet. Its application seems to be attractive for ultrasonic data processing, especially for detection of defects in grainy materials. In NDT it is applied for enhancement of detection of defects (Bettayeb et al 2004).

The Wavelet Transform decomposes signal \( s(t) \) in a sum of elementary contributions called wavelets. The WT is the correlation between the signal and a set of basic wavelets. The daughter wavelets \( \psi_{a,b}(t) \) are generated from the mother wavelet \( \psi(t) \) by dilation and shift operations. The mother wavelet function is used to extract details and information in the time and the frequency domains from the transient signal under analysis.

The WT expansion coefficients \( X_{WT}(a,b) \) of the signal \( s(t) \) is given in equation (4.1)

\[
x_{WT}(a,b) = \int_{-\infty}^{\infty} s(t)\psi_{a,b}(t)dt
\]  

(4.1)

where

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right)
\]
A wavelet transform is the representation of a function by wavelets. The wavelets are scaled and translated copies (known as ‘daughter wavelets’) of a finite-length or fast-decaying oscillating waveform (known as the ‘mother wavelet’). The wavelet transform uses scaling in the time domain to scale a single function in frequency. This function, commonly referred to as the mother wavelet, is used to extract details and information on time and frequency domains from the transient signal under analysis (Kazys et al 2005).

This approach results in a more natural description of the signal, since the size of the window in the time domain is now a function of scaling.

One major advantage afforded by the wavelet transform is the ability to perform local analysis, that is, to analyze a localized area of a large signal. Based on this advantage, wavelet analysis is capable of revealing aspects of data like trends, breakdown points, discontinuities in higher derivatives, and self-similarity. Furthermore, wavelet analysis can often compress or denoise a signal without appreciable degradation.

Wavelet transform are classified into two types into discrete wavelet transforms (DWTs) and continuous wavelet transform (CWT). CWTs operate over every possible scale and translation whereas DWTs use a specific subset of scale and translation values or representation grid.

### 4.4.1 Continuous Wavelet Transform

Let $f(t)$ be any square integrable function. The Continuous Wavelet Transform (CWT) of $f(t)$ with respect to a wavelet $\psi(t)$ is defined in equation (4.2)
\[ W(a,b) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{|a|}} \psi^* \left( \frac{t-b}{a} \right) dt \]  \hspace{1cm} (4.2)

where ‘a’ and ‘b’ are real and * denotes complex conjugation. Thus, the wavelet transform is the function of two variables.

**Properties of CWT**

A wavelet \( \psi(t) \) is simply a function of time ‘t’ that obeys a basic rule, known as the wavelet admissibility condition which is given in equation (4.3).

\[ C_\psi \int_{-\infty}^{\infty} \left| \hat{\psi}(\omega) \right| d\omega < \infty \]  \hspace{1cm} (4.3)

where \( \hat{\psi}(\omega) \) is the Fourier transform. This condition ensures that \( \hat{\psi}(\omega) \) goes to zero quickly as \( \omega \to 0 \). In fact to guarantee that \( C_\psi \leq \infty \), we must impose \( \hat{\psi}(0) = 0 \) which equivalent is given in equation (4.4)

\[ \int_{-\infty}^{\infty} \psi(t) dt = 0 \]  \hspace{1cm} (4.4)

A secondary condition imposed on wavelet function is unit energy. That is,

\[ \int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1 \]  \hspace{1cm} (4.5)

**4.4.2 Discrete Wavelet Transform**

Discrete Wavelet Transform is viewed as a sampling function of the CWT. It is a process of decomposing a signal in to components that are
non-sinusoidal. The DWT analyzes the signal by decomposing it into its coarse approximation and its detailed information, which is accomplished by using successive high pass and low pass filtering operations in the frequency domain. The original signal \( x[n] \) is first passed through a half-band high pass filter \( g[n] \) and a low pass filter \( h[n] \), where \( g[n] \) and \( h[n] \) are quadrature mirror filters (QMF) of each other (Mallat 1989). Various filters have been developed to be used in DWT computation, and among them Biorthogonal wavelets (Bior 4.4) have found widespread use. After the filtering, half of the samples of the two output signals are discarded by down sampling, because the signals now have a bandwidth of \( \pi/2 \) radians instead of \( \pi \). This constitutes one level of decomposition and is expressed mathematically as given in equations (4.6) and (4.7).

\[
y_{\text{high}}(k) = \sum_n x(n).g(2k - n) \tag{4.6}
\]

\[
y_{\text{low}}(k) = \sum_n x(n).h(2k - n) \tag{4.7}
\]

where \( y_{\text{high}}(k) \) and \( y_{\text{low}}(k) \) are the outputs of the high pass and low pass filters, respectively, after down sampling by 2.

The above procedure is repeated for further decomposition of the low pass filtered signals. The high pass filtered signals constitute the DWT coefficients. Such a signal decomposition process of DWT is shown in Figure 4.6. The DWT coefficients at different levels can be achieved by this procedure. For example, for a signal consisting of 4096 samples, the first level contains 2048 samples, the second level contains 1024 samples, and the third level contains 512 samples, and the fourth level contains 256 samples and so on.
The DWT also provides a very effective signal compression and data reduction scheme. Because the energies of most signals are concentrated in a certain frequency band, all other frequencies are represented by very low amplitudes in the transform’s domain, and can be discarded with little or no loss of information. In this respect, DWT not only can be used as an efficient feature extraction scheme for the signal classification, but also provides significant data reduction, thereby reducing the computational burden considerably. Furthermore, this data reduction also may improve the classification accuracy because the excluded samples are most likely to correspond to noise.

**Figure 4.6 Signal decomposition process of DWT**
Especially, Discrete Wavelet Transform (DWT) has been widely used in the ultrasonic signal analysis as a fast algorithm to obtain the wavelet transform of a discrete time signal. This method comes from the idea of calculating wavelet coefficients at only a subset of scales and positions rather than at every possible scale. Moreover, DWT employs a much more efficient method for analyzing non-stationary signals.

4.5 SUMMARY

This chapter explains two signal processing techniques called Fourier Transform and wavelet transform. The two techniques are compared and the reasons for the superiority of the wavelet transform to Fourier Transform as a feature extraction scheme is also justified. Signal decomposition process of Discrete Wavelet Transform is also clearly illustrated.