CHAPTER 2

THE VEHICLE ROUTING PROBLEM AND ITS VARIANTS

2.1 INTRODUCTION

The Vehicle Routing Problem (VRP) is a generic name given to a whole class of problems in which a set of routes for a fleet of vehicles based at one or several depots must be determined for a number of geographically dispersed cities or customers. The objective of this chapter is to provide a brief discussion on the origins of VRP, its Variants, and applications.

2.2 THE ORIGINS OF THE BASIC VRP

2.2.1 The Traveling Salesman Problem (TSP)

One of the simplest, but still NP-hard, routing problems is probably the traveling salesman problem (TSP). In the TSP one is given a set of cities and a way of measuring the distance between each city. One has to find the shortest tour that visits all cities exactly once and returns to the starting node. The problem comes in different flavours depending on what properties the distances satisfy. If the distances satisfy that the distance from city i to city j is the same as the distance from city j to city i for all cities i and j, the problem is said to be symmetric. If this property does not hold, then the problem is said to be asymmetric.

The problem can be formulated as a mathematical model in the following way. Let $G = (V,A)$ be a complete, directed graph where
$V = \{1, \ldots, n\}$ is the set of nodes/cities and $A$ is the set of arcs. To each arc $(i, j) \in A$ is assigned a distance or cost $c_{ij}$. We define binary decision variable $x_{ij}$ that is set to one if and only if arc $(i, j)$ is used in the solution. The problem can be formulated as

$$\min z = \sum_{(i,j) \in A} c_{ij} x_{ij} \tag{2.1}$$

Subject to

$$\sum_{i=1}^{N} x_{ij} = 1 \quad \forall j \in \{2, \ldots, N\} \tag{2.2}$$

$$\sum_{j=1}^{N} x_{ij} = 1 \quad \forall i \in \{2, \ldots, N\} \tag{2.3}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1 \quad \forall S \subset V \tag{2.4}$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \{2, \ldots, N\} \tag{2.5}$$

The objective of the problem minimizes the total distance traveled in (2.1). Each node must be visited exactly once according to (2.2) and (2.3). Sub-tours are eliminated through the introduction of (2.4). The $|S|$ denotes the number of elements in the subset $S$.

The origins of the TSP are discussed in Schrijver (2005). An overview of the TSP algorithms (both exact and heuristic) can be found in Laporte (1992).

### 2.2.2 The Multiple Traveling Salesman Problems (M-TSP)

The multiple traveling salesman problem (M-TSP) is a generalization of the TSP that comes closer to accommodating real world
problems where there is a need to account for more than one salesman (vehicle). In M-TSP, M salesmen are to visit all nodes in the given network in such a way that the total distance traveled by all M salesmen is minimum.

The M-TSP is not studied widely in the literature, probably because it is so closely related to the TSP. The literature about heuristics and exact methods has recently been surveyed by Bektas (2006).

2.2.3 Capacitated Vehicle Routing Problem (VRP)

The basic capacitated Vehicle Routing Problem (CVRP) can be described in the following way. Given a set of homogeneous vehicles each of capacity Q, located at a central depot and a set of customers with known locations and demands to be satisfied by deliveries from the central depot, each vehicle route must start and end at the central depot and the total customer demand satisfied by deliveries on each route must not exceed the vehicle capacity, Q. The objective is to determine a set of routes for the vehicles that will minimize the total cost. The total cost is usually proportional to the total distance traveled if the number of vehicles is fixed and may also include an additional term proportional to the number of vehicles used if the number of routes may vary. A review of research work related to VRP can be found in Cordeau et al (2000, 2002)

2.3 VARIANTS OF VEHICLE ROUTING PROBLEM

In practice, the basic vehicle routing problem is extended with constraints, for instance, on the allowed capacity of the vehicle, length of route, arrival, departure and service time, time of collection and delivery of goods. The extended classes of VRP are VRP with Time Windows (VRPTW), Backhauls VRP (VRPB), Pickup and Delivery VRP (VRPPD),
and Simultaneous Pickup and Delivery VRP (VRPSPD). The main goal in all VRP problems is to obtain the minimal transportation cost. These variants of VRP are discussed in the following sections.

2.3.1 VRP with Time Windows

In a Vehicle Routing Problem with Time Windows (VRPTW) the capacity constraint still holds and each customer i is associated with a time interval \([a_i, b_i]\), called the time window, and with a time duration, \(s_i\), the service time. These constraints restrict the times at which a customer is available to receive a delivery. This problem is often common in real world applications, since the assumption of complete availability over time of the customers made in CVRP is often unrealistic. Time windows can be set to any width, from days to minutes, but their width is often empirically bound to the width of the planning horizon. The presence of time windows imposes a series of precedence on visits, which makes the problem asymmetric, even if the distance and time matrices were originally symmetric. VRPTW is also NP-hard and even to find a feasible solution to VRPTW is an NP-hard problem. A review of research work related to VRPTW can be found in Cordeau et al (2002), Braysy and Gendreau (2005a, 2005b) and Kallehauge (2008).

2.3.2 VRP with Pick-up and Delivery

The pickup and delivery problem (PDP) is a problem of finding a set of optimal routes, for a fleet of vehicles, in order to serve a set of transportation requests. Each vehicle from the fleet of vehicles has a given capacity, a start location, and an end location. Each transportation request is specified by a load to be transported, an origin, and a destination location. In other words, the pickup and delivery problem deals with the construction of
optimal routes in order to visit all pickup and delivery locations, and satisfy precedence and pairing constraints. Precedence constraints deal with the restriction, which each pickup location has to be visited prior to visiting the corresponding delivery location. Pairing constraints restrict the set of admissible routes, so that one vehicle has to do both the pickup and the delivery of the load of one transportation request. A review of various approaches to the solution of VRPPD is presented in Desaulniers et al (2000), Cordeau et al (2007) and Parragh et al (2008).

### 2.3.3 Time Dependent VRP

An interesting extension of the VRPTW in urban environments is the Time Dependent VRPTW, where the arc costs on the graph depend on time. This situation is quite common in most cities, since the time taken to travel from a location to another one depends on the traffic load, which varies with the time of the day. Some article dealing with this type of variant includes Malandraki and Daskin (1992), Ichoua et al (2003) and Donati et al (2008).

### 2.3.4 Dynamic VRP

Another extension to standard VRP, which is also common in many real-world applications, is when the service requests are not completely known before the start of service, but they arrive during the distribution process. This variant is called the Dynamic Vehicle Routing Problem (DVRP). Since new orders arrive dynamically, the routes have to be replanned at run time in order to include them. The articles dealing with the DVRP include those of Gendreau and Potvin (1998) and Montemanni et al (2005). More details on the DVRP can be found in Zeimpekis and Giaglis (2006).
2.3.5 Period Vehicle Routing Problem (PVRP)

The period vehicle routing problem (PVRP) is a variation of the classic vehicle routing problem in which one has to assign deliveries (or pickups) to customer locations in a manner that each location is visited at the required frequency during the planning horizon (usually one week), and the cost of the delivery (or pickup) routes is minimized. All deliveries (or pickups) are from a single depot. Some articles dealing with this variant of the VRP are available in the literature; for example, Cordeau et al (1997) and Mourgaya and Vanderbeck (2007).

2.3.6 VRP with Backhauls

In this variation, there are two subsets of customers: the first subset requires deliveries from the depot and the second subset requires goods to be picked up to be delivered to the depot. Normally all deliveries on each route must be completed before any pick-ups. The total deliveries and the total pick-ups on each route must separately be less than the capacity of each vehicle. Some articles dealing with this variant are Brandao (2006) and Ropke and Pisinger (2006).

2.3.7 Open VRP

The term “Open” is applied to problems where each vehicle is not required to return to the central depot after visiting the final customer, i.e. the vehicle routes are open paths instead of closed circuits. This formulation may be appropriate for a situation where vehicles are hired from a third party and the cost of the hire is based on the distance traveled while the vehicles are loaded. More details are available in Brandao (2004) and Fu et al (2005). An exact method for the Open VRSP is to be found in Letchford et al (2007).
2.3.8 Stochastic VRP

The customer demand in many cases is known before the routes need to be planned, because orders have been received at the central depot. However in some situations, the size of the demand from a customer may be unknown until the vehicle arrives at the customer’s place. This is an example of a stochastic vehicle routing problem where routes must be planned based on the probability distribution of the demand at any customer. In such problems, a strategy needs to be defined that explains what happens when a vehicle runs out of the commodity it is carrying before it has completed all its deliveries, for example, returning to the depot by the shortest route in order to reload. Some articles dealing with this variant are Secomandi (2001) and Chepuri and Mello (2005).

2.3.9 Other Variations

Many other variations of the basic CVRP have been studied that may include some of the features already described. These include multi-depot models where a set of depots replaces the single central depot. If the capacity of the available fleet of vehicles cannot service the required demand, then the objective may be focused on satisfying as many customers as possible using the available fleet. There may also be considerations due to different types of commodities that imply additional constraints. Some of the articles dealing with other variants of the VRP, are for example, Anily and Federgruen (1990) and Baldacci et al (2006).

2.3.10 Arc Routing Problems

These vehicle routing problems are significantly different from all the VRPs described so far. These have all been Node Routing Problems
because the customer demands are all located at points or at the nodes of an underlying graph or network. In Arc Routing Problems, the demands are associated with the links between the nodes, called edges (in the undirected case) or arcs (in the directed case). These represent roads or carriageways in the underlying road network. Such problems arise naturally in several applications such as garbage collection, mail delivery, snow clearing, school bus routing and police patrolling. In these cases, a service must be carried out which involves the vehicle traveling the length of road section performing the required service. This is often done at a different speed from that, when a vehicle is simply traveling along one of the roads without servicing it, which is often referred to as “deadheading”. These problems may also have similar variations to those described for Node Routing Problems such as the existence of time window constraints for the service. A comprehensive survey of Arc Routing Problems is provided in the book edited by Dror (2000).

2.4 APPLICATIONS OF VRP

The following are some examples of the multitude of VRP applications in manufacturing and service operations management:

- routing of automated guided vehicles, which are considered as one of the most appropriate modes for material handling in contemporary flexibly automated production environments
- minimisation of the distribution costs in a multi-facility production system
determination of vehicle routes for material delivery within the premises of a plant operating under a Just-In-Time philosophy
- rolling batch planning.
Diverse applications of VRP with different sets of variants were discussed by Ganesh et al (2007) (Table 2.1). A few real-life examples, which are variants of the classical VRP, are listed below:

- upper limit on the time of delivery, e.g. milk and newspaper distribution
- barred time windows, e.g. urban solid waste removal
- conflicting/competing time windows, e.g. mobile catering
- combined routing and scheduling, e.g. mobile hospital and mobile court
- independent multiple depots, e.g. mail carrier
- interdependent multiple depots, e.g. waste collection.

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<th>Sl. No.</th>
<th>Application</th>
<th>Variant</th>
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<tbody>
<tr>
<td>1</td>
<td>Food distribution</td>
<td>VRP with time windows (VRPTW) VRP</td>
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<tr>
<td>2</td>
<td>Waste collection problem</td>
<td>VRP with crew constraints VRP VRPTW</td>
</tr>
<tr>
<td>3</td>
<td>Milk distribution</td>
<td>VRPTW VRP with heterogeneous fixed fleet VRP</td>
</tr>
<tr>
<td>4</td>
<td>Container transport</td>
<td>VRP VRPTW Period VRP</td>
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Table 2.1 (Continued)

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<tr>
<td>5</td>
<td>City logistics</td>
<td>Dynamic VRP&lt;br&gt;Multi-depot VRPTW (MDVRPTW)&lt;br&gt;VRP</td>
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<td>6</td>
<td>Company employee distribution relations</td>
<td>VRP with load balancing</td>
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<td>7</td>
<td>Public transport</td>
<td>VRP</td>
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<td>8</td>
<td>Soft drink industry</td>
<td>VRP</td>
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<td>9</td>
<td>Brewing industry</td>
<td>VRP</td>
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<td>10</td>
<td>Land transportation of air cargo forwarder</td>
<td>VRP with Backhauls and time windows with heterogeneous fleet of vehicles</td>
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<td>11</td>
<td>School bus routing</td>
<td>VRP with time dead lines&lt;br&gt;VRP</td>
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<td>12</td>
<td>Furniture transport</td>
<td>VRP with heterogeneous fixed fleet</td>
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<td>13</td>
<td>Sugarcane transport</td>
<td>MDVRPTW</td>
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<td>14</td>
<td>Retail distribution</td>
<td>VRP</td>
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<td>15</td>
<td>Fresh meat distribution</td>
<td>Multi-depot VRP</td>
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<td>16</td>
<td>Mail carrier</td>
<td>VRP</td>
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<td>17</td>
<td>Ship routing</td>
<td>Multi-trip VRP</td>
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<td>18</td>
<td>Emergency planning</td>
<td>VRP with mixed delivery and pickup</td>
</tr>
<tr>
<td>19</td>
<td>Live stock distribution</td>
<td>VRP</td>
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2.5 SUMMARY

In this chapter, the origin and different variants of the routing problem were discussed. Also, the applications of the vehicle routing problem in literature were recapitulated.