CHAPTER 3

DEVELOPMENT OF MATHEMATICAL MODEL

3.1 INTRODUCTION

Turning is a metal cutting process in which the cutting tool enters and leaves the work piece. In CNC machine tools, the finished component is obtained by a number of rough passes and finish passes. In preparing for the finishing cut, the roughing operation is carried out to machine the part to a size that is slightly larger than the desired size. The finishing cut is called single-pass contour machining, and is machined along the profile contour. In this thesis, a turning centre with control system is used. Roughing stages and a finished stage are considered to machine the component from the bar stock. The first roughing stage consists of \((n-1)\) passes where \(n\) is the total number of roughing passes, and the last pass of roughing removes the material along the profile contour. In the finish stage material (amount of finish allowance) is removed along the contour of the profile.

Machining parameter optimization models are mathematical models formulated for realistic machining processes. These models have objective functions based on certain economic criteria and subject to various practical constrains from machining conditions and quality specifications. The formulation of process models requires the knowledge of mathematical equations to represent the relations of economical and physical parameters for the machining process and the knowledge on the whole machine-tool-workpiece system.

In this chapter, machining models, based on the minimum production cost, minimum production time and minimum tool wear criterion are proposed for multi-pass turning in single-tool applications. The total
depth of cut to be removed, in one finishing pass and $n$ roughing passes, is cut by the same tool. Multi-pass machining operations are governed by complex machining conditions. Some formulations of these conditions presented in this thesis can be found in the existing literature. Others are developed in this chapter.

3.2 OBJECTIVE FUNCTIONS

In the optimization of machining parameters, objective functions are mathematical formulations governed by certain production criteria. They are the basis on which machining parameters are optimized. The following three objective functions considered in this thesis are

3.2.1 Minimum production cost

Earlier developed machining models cannot be used for profile machining problems. The length of cutting path is calculated for each pass in the roughing and finishing stages. The objective of this model is to minimize the production cost which includes machining cost, machine idle cost, the tool replacement cost and the tool cost.

If material cost is not considered, unit production cost $U_C$ (Rs/piece) can be expressed by Chen and Su (1998).

$$U_C = C_M + C_I + C_R + C_T \quad (3.1)$$

where

$C_M$ _ cutting cost/ machining cost (Rs/piece)
$C_I$ _ machine idle cost (Rs /piece)
$C_R$ _ tool replacement cost (Rs /piece)
$C_T$ _ tool cost (Rs /piece)
The expression in Equation (3.1) has been widely accepted by many researchers in this field. Each cost term in Equation (3.1) is analyzed as follows.

### 3.2.1.1 Machining Cost per Unit Piece—CM

Machining cost CM is based on actual machining time $t_m$ (min) and labor cost per unit time, $k_o$ (Rs/min), including overhead. It is given as Shaw (1984).

$$CM = k_o \cdot t_m$$  \hspace{1cm} (3.2)

By dividing the cutting process into one finish pass and $n$ rough passes, machining time $t_m$ was expressed by Shin and Joo (1992) as

$$t_m = t_{ms} + t_{mr}$$  \hspace{1cm} (3.3)

where $t_{ms}$ (min) and $t_{mr}$ (min) are finishing time and total roughing time, respectively.

Hitomi (1979) represented finish cutting time $t_{ms}$ as

$$t_{ms} = \frac{\pi D_w L_t}{1000V_s f_s}$$  \hspace{1cm} (3.4)

where $D_w$ (mm) is the diameter of the workpiece in turning; $L_t$ (mm) is cutting travel length for each turning passes; $V_s$ (rpm) and $f_s$ (mm/rev in turning) are cutting speed and feed rate for finishing operations.

Rough machining time $t_{mr}$ can be obtained by summing the cutting time for all roughing passes required.

$$t_{mr} = \sum_{t=1}^{n} \frac{\pi D_w L_t}{1000V_s f_s}$$  \hspace{1cm} (3.5)
where \( n \) is the number of roughing passes: \( V_{ri} \) (rpm) and \( f_{ri} \) (mm/rev) in turning, cutting speed and feed rate for the \( i^{th} \) roughing pass.

Therefore, machining time \( t_m \) can be expressed by

\[
t_m = \frac{\pi D w L t}{1000 V_{ri} f_{ri}} - \sum_{i=1}^{n} \frac{\pi D w L t}{1000 V_{ri} f_{ri}}
\]  

(3.6)

\( L_t \) can be calculated by

\[
L_t = L + 3
\]  

(3.7)

where \( L \) (mm) is the length of the workpiece in turning and 3 (mm) is the recommended extra travel length of the cutter at the ends of each turning passes.

Therefore, machining cost \( C_M \) can be calculated by

\[
C_M = K_o \frac{\pi B w L t}{1000 V_{ri} f_{ri}} + \sum_{i=1}^{n} \frac{\pi B w L t}{1000 V_{ri} f_{ri}}
\]  

(3.8)

### 3.2.1.2 Machine Idle Cost per Unit Piece – CI

The machine idle cost CI is expressed by Shin and Joo (1992).

\[
CI = k_o \ t_i
\]  

(3.9)

Machine idling time \( t_i \) (min) can be further divided into a constant term \( t_p \) due to workpiece loading and unloading operations and a variable term \( t_i \) due to tool idle motion:

\[
t_i = t_p + t_i
\]  

(3.10)

where \( t_p \) (min/piece) is preparation time for loading and unloading a workpiece.
The idle tool motion time \( t_i \) (min) can be expressed by dividing tool motion into \( n \) rough passes and one finish pass as given below:

\[
t_i = n (h_1L_t + h_2) + (h_1L_t + h_2)
\]

where \( h_1 \) (min/mm) and \( h_2 \) (min) are constants related to tool travel and approach/depart time.

Therefore, machine idle cost \( CI \) can be expressed by

\[
CI = k_o [t_p + n (h_1L_t + h_2) + (h_1 + L_t + h_2)]
\]

3.2.1.3 Tool Replacement Cost per Unit Piece – CR

The tool replacement cost \( CR \) is expressed as given by Shaw (1984).

\[
CR = K_o t_e \frac{tm}{T}
\]

where \( t_e \) (min/edge) is tool change time: \( T \) (min) is tool life.

3.2.1.4 Tool Cost per Unit Piece – CT

The tool cost \( CT \) can be given by

\[
CT = K_t \frac{tm}{T}
\]

where \( k_t \) (Rs/edge) is tool cost.

By substituting Eqs. (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9), (3.10), (3.11), (3.12), (3.13) and (3.14) into equation (3.1), for the respective parameters, the objective function to minimize the total unit production cost can be written as
Minimize

\[ UC = CM + CI + CR + CT = UC_s + \sum_{i=2}^{n} UC_i + k_o t_p \]  \hspace{1cm} (3.15)

where

\[ UC_s = \left[ K_o + \frac{K_t}{T_s} + \frac{K_{ote}}{T_s} \right] \frac{\pi D_w L_t}{1000 \pi s_f s} + K_o (h_1 L_t + h_2) \]  \hspace{1cm} (3.16)

\[ UC_i = \left[ K_o + \frac{K_t}{T_{rl}} + \frac{K_{ote}}{T_{rl}} \right] \frac{\pi D_w L_t}{1000 \pi r_f r_{rl}} + K_o (h_1 L_t + h_2) \]  \hspace{1cm} (3.17)

### 3.2.2 Minimum operation time

The machining time criteria is used in this work for finding the performance of the machining operations under practical constraints. The length of cutting path is calculated for each pass in the roughing and finishing stages. Then the cutting time for each pass is calculated for these stages. The objective of this model is to minimize the cutting time. To measure the entire time required to finish the workpiece is represented as Stephenson et al. (1997).

\[ t_p = T_m + t_e (T_m / t_i) + [T_i] \]  \hspace{1cm} (3.18)

\[ = (L_j + 3/f_s n) + t_e ((L_j + 3/f_s n)/f_s d_s n) + [T_i] \]  \hspace{1cm} (3.19)

where,

- \( t_p \) = Unit time per workpiece (min)
- \( T_m \) = Cutting/(machining time) time
- \( t_e \) = Time required to exchange a tool (min)
- \( t_i \) = Tool life (min)
- \( T_i \) = Machine idle time
\[ L_j = \text{Length of the travel} \]
\[ f = \text{feed} \]
\[ n = \text{speed} \]
\[ d = \text{depth of cut} \]

**Cutting time:**

\[ T_m = (L_j + 3/f \cdot n) \quad (3.20) \]

where,

\[ L_j = \text{Length of the travel} \]
\[ f = \text{feed} \]
\[ n = \text{speed} \]

**Tool life (min)**

\[ t_l = f \cdot d \cdot n \quad (3.21) \]

\[ t_l = \text{Tool life (min)} \]
\[ f = \text{feed} \]
\[ n = \text{speed} \]
\[ d = \text{depth of cut} \]

### 3.2.3 Minimum tool wear

The tool will be giving unsatisfactory performance during machining process due to wear and the tool is subjected to three important factors such as forces, temperature and sliding action which loss in dimensional accuracy, increased surface roughness. This will result in loss of production and cost of replacing. To minimize the tool wear the empirical relation between tool wear and the machining variables is given by Bhattacharya (1970),

\[ V_b = C \cdot V^{K_1} f^{K_2} d^{K_3} \quad (3.22) \]
Where,

$K_1$, $K_2$ and $K_3$ are the parameters and $C$ is the constant obtained in modeling. The parameter $V$ is the cutting speed in meter per minute, $f$ is the feed rate in millimeters per revolution, and $d$ is the depth of cut in millimeters.

Thus, the model for tool wear is

$$T_W = 0.33349 \ V^{0.14804} \ f^{0.49116} \ d^{0.28979} \quad (3.23)$$

3.3 MACHINING CONSTRAINTS

In the optimization of machining parameters, physical limitations on cutting conditions due to the characteristics of the machine-tool-workpiece system should be identified from previous experience and taken into account as constraints in the optimization process. When formulating the models in this thesis, it was considered:

1. Parameter constraints
2. Cutting force constraints
3. Cutting power constraints
4. Dimensional accuracy constraint
5. Surface finish constraint
6. Roughing and finishing parameter relations

3.3.1 Parameter constraints

Let $V_{\min}$ and $V_{\max}$ (rpm) be minimum and maximum allowable cutting speeds respectively. $f_{\min}$ and $f_{\max}$ (mm/rev) be minimum and maximum allowable feed rates respectively. $d_{s_{\min}}$ and $d_{s_{\max}}$ (mm) minimum and maximum recommended depths of cut in the finishing pass, respectively. $d_{r_{\min}}$ and $d_{r_{\max}}$ (mm) minimum and maximum recommended depths of cut in the roughness passes respectively.
For the finishing pass,

\[
V_{\text{min}} \leq V_s \leq V_{\text{max}} \\
f_{\text{min}} \leq f_s \leq f_{\text{max}} \\
d_{s\text{min}} \leq d_s \leq d_{s\text{max}}
\]  
(3.24)

For the roughing passes, the lower and upper limits are,

\[
V_{\text{min}} \leq V_{ri} \leq V_{\text{max}} \\
f_{\text{min}} \leq f_{ri} \leq f_{\text{max}} \\
d_{r\text{min}} \leq d_{ri} \leq d_{r\text{max}}
\]  
(3.25)

### 3.3.2 Cutting force

Cutting force constraint is placed to limit the deflection of the workpiece, holding device and cutting tool to prevent chatter. It is expressed by Chen and Su (1998)

For the finish turning pass

\[
F_s = K_1 f_s^\mu d_s^v \leq F_{\text{max}}
\]  
(3.26)

where \(K_1\), \(\mu\), \(V\) are constant pertaining to a specific tool - workpiece combination, \(F_s\) and \(F_{\text{max}}\) are cutting force and maximum allowable cutting force (kgf), respectively.

Similarly for rough turning pass,

\[
F_r = K_1 f_r^\mu d_r^v \leq F_{\text{max}}
\]  
(3.27)

### 3.3.3 Power constraints

Cutting power consumption should not exceed available power of the machine tool. It can be derived by multiplying cutting force and cutting speed. Cutting power can be expressed by
\[ P = \frac{KLfNv}{6120q} \leq P_{\text{max}} \]  
(3.28)

where \( \eta \) is the efficiency of the machine tool; \( P \) and \( P_{\text{max}} \) are cutting power (kW) and the maximum power of the machine tool (kW), respectively.

### 3.3.4 Roughing and finishing parameter relations

The total depth of cut, \( d_1 \), should be equal to the depth of finish cut, \( d_s \), adding the total depth of rough cuts, \( \sum_{i=1}^{n} d_i \). It is expressed by,

\[ d_1 = d_s + \sum_{i=1}^{n} d_i \]  
(3.29)

### 3.3.5 Dimensional accuracy constraint

The regression relation for calculating the dimensional accuracy is given by Jang (1992),

\[ \delta = 100.66 f^{0.9709} d^{0.4905} V^{-0.2848} \]  
(3.30)

where \( \delta \) is the dimensional accuracy, \( f \) is the feed rate per revolution, \( d \) is the depth of cut, and \( V \) is the cutting speed.

### 3.3.6 Surface finish constraint

The maximum allowable surface roughness is calculated as given below. Surface roughness is influenced by the feed and the nose radius of the tool is represented as Shin and Joo (1992).

\[ \frac{f_s^2}{r} \leq R_{\text{max}} \]  
(3.31)

where \( r \) is the nose radius of cutting tool (mm), \( R_{\text{max}} \) is maximum allowable surface roughness (\( \mu \)) and \( f_s \) is the feed.