CHAPTER 3

THEORETICAL BACKGROUND

3.1 FATIGUE

The word fatigue originated from the Latin expression *fatigare*. Although it is commonly associated with physical and mental weariness in a person, the word fatigue has become a widely accepted terminology in engineering vocabulary for damage and failure of materials under cyclic loads. Fatigue is defined as ‘the changes in properties which can occur in a metallic material due to the repeated application of stresses or strains, although usually this term applies specially to those changes which can lead to cracking or failure’ as quoted from General Principles for Fatigue Testing of Metals, published by International Organization for Standardization in Geneva in 1964. This description is also generally valid for fatigue of non metallic materials.

Fatigue failures occur in many different forms. Mere fluctuations in externally applied stresses or strains result in mechanical fatigue. Cyclic loads acting in association with high temperature cause creep-fatigue, when the temperature of cyclically loaded component also fluctuates; thermomechanical fatigue is induced. Recurring loads imposed in the presence of a chemically aggressive or embrittling environment gives rise to corrosion fatigue. The other types include sliding contact fatigue, rolling contact fatigue and fretting fatigue. The majority of the failures of the machinery and structural components can be attributed to one of the above fatigue processes. Such failures generally take place under the influence of cyclic loads whose peak values are considerably smaller than safe loads.
Fatigue is a branch of study which encompasses many scientific disciplines and which offers a rich variety of phenomena for fundamental and industrial research. From published reports, research on fatigue of materials can be traced back to the first half of the nineteenth century. Since that time, scores of scientists and engineers have made pioneering contributions to understanding of fatigue in a wide variety of metallic and non metallic, brittle, monolithic and composite materials.

Interest in study of fatigue began to expand with the increasing use of ferrous structures, particularly bridges in railway systems. The first detailed research effort into metal fatigue was initiated in 1842 following a railway accident near Versailles in France which resulted in the loss of lives (The Times of London, May 11, 1842, comprehensive description of accidents, Smith, 1990). The cause of the accident was traced to fatigue failure originating in the locomotive front axle. As early as 1843, Rankine, a British railway engineer who later became famous for his contributions to mechanical engineering, recognized the distinctive characteristics of fatigue fractures and noted the dangers of stress concentrations in machine components. In 1849, the British Government commissioned Hodgkinson to study the fatigue of wrought and cast iron used in railway bridges. The report of this commission described alternating bending experiments on beams whose mid points were repeatedly deflected by a rotating cam. During this period, research on fatigue fracture was also documented in the work of Braithwaite (1854) who employed the term fatigue exclusively to denote the cracking of metals under repeated loading.

Wohler conducted a systematic investigation on fatigue failure during the period 1852-1869 in Berlin, where he established an experiment station. His work also led to the characterization of fatigue behavior in terms of stress amplitude-life (S-N) curves and to the concept of fatigue “endurance limit”. In 1874, the German engineer Gerber began developing methods for
fatigue design: his contribution included the development of methods for fatigue life calculations for different mean levels of cyclic stress. Interpretations of fatigue mechanism based on the old crystallization theory were laid to rest by the pioneering work of Ewing and Rosehain (1900) and Ewing and Humphrey (1903). These researchers investigated the fatigue of Swedish iron and published optical micrographs of cyclic damage on the specimen surface. It was convincingly shown that slip bands developed in many grains of the polycrystalline material. These slip bands broadened with progression of fatigue and led to the formation of cracks; catastrophic failure of the specimen was instigated by the growth of a single dominant flaw.

In 1910, Basquin proposed empirical laws to characterize the $S-N$ curves of metals. He showed that log-log plot of stress versus the number of fatigue cycles resulted in a linear relationship over a large range of stress. Several others contributed to the understanding of fatigue of materials.

There are different stages of fatigue damage in an engineering component where defects may nucleate in an initially undamaged section and propagate in a stable manner until catastrophic fracture ensues. For this most general situation, the progression of fatigue can be broadly classified into the following stages:

1. Substructure and microstructural changes which cause nucleation of permanent damage.
2. The creation of microscopic cracks.
3. The growth and coalescence of microscopic flaws to form dominant cracks which may eventually lead to catastrophic failure.
4. Stable propagation of the dominant microcrack.
5. Structural instability or complete fracture.
Classical approaches to fatigue design involve the characterization of total fatigue life to failure in terms of cyclic stress range (S-N curve) or (plastic or total) strain range. In these methods the number of stress or strain cycles necessary to induce fatigue failure in initially uncracked laboratory specimens is estimated under controlled amplitudes of cycles or strains. The resulting fatigue life incorporates the number of cycles to initiate dominant crack and to propagate until catastrophic failure occurs.

Since the crack initiation life constitutes a major component of the total fatigue life in smooth specimens (Figure 3.1) under high cycle, low stress fatigue situations has been traditionally characterized in terms of stress range. However the stresses associated with low-cycle fatigue are generally high enough to cause appreciable plastic deformation prior to failure.

Figure 3.1 A typical S-N curve
The other approaches to estimate the fatigue life are safe-life, fail-safe and the damage tolerant approach. The stress-based and the strain-based approaches have found widespread applications, particularly in the design of many fatigue-critical components for automobiles and other surface vehicles. The terminology of fatigue is illustrated in the Figures (3.2 to 3.4).

Figure 3.2 Cyclic stresses for fluctuating load

Figure 3.3 Pulsating loads

Figure 3.4 Alternating loads
Figure 3.5 illustrates stress life plot (S-N) under constant amplitude loading conditions. Metals exhibit a plateau in the stress life plot typically beyond about $10^6$ fatigue cycles. Below this plateau level the specimen may be cycled indefinitely without causing failure. This stress amplitude is known as the fatigue limit or endurance limit $\sigma_e$. This endurance limit is 35-50 % of tensile strength.

The empirical descriptions of fatigue life pertain to fully reversed fatigue loads where the mean stress of the fatigue cycle $\sigma_m$ is zero. However, fully reversed stress cycles with zero mean stress are not always representative of many applications. The mean level of the imposed fatigue cycle is known to play an important role in influencing the fatigue behavior.

The fatigue loading of sinusoidal waveform is schematically illustrated in Figures 3.2 to 3.4. Defining

\[
\text{Stress range } \Delta \sigma = S_{\text{max}} - S_{\text{min}} \tag{3.1}
\]

\[
\text{Stress amplitude } S_a = \frac{(S_{\text{max}} - S_{\text{min}})}{2} \tag{3.2}
\]

\[
\text{Mean stress } S_m = \frac{(S_{\text{max}} + S_{\text{min}})}{2} \tag{3.3}
\]
The mean stress is also characterized in terms of the load ratio, $R = \frac{S_{\text{min}}}{S_{\text{max}}}$ defining $R = -1$ for fully reversed loading, $R = 0.1$ for tension and $R = 1$ for static loading.

When the mean stress amplitude from a uniaxial fatigue test is plotted as a function of number of cycles to failure, the resultant $S$-$N$ curves are generally a strong function of the applied mean stress level. Mean stress effects in fatigue can be represented in terms of constant fatigue life diagrams. Here different combinations of the stress amplitude and mean stress providing a constant fatigue life are plotted. Most well known among these models are due to Gerber (1874); Goodman (1899); and Soderberg (1939). The life plots represented by Soderberg provide a conservative estimate of fatigue life for most of the engineering alloys. The Goodman plot matches the experimental data quite closely for brittle materials. For compressive mean stress however it is non conservative. Gerber’s relation is good for ductile materials for tensile mean stresses (Figure 3.6).

![Figure 3.6 Effect of mean stress](image)
Understanding the fatigue behavior of composites (Chawla, 1988) is of vital importance and many high volume applications of composite materials involve cyclic loading situations. It would be fair to say that this understanding of the fatigue behavior of composites has lagged that of other aspects such as strength or stiffness. The major difficulty in this regard is the applications of conventional stress based approaches. The main reason for this difficulty is the inherent non-homogeneity and anisotropic nature of composites. This results in damage mechanisms in composites being very different from those encountered in conventional, homogenous or monolithic materials. The fracture behavior of composites is characterized by a multiplicity of damage modes, such as matrix crazing, matrix cracking, fiber fracture, delamination, debonding, void growth and multidirectional cracking. These failure modes appear early in the fatigue life of composites. Progressive loss of stiffness during fatigue loading is very different from that of the conventional metals.

A unique feature of a fiber reinforced composite material is that it exhibits a gradual softening or loss of stiffness due to appearance of microscopic damages long before any visible damage occurs. As a result the stress decreases and there is a loss in residual strength of the material. Instead of specimen failure, many fatigue tests for composites are performed until the specimen stiffness or residual strength decreases to a predetermined level. Thus, cycles to failure may not always represent the specimen life at complete fracture.

Many investigators have attempted to describe $S$-$N$ plot for various fiber reinforced composites by a straight line

$$S = \sigma_u (m \log N + b)$$

(3.4)

Where $\sigma_u$ = maximum fatigue stress
There are a number of sources of uncertainty in the analysis of fatigue results in general and in the use of stress-life approach. These arise from:

1. Uncertainties and/or errors in the estimation of material properties which include microstructural variability from one specimen or batch to another as well as experimental errors in the measurement of properties in the same batch of specimens.

2. Uncertainties in modeling of applied stresses, for a given service condition and environment. This variability stems from two sources: (i) variability in stress amplitudes during a known service cycle as a consequence of such factors as vibrations and (ii) lack of knowledge about the exact distribution of stress cycles which occur over the design.

3. Uncertainties in a priori estimation of environment and in the ensuing variation in loading intensity.

4. Uncertainties in modeling, predictions and life estimates for fatigue processes

### 3.2 RELIABILITY STUDY

Such uncertainties are analyzed using known statistical approaches to derive the level of reliability or probability of failure. A parameter which is widely used to describe the uncertainty is the coefficient of variation. A
distribution in the value of the random variable is usually characterized in terms of normal distribution, log-normal distribution or Weibull distributions. The fatigue strength listed from experiments representing the arithmetic mean derived from multiple experiments. In brittle solids such as polymers with considerable microstructural variability arising from processing, the extent of scatter in the fatigue data may be large as a result of large scatter in microscopic flaw size distribution. Consequently, different sets of experiments conducted on the same material may not give the same arithmetic mean of the critical strength parameter. To address this issue, Weibull proposed the concept of a probability of failure \( P \), at a given failure strength, \( \sigma_f \) normalized by an average value of critical stress \( \sigma_{cr,ave} \). At low values of \( \sigma_f \), \( P \) will tend to zero while at high values of \( \sigma_f \), \( P \) will tend to one. Weibull defined the failure probability as

\[
P = 1 - P_s = 1 - \exp \{ - (\sigma_f / \sigma_{f,0})^{m_w} \} \tag{3.5}
\]

Where \( m_w \) is known as the weibull modulus and \( \sigma_{f,0} \) is a reference strength, \( P \) represent the fraction of total number \( N_s \) of the identical test specimens in a batch for which the failure strength falls below \( \sigma_f \). Similarly \( P_s \), the probability of survival, represents the fraction of the total number \( N_s \) of identical test specimens in a batch for which the failure strength exceeds \( \sigma_f \). When \( \sigma_f / \sigma_{f,0} = 1 \), \( P = 1 - \exp (-1) = 0.63 \). In other words, the reference stress \( \sigma_{f,0} \) represents the stress batch at a stress level \( \sigma_f \) or lower is 63.3%.

Rearranging the above equation to give

\[
1/(1-P) = \exp \{ (\sigma_f / \sigma_{f,0})^{m_w} \} \tag{3.6}
\]

Taking logarithms on both sides

\[
\ln (1/(1-P)) = (\sigma_f / \sigma_{f,0}) \tag{3.7}
\]
Taking logarithm once again, to give

\[ \ln \{ \ln \left( \frac{1}{1-P} \right) \} = m_w \ln \left( \sigma_f \sigma_{f,o} \right) \]  \hspace{1cm} (3.8)

It is apparent that a plot of the double logarithm of \( \frac{1}{1-P} \) (ordinate) against the logarithm of \( \sigma_f \) (abscissa), based on experiment conducted on a number of identical specimens yields a straight line whose slope is \( m_w \) and whose intercept with ordinate is \( -m_w \ln \sigma_{f,o} \). Such a plot is referred to as Weibull diagram. The overall objective of incorporating probabilistic analysis into fatigue design is to ensure that a low probability exists for a combination of higher average cyclic stress amplitude and a lower average fatigue endurance limit stress to cause failure.

Five most important functions in reliability and life testing are the following: (1) failure probability density function, (2) reliability function, (3) conditional reliability function, (4) failure rate function and (5) mean life function. The failure probability density function enables the determination of the number of failures occurring over a period of time referred to the original, total population. The reliability function enables the determination of the probability of success of any unit in undertaking a mission of a prescribed duration or operating for the desired function period. The conditional reliability function enables the determination of the probability of success of any unit in undertaking a new mission given the unit already survived the previous mission. The failure rate function enables the determination of the number of failures occurring per unit time referred to the size of population existing at the beginning of the period for which the failure rate is to be calculated. The mean life provides the average time of operation to a failure.

The work of a reliability engineer starts with observations of time to failure, cycles to failure etc, while they operate in specified applications and operating environments according their specifications. Such observations
are called data and are subject to central tendencies and deviations from their mean value caused by variations in raw materials, manufacturing tools and processes, workmanship, quality control and outgoing quality levels, experimental and human errors that prevail for the units whose reliability wise performance is being monitored. The best scientific way of obtaining data is to reduce the data into a meaningful condensed form, using statistical distributions, the probability density function, frequency distribution and by estimating the confidence levels.

The probability density for Weibull distribution is given by

\[ f(T) = \beta \eta (T - \gamma)^{\beta - 1} \exp(-(T - \gamma)^\beta) \]  \hspace{1cm} (3.9)

where \( T \) can be anything like age, number of cycles to failure etc, \( \beta \) = shape parameter \( \eta \) = scale parameter \( \gamma \) = location parameter

![Figure 3.7 Probability density function](image)
Figure 3.7 shows how the shape of the Weibull probability density function changes as $\beta$ changes and the change in the parameter $\eta$ has the effect on the distribution as a change on the abscissa scale. If $\eta$ is increased while $\beta$ and $\gamma$ are kept the same, the distribution gets stretched out to the right. The location parameter $\gamma$ locates the distribution along the abscissa. These parameters can be estimated by the following methods representing the data: (1) least squares, (2) matching moments, and (3) maximum likelihood estimators. A three parameter Weibull distribution will represent the fatigue data, while a two parameter fit is used for calculating the parameters using both least squares method and WEIBULL ++ 7, which is a powerful tool to estimate the reliability of the fatigue data.

3.3 LINEAR ELASTIC FRACTURE MECHANICS

Conventional linear elastic fracture mechanics deals with homogenous, isotropic materials and has been highly successful because so many useful materials are reasonable approximations to these assumptions.

The use of a stress analysis in modern design procedures ensures that in normal service very few engineering components fail because they are overloaded. However, weakening of the component by such mechanisms as corrosion or fatigue-cracking may produce a catastrophic fracture and therefore in some instances, the fracture properties of the component are the most important consideration. The study of how materials fracture is known as Fracture Mechanics and the resistance of a material to fracture is colloquially known as “toughness”.

No structure is entirely free of defects and even on a microscopic scale these defects act as stress-raisers which initiate the growth of cracks. The theory of Fracture Mechanics therefore assumes the pre-existence of cracks and develops criteria for the catastrophic growth of these cracks. In a
stressed body, a crack can propagate in a combination of the three opening modes shown in Figure 3.8. Mode-I represents opening in a purely tensile field while Mode-II and Mode-III are in-plane and anti-plane shear modes respectively. The most commonly found failures are due to cracks propagating predominantly in Mode-I, and for this reason materials are generally characterized by their resistance to fracture in that mode.

![Fracture modes](image)

(a) Mode-I          (b) Mode-II       (c) Mode-III

**Figure 3.8 Fracture modes**

Fracture can also be phenomenologically classified according to macroscopic deformation before fracture into three categories brittle, semi-brittle (or semi-ductile) and ductile fracture. Fracture without any macroscopic plastic deformation or fracture in the elastic state prior to yielding is called brittle fracture. Semi-brittle fracture accompanies local plastic deformation around stress concentrators such as notches or inclusions. Ductile fracture occurs after uniform plastic deformation.

Another description of these characteristics is the dependence of fracture resistance on the size of a notch or crack, as shown in Figure 3.9. The brittle material C exhibits rather high fracture strength without a notch or crack, but the reduction of strength is considerable with an increasing notch depth. The material B exhibits a constant strength no matter what the notch depth. This is ductile fracture independent of the presence of a notch or crack. The strength of the material A remains nearly constant up to a certain critical
notch depth and then decreases with an increasing notch depth. This fracture mode, which is in between brittle and ductile fracture, is called semi-brittle or semi-ductile.

![Figure 3.9 Material classifications according to the influence of the crack length on fracture toughness](image)

Griffith (1921) considered that fracture produces a new surface area, and that for fracture to occur, the increase in energy required to create the new surface must be balanced by a decrease in elastically stored energy in the sample. To explain the large discrepancy between the measured strength of materials and that based on theoretical considerations, he postulated that the elastically stored energy is not distributed uniformly throughout the specimen or sample, but is concentrated in the neighborhood of small cracks. Fracture occurs due to these cracks which originate from pre-existing flaws.

The growth of any crack is usually associated with an amount of work, $dW$, being done on the system by external forces and a change, $dU$, in the elastically stored energy, $U$. The difference between these quantities, $dW-dU$, is the energy available for the formation of a new surface. A crack (Figure 3.10) of length, $a$, grows when:
where $dU/da$ is the surface free energy per unit area and $dA$ is the associated increment of surface area. If there is no change in the overall extension when the crack propagates, $dW=0$ and:

$$-(dU/da) \geq \sigma_F dA/da$$

(3.11)

Equation (3.12) allows the fracture stress $\sigma_F$, of a material to be defined in terms of crack length by the relationship:

$$\sigma_F = (2E/\pi a)^{0.5}$$

(3.12)

where $E$ is young’s modulus under plane stress conditions. For an infinite sheet with a central crack of length, $a$, subjected to a uniform stress $\sigma$ Irwin showed that

$$K = \sigma(\pi a)^{0.5}$$

(3.13)

Irwin postulated that when $\sigma$ reaches the fracture stress, $\sigma_F$, $K$ takes a critical value

$$K_c = \sigma_F (\pi a)^{0.5}$$

(3.14)
where the fracture toughness of the material can be defined by the value of $K_c$ called stress intensity factor which defines the stress field at fracture zone. Equation (3.14) which can be written as

$$\sigma_F = (K_c^2 / \pi a)^{0.5} \quad (3.15)$$

Being identical in form to Equation (3.12), this is Griffith’s formulation. The strain energy release rate, $G$ is the energy available for unit increase in crack length (Figure 3.10).

$$G = dW/dA - dU/dA \quad (3.16)$$

Generally, in plane stress conditions, the plastic zone crack tip is produced by shear deformation through the thickness of the specimen. Such deformation is enhanced if the thickness of the specimen is reduced. However, if the specimen thickness is increased then the additional constraint on through thickness yielding produces a triaxial stress distribution so that approximate plane strain deformation occurs with shear. There is usually a transition from plane stress to plane strain conditions as thickness is increased (Figure 3.11).
As $K_{ic}$ values are generally quoted for plain strain, it is important that this condition prevails during fracture toughness testing. Even on the thickest specimen, a region of plane stress yield is always present on the side surfaces because no triaxial stress can exist there.

Since polymers are viscoelastic materials, strict Linear Elastic Fracture Mechanics (LEFM) theory cannot be applied and the total work of fracture must include the various forms of plastic deformation, which appear prior to and during failure. In particular, three points should be considered:

a) Because of the relatively low yield stress values of many plastics, plastic deformation at the crack tip is far more likely to occur.

b) While a small degree of dissipative energy can be accommodated in the overall work of fracture, it is obvious that as this assumes greater significance, there is much greater possibility that a fracture mechanics approach will lose its general validity.

c) Plastic properties such as fracture toughness and yield stress are dependent on many variables related to fracture testing, and for a given material, the test conditions necessary to ensure validity are therefore quite restricted. The bottom line is that the major problem in applying LEFM theory to polymers is to assess the extent to which the plastic deformation zone at the crack tip influences the resulting fracture behavior.
3.4 J-INTEGRAL

J-Integral was originally defined by Rice as a contour integral independent of the path, which expresses the energy per unit area necessary to create new fracture surfaces in a loaded body containing a crack. From load-displacement curves of two bodies with initial crack lengths $a$ and $(a+da)$, as indicated in Figure 3.12, if the crack propagation takes place in point S and S’ for the first and second body respectively, the area between the two curves (shadowed zone) is the energy necessary to produce a crack surface. This can be expressed as

$$J = -1/B \left( dU/da \right)$$  \hspace{1cm} (3.17)

The resulting fracture criterion is $J \geq J_c$, $J_c$ being the critical value independent on both the crack length and sample geometry, the Equation (3.17) can be expressed as

$$J = J_e + J_p = \eta_e \frac{U_e}{B(W-a)} + \eta_p \frac{U_p}{B(W-a)}$$  \hspace{1cm} (3.18)

Where the integrals $J_e$ and $J_p$ are elastic and plastic contribution of the whole $J$ value. $U_e$ and $U_p$ are also the elastic and plastic components of the total energy $U$. Moreover $\eta_e$ and $\eta_p$ are elastic and plastic work factors respectively. It is assumed that the load displacement curves are independent of the path, which is not strictly correct.
Figure 3.12 Elastic plastic behavior: (a) Decrease of potential energy due to crack growth, (b) Separation of elastic and plastic contribution

In tough materials the crack tip gets blunted prior to the crack propagation process. Traditionally a blunting line can be defined to estimate crack blunting before the crack propagation.

\[ J = 2\sigma_s \Delta a \]  \hspace{1cm} (3.19)

Rice (1973) proposed that a single specimen with a notch can be used to experimentally determine the \( J \)-Integral. Therefore a graphical interpretation of the load versus displacement curve will yield the \( J \)-Integral.

\[ J_{IC} = 2 \frac{U_{tcr}}{Bb} \]  \hspace{1cm} (3.20)

where

- \( U_{tcr} \) - total critical strain energy
- \( B \) - Specimen thickness
- \( b \) - \( W-a \)
- \( W \) - width of the specimen
- \( a \) - notch length
3.5 INTERLAMINAR FRACTURE TOUGHNESS $G_{IC}$

Extensive range (Klaus Friedrich, 1989) of tests is designed to quantify toughness of fiber reinforced composites. These tests are generally identified by the type of loading that is Mode-I, Mode-II, Mode-III and Mixed-Mode and by the orientation of the composite with respect to the crack propagation plane. The Mode-I interlaminar fracture testing has received more attention, due to the inherent weakness of the interply layer to the delamination under opening mode (cleavage loading). One test specimen dominates this class of test, the double cantilever beam (DCB) specimen. Cleavage tests have proved useful in the investigation of a very wide range of materials. In mid 1960s several authors were using fracture mechanics approach to examine the fracture toughness of adhesive joints. Specimens consisting of a bond line between two metal arms proved well-suited to measuring $G_{IC}$.

The principal requirement for a DCB specimen is a means of load application and a way of ensuring that the delamination propagates along the mid plane. In most cases, these requirements are satisfied by bonding of hinges (Figure 3.13) to the specimen and the introduction of a starter crack at the mid-thickness of the laminate panel before molding. This specimen would have a width of 20-25 mm and the length of the specimen of about 150 mm, with thickness of 2 to 3 mm.

![Figure 3.13 A DCB specimen with aluminum hinges](image)
The methods applied to interpret the data recorded during Mode-I may be grouped into two categories, compliance methods and direct energy methods. The compliance methods are all based on the Irwin- Kies equation

\[ G_{ic} = P_c^2 l/2B \frac{dC}{da} \]  \hspace{1cm} (3.21)

where \( P_c \) is the critical load, \( B \) the width, \( C \) compliance and \( a \) the crack length.

\[ C = \frac{\delta}{P} \]  \hspace{1cm} (3.22)

Plotting the curves between \( C \) versus \( a \) in a log- log form and fitting the data for crack length with a slope equal to 3 by linear regression, the slope of this curve is \( A_1 \). Then the critical load \( P_c \) and \( a \) is plotted for different crack length in a log-log form. This line is a fit with a slope equal to -1 and the slope of this curve is \( A_2 \). Determining the strain energy release rate as:

\[ G_{IC} = \frac{3(A_1 A_2^2)}{2w} \]  \hspace{1cm} (3.23)

Data analysis for finding \( G_{IC} \) requires fitting a straight line of a known slope to experimentally recorded data. The equation for the straight line is

\[ y = m x + c \]  \hspace{1cm} (3.24)

where \( m \) is slope which is known and \( c \) is to be determined. The line is best fitted so as to minimize the sum of square of the distance from data points to the line. Consider a point \((x_i, y_i)\) and if the distance of the point from the line is \( d_i \), the sum \( D \) of the square of the distance for \( n \) points is

\[ D = \sum d_i^2 = \sum (m x_i - y_i + c)^2 / m^2 + 1 \]  \hspace{1cm} (3.25)
For minimum $D$, leading to

$$c = (\sum m x_i + \sum y_i)/n$$ \hspace{2cm} (3.26)

Where $n$ is the number of sample points.

Assigning $m = 3$ and $c = 1$ $c$ can be estimated leading to the two constants $A_1$ and $A_2$ and finally to $G_{IC}$.

### 3.5.1 Impact Study

The impact property of a material represents the capacity to absorb and dissipate energy under impact shock loading. In practice the impact condition may range from accidental dropping of the tools to high collision and the response of the structure may range from localized damage to total disintegration.

### 3.6 INTERLAMINAR FRACTURE TOUGHNESS $G_{IC}$

Based upon the application of interlaminar fracture mechanics concepts and the emergence of new material systems exhibiting superior Mode-I fracture toughness, emphasis has now shifted to understanding the interlaminar behavior in Mode-II. The end notched flexure (ENF) specimen (Figure 3.14) has emerged as the most frequently used test method to measure the Mode-II critical strain energy release rate. The mechanics of the ENF geometry is quite complex because of the presence of a crack in the finite domain. An elasticity solution incorporating the crack tip singularity is currently not available. In the absence of an elasticity solution approximate analytical models have been derived (Klaus Friedrich, 1989): analytical expressions for compliance and strain energy release rate based upon beam theory, a higher order beam theory based upon Reissner’s variational
principles and a shear deformation plate theory. All these methods assume linear small deflections.

\[
C = \frac{(2L^3 + 3a^3)}{8E_1wh^3}
\]  
\hspace{2cm} (3.27)

Where \(E_1\) is the flexural modulus in the axial direction of the beam, \(h\) is thickness, \(a\) is the initial notch length, \(2L\) is the distance between support, and

\[
G_{IIc} = \frac{(9P^2 Ca^2)}{2w(2L^3 + 3a^3)}
\]  
\hspace{2cm} (3.28)

However compliance is not evaluated using above Equation (3.27) due to uncertainty in the modulus of the laminate. It can be determined from the load-displacement record.

Generally the Mode-II delamination is obtained by the normalized values of shear deformation theory and beam theory and it is given by
\[ \frac{G^{SH}}{G^{BT}} = 1 + 0.2 \frac{E_{11}}{G_{13}} (1 + 0.2 (h/a)^2 ) \] (3.29)

Where \( E_{11}, \; G_{13} \) are material moduli, \( h/a = 0.5 \) which is the ratio of thickness to crack length.

### 3.7 MIXED-MODE FRACTURE TOUGHNESS

The Arcan fixture (Figure 3.15) and specimen geometry were developed by Arcan and his co-workers (Arcan et al. 1976) in an attempt to produce uniform plane stress in the test section. A state of pure shear can be obtained by varying the loading angle and a combined stress state is achieved in the test section. Jurff and Pipes (1982) used this Arcan fixture to produce fracture mechanics data for adhesively bonded composite joints but the samples were bonded to the fixture. But in this present work this Arcan fixture (Figure 3.15) is modified to hold a single edge notched specimen by means of bolts and nuts.

![Figure 3.15 Modified Arcan fixture](image)

The modified Arcan fixture was made of two pairs of 6.4 mm thick stainless steel parts. Each pair was equal to one half of the original Arcan fixture and was mounted on a circular disk. A 3.2 mm deep trapezoidal cut
out was machined in each part to host the specimen. As a result a specimen of up to 6.4 mm thickness could be tested using an Arcan fixture. Three holes were drilled at the cut out section in each part for tightening two parts together with bolts. In addition, the bolts could prevent the specimen from slipping and provide a smooth load transfer. In each part, several 6.4 mm diameter holes were drilled near the edge along the circumference of the plate. These holes were designed for load application through a pin yoke system and for tightening each pair of fixture plates together. They were located at angles starting at 0 degrees to 90 degrees and the angles were varied in steps of 22.5 degrees. The fixture and the specimen are shown in Figure 3.16. The critical fracture toughness ($K_{IC}$ & $K_{IIIC}$) can be estimated and this fixture is particularly suitable for mixed mode fracture also. An advantage is that this metal fixture is reusable.

For a given angle $\alpha$ far field normal and shear stresses may be determined

$$\sigma_x = \sigma_A \sin \alpha$$  \hspace{1cm} (3.30)

$$\tau_x = \sigma_A \cos \alpha$$  \hspace{1cm} (3.31)

**Figure 3.16** Modified Arcan fixture with sample
Where the stress is defined as applied load $P$ divided by crosssectional area $A$ of the composite specimen. Based on the normal and shear components, the stress intensity factors $K_I$ and $K_{II}$ associated with the opening and shearing mode may be determined

$$K_I = (P \sin \phi \ t) f_I(a/w)$$

$$K_{II} = (P \cos \phi \ t) f_{II}(a/w)$$

Where $f_I$ and $f_{II}$ are the correction factors for the fixtures according to Jurf and Pipes

$$f_I(a/w) = 1.12 - 0.231(a/w) + 10.35 (a/w)^2 - 21.27 (a/w)^3 + 30.39 (a/w)^4$$

(3.34)

$$f_{II}(a/w) = (1.122 - 0.56 (a/w) + 0.085 (a/w)^2 + 0.18 (a/w)^3) / (1 - (a/w))^{1/2}$$

(3.35)

and

$a/w$ is the ratio of notch length to crack length.

One major issue in all fracture toughness specimens is pre-cracking. One method is to use a razor blade and the other method is to use a starter film. These starter films are frequently employed to initiate delamination and they are Teflon and Polyimide films. But folded aluminum foil can be used for composite at high temperature. Precracking could also be obtained by means of fatigue loading. In this present work a sharp precrack was obtained by using a jeweler’s saw. The response of this region will depend both on its dimension and on the toughness of the neat matrix.
3.8 FRACTROGRAPHY STUDY

Non-destructive tests (Klaus Friedrich, 1989) in great variety are in worldwide use to detect variations in structure, minute changes in surface finish, the presence of cracks or other physical discontinuities, to measure the thickness of materials and coatings and to determine other characteristics of industrial products. In the 1930s, nondestructive testing, where it had been heard of at all, was generally considered an evil. Later it became a necessary evil. For a number of years now NDT has been a necessary aid in tens of thousands of shops in a multitude of industries.

Nondestructive testing is not confined to crack detection. Other discontinuities include porosity, wall thinning from corrosion and many sorts of disbonds. Nondestructive material characterization is a growing field concerned with material properties including material identification and microstructural characteristics—such as resin curing, case hardening and stress that have a direct influence on the service life of the test object. Nondestructive testing has also been defined by listing or classifying the various methods. This approach is practical in that it typically highlights methods in use by industry.

Ensuring product reliability is necessary because of the general increase in performance expectancy of the public. The manufacturer expects the lathe, punch press or fork lift to stand up for years of continuous work even under severe loads. But reliability merely for convenience and profit is not enough. Reliability to protect human lives is a valuable end in itself. The railroad axle must not fail at high speed. The front spindle of the intercity bus must not break on the curve. The aircraft landing gear must not collapse on touchdown. The mine hoist cable must not snap with people in the cab. Such critical failures are rare indeed. And this is most certainly not the result of mere good luck. In large part it is the direct result of the extensive use of
Nondestructive testing and of the high order of nondestructive testing ability now available.

Non-destructive Testing is not just a method for rejecting substandard material; it is also an assurance that the supposedly good is good. The technique uses a variety of principles; there is no single method around which a black box may be built to satisfy all requirements in all circumstances. The methods most commonly used in industry are: Magnetic Particle Crack Detection, Dye Penetrant Testing, Eddy Current and Electromagnetic Testing, Radiography, Thermography, Ultrasonic Flaw Detection.

3.8.1 Flaw Detection by X-Ray Radiography

30% of all film radiography could be replaced by today's technologies in the field of digital radiography. The choice to go digital depends on cost, quality requirement, workflow and throughput. Digital images offer a lot of advantages in terms of image manipulation and workflow. But despite the many advantages, a lot of considerations are needed before someone can decide to convert his organization from conventional to digital radiography.

A heterogeneous beam of X-rays is produced by an X-ray generator and is projected towards an object. According to the density and composition of the different areas of the object a proportion of X-rays are absorbed by the object. The X-rays that pass through are then captured behind the object by a detector (film sensitive to X-rays or a digital detector) which gives a 2D representation of all the structures superimposed on each other (Figure 3.17).
3.8.2 Flaw Detection by Infra-Red Thermography

Every body with a temperature higher than the absolute zero emits electromagnetic radiations in a spectrum which is in the infra-red (IR) region. These radiations depend on the surface temperature of the body; therefore, it is possible to measure the surface temperature of a material without any contact need, by detecting the emitted radiations.

This is the base principle of the IR-thermography (Prasad and Nair, 2011) a non destructive technique (NDT) for the evaluation, by image analysis, of the surface temperature of a body. This technique is used for non destructive inspections (NDI) on various types of materials and on large areas of engineering components. In recent years, active infrared (IR) thermography has emerged as a widely used method for nondestructive testing. Thermography offers noncontact, wide area detection of subsurface defects, and can be used as an alternative or complement to conventional inspection technologies. Heat flow in a material is altered by the presence of some types of anomalies. These changes in heat flow cause localized temperature differences in the material. Slow heating of part reveals these anomalies.
Although the changes in the sample surface temperature are measured by an infrared camera, in most cases, the primary limits of detectability of subsurface features or anomalies are not due to the optics or pixel density of the infrared detector. Rather, the most severe constraints on thermography are usually due to the fact that signals occur as a result of a diffusion process specifically, the diffusion of heat through the sample. The net effect of thermal diffusion on the NDT process is that the minimum detectable feature size increases with the depth of the anomaly or feature, and the time required to detect a feature is proportional to the square of its depth.

Regardless of the excitation technique employed or the sample material involved, in most modern thermographic practice, changes in the surface temperature are detected by an infrared camera. This requires that the surface of the sample is an effective emitter of infrared energy. Emissivity is the ratio of the infrared energy radiated by an object at a given temperature to the energy emitted by a perfect radiator (emissivity = 1), or blackbody, at the same temperature, and thus, is an excellent measure of how effectively an object emits infrared energy. Emissivity is a property of the sample surface only.

With composite construction, where damage was likely to be undetectable at the surface, visual testing techniques were of little value, so alternatives capable of wide-area testing were sought. Unlike metals, in many respects composite materials were ideally suited for thermographic (Figure 3.18) testing with the newly introduced infrared cameras.
3.8.3 Sub-surface Detection by Scanning Electron Microscope

The macroscopic appearance of fracture and mechanical properties depend upon which of these mechanisms dominate the overall fracture process. In particular, the following failure mechanism is of special importance:

1. Matrix deformation and fracture
2. Fiber/matrix debonding
3. Fiber pull out and
4. Fiber fracture

All these mechanisms consume energy and contribute to toughness of the composite. Which of them and to what extent actually occur during the failure of the material depends largely on the partial properties of the three microstructural elements of the composite namely matrix, fiber and interphase and on the geometrical arrangement and form of the reinforcing components.

To study the fracture surface details at higher magnification, smaller specimens are required from the fracture area and are viewed through Scanning Electron Microscope (SEM). For better comparisons each
specimens is adjusted in the SEM specimen holder in a way that the crack
detection of the fracture surface section seen on the SEM screen is always the
same. For receiving an optimum illumination and contrast as well as aesthetic
appearance, the specimens are usually tilted by a certain angle.