Wavelet filters supporting spatial-frequency decomposition discussed in the previous chapter is chosen in our work as the key method for texture characterization. This chapter reviews the wavelet decomposition in general and develops the texture characterization technique using the Discrete Wavelet Transform. Texture features are computed from the energy of the wavelet sub-bands. Simple classification experiments are conducted utilizing the energy measures of the wavelet sub-bands.

3.1 Wavelets: A Tool for Multi-resolution Analysis

Discrete one-dimensional or multi-dimensional transforms represent mathematical tools for efficient signal and image analysis [51] [72] [94] [96] [97] [147] [148] [149] [170]. Fourier analysis has an enormous impact in engineering, science, and mathematics. For example most medical blood pressure transducers are designed based on Fourier analysis. The mathematical theory of wavelets is not very old, yet the wavelet transform has already become a fundamental tool in signal and image processing [51] [72] [97]. It is recognized that a global Fourier transform gives good information of the spectrum of the signal [34]. However unlike the wavelet transform it cannot easily detect high frequency bursts [51] [52] [72]. Non-stationary signals which are unpredictable like a speech signal or an EEG signal, are easily analyzed using wavelets which necessitate the notion of frequency analysis that is local in time (time-scale analysis) [47] [90]. An important aspect of these transforms is the chance to extract relevant information from a signal or the underlying process, which is actually present but hidden in its complex representation [97].

The wavelet paradigm is a commonly used mathematical framework that has unified the theories of continuous and discrete spatial-frequency and multi-resolution signal analysis [97]. The main contrast between wavelet transforms and the previously developed spatial-frequency...
decompositions (such as the short time Fourier transform) is that wavelet transforms use short analysis windows at high frequencies and long analysis windows at low frequencies. This makes intuitive sense as lower frequencies require measurements over longer distances for effective analysis. Conversely, higher frequencies need only small windows for analysis [147].

3.1.1 Wavelet Families
Wavelet transform decomposes a signal into a set of basis functions [98] [149]. These basis functions are the wavelets. They are obtained from the mother wavelet by dilation and shifting [98] [149]. There are a number of basis functions that can be used as the mother wavelet for wavelet transformation [51][72][98][99][139][149]. Figure 3.1 illustrates some of the commonly used wavelet functions [51] [72] [149]. Haar wavelet is one of the oldest and simplest wavelet [98] [149]. Therefore, any discussion of wavelets starts with the Haar wavelet. Daubechies wavelets are the most popular wavelets. They represent the foundations of wavelet signal processing and are used in numerous applications. These are also called Maxflat wavelets as their frequency responses have maximum flatness at frequencies 0 and $\pi$. This is a very desirable property in some applications [31][51][98][149].

![Wavelet families](image)

Fig. 3.1: Wavelet families (a) Haar (b) Daubechies4 (c) Coiflet1 (d) Symlet2 (e) Meyer (f) Morlet (g) Mexican Hat.
The Haar, Daubechies, Symlets and Coiflets are compactly supported orthogonal wavelets[31][51]. These wavelets along with Meyer wavelets are capable of perfect reconstruction. The Meyer, Morlet and Mexican Hat wavelets are symmetric in shape [31] [51]. The wavelets are chosen based on their shape and their ability to analyze the signal in a particular application [31][51]. Since the mother wavelet produces all wavelet functions used in the transformation through translation and scaling, it determines the characteristics of the resulting Wavelet Transform. Therefore, the details of the particular application are taken into account and the appropriate mother wavelet is chosen in order to use the Wavelet Transform effectively [98] [149].

### 3.1.2 The Continuous Wavelet Transform

The Continuous Wavelet Transform (CWT) provides a time-scale description similar to that of the Short Time Fourier transform (STFT) with a few important differences: frequency is related to scale and the CWT is able to resolve both time and scale (frequency) events better than the STFT [13] [142]. Wavelet series are thus constructed with two parameters namely scale and translation [98] [149]. These parameters make it possible to analyze a signal behavior at a dense set of time locations and with respect to a vast range of scales, thus providing the ability to zoom in on the transient behavior of the signal. The CWT is expressed by (3.1), where \( x(t) \) is the signal to be analyzed, \( W(t) \) is the mother wavelet or the basis function [13] [142] [149]. All the wavelet functions used in the transformation are derived from the mother wavelet through translation (shifting) and scaling (dilation or compression).

\[
X_w(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} W\left( \frac{t-b}{a} \right) x(t) \, dt
\]  

(3.1)

The mother wavelet used to generate all the basis functions is designed based on some desired characteristics associated with that function [98] [149]. The translation parameter \( b \) relates to the location of the wavelet function as it is shifted through the signal [98] [149]. Thus, it corresponds to the time information in the Wavelet Transform. The scale parameter \( a \) is defined as \( |1/\text{frequency}| \) and corresponds to frequency information. Scaling either dilates (expands) or compresses a signal. Large scales (low frequencies) dilate the signal and provide detailed information hidden in the signal, while small scales (high frequencies) compress the signal and provide global information about the signal [13] [31]. Notice that the Wavelet Transform merely performs the convolution operation of the signal and the basis function. The above analysis
becomes very useful as in most practical applications, high frequencies (low scales) do not last for a long duration, but instead, appear as short bursts, while low frequencies (high scales) usually last for the entire duration of the signal.

### 3.1.3 The Discrete Wavelet Transform

By restricting to a discrete set of parameters, we get the Discrete Wavelet Transform (DWT) which corresponds to an orthogonal basis of functions all derived from a single function called the mother wavelet \([98][149][152]\). CWT is redundant since the parameters \((a, b)\) are continuous and thus it’s necessary to discretize the grid on the time-scale plane corresponding to a discrete set of continuous basis functions. This leads us to a question: how is it possible to discretize the wavelet in 3.1? In theory \(a_j = a_0^j\) and \(b_k = k b_0 a_0^j\) where \(j, k \in \mathbb{Z}, a_0 > 1, b_0 \neq 0\). The discrete form of the wavelet is shown in 3.2, where \(j\) controls the dilation and \(k\) controls the translation.

\[
W_{j,k}(t) = \frac{1}{\sqrt{a_j}} W\left(\frac{t-b_k}{a_j}\right)
\]

(3.2)

The DWT is another way to decompose a time series into a sequence of components with different scales. The original signal is reconstructed from these components. The DWT based on sub-band coding is found to yield a fast computation of Wavelet Transform. It is easy to implement and reduces the computation time and resources required \([152]\). Wavelet analysis \([66][97][99][113][118][152]\) is simply the process of decomposing a signal into shifted and scaled versions of the mother (initial) wavelet. An important property of wavelet analysis is perfect reconstruction, which is the process of reassembling a decomposed signal or image into its original form without loss of information \([66][99][118][142]\). For decomposition and reconstruction two types of basis functions are normally used. They are:

Scaling function \(\phi_{j,k}(t) = 2^{-\frac{j}{2}} \phi_0\left(2^{-j} t - k\right)\)

(3.3)

Wavelet function \(W_{j,k}(t) = 2^{-\frac{j}{2}} \psi_0\left(2^{-j} t - k\right)\)

(3.4)

where \(j\) stands for dilation or compression and \(k\) is the translation index. Every basis function \(W\) is orthogonal to every basis function \(\Phi\). Wavelets are defined over a finite interval and have an
average value of zero \[13][31][66][99][118][139][152]. An example of a simple wavelet function is called the Haar wavelet. The Haar mother wavelet \(W(t)\) and scaling function \(\phi(t)\) are defined as follows:

\[
\phi(t) = \begin{cases} 
1 & 0 \leq t \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]

\[
W(t) = \begin{cases} 
1 & 0 \leq t \leq \frac{1}{2} \\
-1 & \frac{1}{2} \leq t \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]  

(3.5)

3.1.4 Implementation of DWT on signals

In CWT, the signals are analyzed using a set of basis functions which relate to each other by simple scaling and translation \[118] \[139]. In the case of DWT, a time-scale representation of the digital signal is obtained using digital filtering techniques. The signal to be analyzed is passed through filters with different cut-off frequencies at different scales and they divide the signal frequency band into sub-bands \[99][113][152].

![One-dimensional signal decomposition](image)

Fig. 3.2: One-dimensional signal decomposition

In Figure 3.2, \(L\) and \(H\) represent the low-pass filter (LPF) or scaling function and high-pass filter (HPF) or wavelet function respectively. The wavelet filter coefficients are obtained from
alternating flip of scaling filter coefficients. Short filter results in faster computation of convolutions. These filters split a signal’s bandwidth in two halves. This provides the coefficients $c_j(k)$ and $d_j(k)$ for the decomposition of the signal into its scaling function and

![Diagram](image1)

Fig. 3.3: First step in computing the 1-d wavelet transform: low pass filtering (in increments of two) on an input signal $x$.

Fig. 3.4: Second step in computing the 1-d wavelet transform: high pass filtering (in increments of two) on an input signal $x$.

wavelet function components or approximation (low frequency component) and detail coefficients respectively (high frequency component) [152]. Figure 3.3 shows the first step in
computing the DWT namely the low-pass filtering. As is seen, the filter is moved in increments of two along the length of the input vector, \( x \). This is equivalent to down-sampling by a factor of two. The resultant vector produced is half as long as the original input signal \( x \). Next, as shown in Figure 3.4, the input vector \( x \) is convolved with a discrete time HPF in the same manner as the LPF. However, the result of these convolutions is placed in the second half of the \( w \) vector. This is considered as first level wavelet decomposition \([1][118][161]\). This process is continued again and again, each time working with the previous generated vector as the input to the next level as illustrated in Figure 3.5. Because each level decreases the length of the workable vector by a factor of two, this process cannot be continued forever.

![Diagram of three level wavelet transform](image)

The resulting coefficients are

\[
c(n) = \sum_{n=0}^{L-1} h_0(k)x(n-k)
\]

(3.6)

\[
d(n) = \sum_{n=0}^{L-1} h_1(k)x(n-k)
\]

(3.7)

where \( c(n) \) represent the approximation coefficients for \( n = 0, 1, 2 \ldots, L - 1 \) and \( d(n) \) the detail coefficients. \( h_0 \) and \( h_1 \) are the coefficients of the discrete-time filters \( L \) and \( H \) respectively.

\[
\{h_0(n)\}_{n=0}^{L-1} = (h_0(0), h_0(1), \ldots, h_0(L - 1))
\]

(3.8)

\[
\{h_1(n)\}_{n=0}^{L-1} = (h_1(0), h_1(1), \ldots, h_1(L - 1))
\]

(3.9)
The inverse discrete wavelet transform (IDWT) reconstructs a signal from the approximation and detail coefficients derived from decomposition [13][99][118]. The right side of Figure 3.6 shows the reconstruction process. The process of IDWT differs from the DWT in that it requires up-sampling and filtering, in that order. Up-sampling, also known as interpolating means the insertion of zeros between samples in a signal.

3.1.5 Two-dimensional Discrete Wavelet Transform

Digital images are 2D signals that require two-dimensional discrete wavelet transform (2D DWT) for processing [1][161]. The 2D DWT analyzes an image across rows and columns in such a way as to separate horizontal, vertical and diagonal details. In the first step the rows of an $N \times N$ image $A$ are filtered using LPF and HPF as explained above, to obtain two resultant images $B$ and $C$ of size $N/2$ each [2][66][97][118]. In the second step, the columns of $B$ are filtered with the same LPF and HPF to give two resulting images of size $N/4$. The process is repeated on image $C$. This process completes a single level decomposition [1][118]. These operations correspond to the following filtering processes:

1. 4 point average or 2D low-pass filter (Lo-Lo)
2. Average horizontal gradient or horizontal high-pass and vertical high-pass filter (Hi-Lo)
3. Average vertical gradient or horizontal low-pass and vertical high-pass filter (Lo-Hi)
4. Diagonal curvature or 2D high-pass filter (Hi-Hi)
Fig. 3.7: A one-level 2D DWT decomposition

Fig. 3.8: A one-level 2D DWT decomposition in detail

Fig. 3.9: Impulse response of the 2D wavelet transform filter
So each level decomposition leads to 4 different sub-bands namely HH, HL, LH and LL. This is illustrated in Figure 3.7 and Figure 3.8. At each scale the sub-bands are sensitive to frequencies at that scale and the LH, HL and HH sub-bands are sensitive to vertical, horizontal and diagonal frequencies respectively [1][66][118][161].

![Diagram](image)

Fig. 3.10: Frequency response of a 2D wavelet transform filter

The impulse response and frequency response of the filters are shown in Figures 3.9 and 3.10 respectively. For multi-level decomposition, the procedure is repeated on the LL band as shown in Figure 3.11 for 2-level decomposition.

![Diagram](image)

Fig. 3.11: 2 levels of wavelet decomposition.
Figure 3.12(b) shows a 2-level DWT decomposition of the Lena test image in Figure 3.12 (a). This image also shows the scale and orientation selectivity of the DWT. High energy sub-band regions pick out the texture content at different scales and orientations. It is clear from Figure 3.12 (b) that most of the energy is contained in the LL sub-band and the least energy is in the HH sub-band. The HL sub-band contains the near vertical edges and the LH sub-band contains the near horizontal edges. Table 3.1 lists the energy values for a single level DWT decomposition of the Lena image expressed as a percentage of the total energy for the various sub-bands.

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>HL</th>
<th>LH</th>
<th>HH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>96.5%</td>
<td>2.2%</td>
<td>0.9%</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

3.2 Choice of Wavelet Filter
Villasenor et al. presented guidelines for wavelet basis selection for application in wavelet image compression [113][139][151][153]. Very little attention was paid to filter choice for texture classification and analysis. When applying the DWT for texture analysis tasks, the filters are chosen to ensure an effective characterization of the spatial-frequency content [1] [118]. Several filter characteristics are considered important to the choice of filter. These characteristics include [139]:

Fig. 3.12: (a) Original Lena image  (b) 2 level DWT on Lena image
**Regularity:** This is defined as the number of times the wavelet basis is continuously differentiable. This shows the bases’ “smoothness” and usually reflects the frequency localization of the basis.

**Compact Support:** Spatial localization of wavelet filters is important in texture characterization.

**Shift Invariance:** The sub-sampling necessary for the critical decimation of the DWT results in shift variance. However, the amount of shift variance varies from filter to filter. Shift invariance is important for properly characterizing textures as the representation should not be dependent on the position of the input signal.

**Number of Vanishing Moments:** This reflects the ability of the filter to attenuate specific frequencies. This is important in avoiding aliasing.

However, few authors also concluded that filter choice does not have a significant effect on the number of correctly identified textures in classification experiments [19] [38]. To highlight, De Brunner and Kadiyala [38] conducted a study of over twenty wavelet bases for texture classification. They concluded that the maximum difference in classification rate from the best wavelet basis to the worst was less than 3% (with classification rates of well over 90%). They also suggested that equivalent or better performances could be achieved by very compact support filters when compared to the previously recommended 16 tap Battle-Lemarie filters [19].

The need of compact support filters was also pointed by Unser [148] who concluded that the spatial localization of the filters themselves is often more important than the nature (smoothness / regularity) of the resulting wavelets. Biorthogonal (i.e. linear phase) wavelets were compared to orthogonal filters by Mojsilovic et al. [106] and Ma and Manjunath [96]. Ma et al. concluded that the biorthogonal wavelets gave very similar but slightly worse classification results to the orthogonal wavelets. However Mojsilovic et al. concluded the opposite i.e. the orthogonal wavelets gave very similar but slightly worse classification results compared to the biorthogonal wavelets. This is because of the increased shift variance of the orthogonal wavelets compared to the biorthogonal wavelets [106]. They also mention that even length biorthogonal wavelets performed better than odd length biorthogonal wavelets. This was attributed to the relatively greater shift invariance of the even length filters. So it follows that the majority of the work described above has provided inconclusive or at worst contradictory results. However, there is an understanding that filters are to be very spatially localized but with the maximum possible frequency localization. The even length biorthogonal filters bior3.1 is therefore adopted for the
subsequent experiments within this chapter. This filter set is adopted since it gives the second best classification performance within [106] but is considerably more compact than the best performing filters. Moreover this filter set is reported to be more suitable for texture analysis with the shift invariance considered more important than regularity [106].

3.3 Texture features

An image is considered as the combination of different texture regions and the image features associated with these texture regions are used in the identification of regions. The most difficult step in the classification of pictorial data is the definition of a set of meaningful features describing the data. In the search for meaningful features it is natural to look towards the type of features which humans use in interpreting pictorial data. Due to the resemblance between multi-resolution filtering techniques and human visual processing, wavelet transform techniques are often used for texture characterization through the analysis of spatial-frequency content. These techniques are extensively used to extract texture features because different textures have different energy or statistics distribution in the frequency domain. Hence statistical distributions or sub-bands energy are normally used as feature vectors during feature selection phase.

The wavelet coefficients are the elementary texture features that can be used for further processing. But managing the coefficients of multiple sub-bands for multi-level decomposition becomes difficult. Hence derivation of statistics from these coefficients is the practical approach to feature extraction. The simplest forms of statistics that can be formed from the coefficients of the sub-bands are the mean, variance and entropy. Directionality is an important characteristic for texture images. So computation of different directionalities namely ‘the vertical directionality’, ‘the horizontal directionality’ and ‘the diagonal directionality’, to represent directional information of the texture is another statistic. Similarly contrast, for the purpose of denoting the change of gray-levels in a texture, is commonly defined for each pixel as an estimate of the local variation in a neighborhood. The calculation of the contrast is implemented in the wavelet decomposed approximation sub-band, denoted as LL, containing the low frequency and reflecting the global information of texture. Another fundamental statistic, coarseness, the most fundamental texture feature, is the granularity measurement of texture. Researchers usually identify the texture by “coarseness” [19][106][116][148]. The energy measure from the sub-bands is used in our initial experiments as it is one of the simplest methods.
and the most extensively used in the classification experiments [56][38][96][148][153]. A characterizing feature vector for a texture is formed from sub-band property measures as shown in Figure 3.13. Energy feature from each sub-band is calculated as

\[ e_s = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} x_s(i,j) \]  

(3.10)

where \( N \) is the width of the channel and \( i \) and \( j \) are the rows and columns of the channel and \( x_s \) is the absolute value of the wavelet coefficient [113]. Each energy measure characterizes the magnitude of frequency content at the orientation and scale of each sub-band.

![Wavelet Transform and Feature Vector](image)

*Fig. 3.13: Schematic of typical feature extraction process using the wavelet transform*

### 3.4 Test Data used in our experiments

The Brodatz textures are the most commonly used texture dataset, especially in the computer vision and signal processing community for evaluating texture recognition algorithms [12]. The reason is that most of the Brodatz textures are photographed under controlled lighting conditions and hence the images are of very high quality (if you scan them carefully). In addition, they expose the textures very well, so that irrelevant information such as noise etc is not there. It contains 111 different texture classes. Each class is represented by only one sample, which is then divided into 9 sub-images, non-overlapping, to form the database. Thus, there are 999 images altogether with a resolution of 215x215. Our initial experiments on texture classifications are performed on a set of 30 Brodatz textures listed in Table 3.2. These textures are chosen as they contain adequate texture information including textures with orientation.
Table 3.2: Textures used in the experiments

<table>
<thead>
<tr>
<th>Texture ID</th>
<th>Texture Description</th>
<th>Texture ID</th>
<th>Texture Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D3</td>
<td>Reptile skin</td>
<td>D57</td>
<td>Handmade paper</td>
</tr>
<tr>
<td>D4</td>
<td>Pressed cork</td>
<td>D65</td>
<td>Handwoven oriental rattan</td>
</tr>
<tr>
<td>D6</td>
<td>Woven aluminium wire</td>
<td>D68</td>
<td>Wood grain</td>
</tr>
<tr>
<td>D9</td>
<td>Grass lawn</td>
<td>D74</td>
<td>Coffee beans</td>
</tr>
<tr>
<td>D11</td>
<td>Homespun woolen cloth</td>
<td>D77</td>
<td>Cotton canvas</td>
</tr>
<tr>
<td>D16</td>
<td>Herringbone</td>
<td>D78</td>
<td>Oriental straw cloth 3</td>
</tr>
<tr>
<td>D19</td>
<td>Woolen cloth with soft tufts</td>
<td>D79</td>
<td>Oriental grass fiber cloth</td>
</tr>
<tr>
<td>D21</td>
<td>French Canvas</td>
<td>D82</td>
<td>Oriental straw cloth 4</td>
</tr>
<tr>
<td>D24</td>
<td>Pressed calf leather</td>
<td>D83</td>
<td>Woven matting</td>
</tr>
<tr>
<td>D29</td>
<td>Beach sand</td>
<td>D84</td>
<td>Raffia looped to a high pile</td>
</tr>
<tr>
<td>D34</td>
<td>Netting</td>
<td>D92</td>
<td>Pig skin</td>
</tr>
<tr>
<td>D36</td>
<td>Netting</td>
<td>D95</td>
<td>Brick wall</td>
</tr>
<tr>
<td>D52</td>
<td>Oriental straw cloth 1</td>
<td>D102</td>
<td>Cane</td>
</tr>
<tr>
<td>D53</td>
<td>Oriental straw cloth 2</td>
<td>D103</td>
<td>Loose burlap</td>
</tr>
<tr>
<td>D55</td>
<td>Straw matting</td>
<td>D105</td>
<td>Cheese cloth</td>
</tr>
</tbody>
</table>

In the succeeding experiments on texture based segmentation, all textures from the Broadatz album are used. For the medical images, we use a set of Magnetic Resonance (MR) images, Ultra sound (US) images and Computed Tomography (CT) images of the brain, stomach, knee etc. These images are obtained from a Medical Research Institute of Thiruvananthapuram. In the last phase of our work, images from the Corel dataset [28] are also used for evaluation. The use of human segmented images for relative comparison of the machine segmented images is a common method of evaluation of segmentation algorithms [58][146]. The availability of the human segmented images in this dataset is the reason for adding this benchmark to our test dataset.

3.5 Texture Classification Experiments

A common approach to model the statistical variation of sub-band energies for texture classification is to use many example images of each texture [113]. The variations of features extracted from each image are then used by classifiers such as neural networks [86] or Bayes classifier [148] to produce a trained classification system. So, for our experiments, we used the set of textures listed in Table 3.2, digitized with 512 x 512 pixels and 256 gray levels. For training and classification, from each texture, 100 samples of size 128 x 128 are chosen resulting
in 3000 images database. These 3000 texture images are separated to two sets, the training set with 1500 images and the test set with 1500 images, respectively. The mean of each image is removed before the processing.

A simple dyadic wavelet transform is used, to 3 levels of decomposition (for suitable number of features) for implementing the classification in the experiments with DWT.

There are therefore 10 resulting sub-bands and 10 energy measures as depicted in Figure 3.14. A simple set of texture classification experiments is conducted to test the developed methods and metrics. Feature extraction of the training and test images is implemented as per algorithm 3.1.

**Algorithm 3.1: Feature extraction**

1. **Subject the grayscale image to a 3 level DWT decomposition using the even length biorthogonal filters bior3.1.**
2. **Calculate the energy ($E_k$) of the image sub-bands.**
3. **Form the feature vector from the energy values.**

The classification experiment comprises of 2 stages, the learning phase and the testing phase as explained in algorithm 3.2. In the learning phase, each sample texture is trained and represented with a mean feature vector and covariance matrix. The testing phase utilizes this information for mapping the unknown sample to the appropriate texture.

**Algorithm 3.2: Classification Algorithm with J Features.**

**Learning phase**

1. **Given $m$ samples obtained from the same texture, generate the feature vector for each sample using algorithm 3.1, giving $m$ feature vectors for each texture.**
(2) Generate the mean feature vector from the \( m \) feature vectors
(3) Compute the mean and covariance matrix from the mean feature vector.
(4) Repeat the process for all textures.

**Classification phase**

(1) Decompose an unknown texture \( x \) with the DWT and construct its energy feature vector.
(2) Compute the mean and covariance matrix of the energy feature vector.
(3) Calculate the distance measure between the unknown texture and the trained textures.
(4) Assign the unknown texture to texture \( i \), if \( D_i < D_j \) for all \( j \neq i \).

Several distance functions are used in classification. In particular, we consider four such functions and list them in Table 3.3, where \( C_i \) is the covariance matrix of the feature set for texture \( i \). The Mahalanobis distance is often used as the metric for a minimum distance classifier for texture classification [116]. The Mahalanobis distance is a useful measure of similarity if some statistical properties of textures are known. In particular, if \( C_i \) is a diagonal matrix or, equivalently, features are independent of each other, the Mahalanobis distance reduces to the form

\[
D_{4,i} = \sum_{j=1}^{J} \frac{(x_j - m_{i,j})^2}{c_{i,j}}
\]  

(3.11)

where \( c_{i,j} \) is the variance of feature \( j \) and class \( i \).

<table>
<thead>
<tr>
<th>Distance Function</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euclidean Distance</td>
<td>( D_{1,i} = \sum_{j=1}^{J} (x_j - m_{i,j})^2 )</td>
</tr>
<tr>
<td>Bayes Distance</td>
<td>( D_{2,i} = (x - m_i)^T C_i^{-1} (x - m_i) + \ln</td>
</tr>
<tr>
<td>Mahalanobis Distance</td>
<td>( D_{3,i} = (x - m_i)^T C_i^{-1} (x - m_i) )</td>
</tr>
<tr>
<td>Simplified Mahalanobis Distance</td>
<td>( D_{4,i} = \sum_{j=1}^{J} \frac{(x_j - m_{i,j})^2}{c_{i,j}} )</td>
</tr>
</tbody>
</table>

Furthermore, if the joint density function of features is available, more sophisticated distance measures such as the one based on the Bayes decision rule can be used. By the law of large
numbers, we assume that the density function of features is Gaussian so that the Bayes decision function assumes the form

\[ D_2(x, m_i, C_i) = (x - m_i)^T C_i^{-1} (x - m_i) + \ln |C_i|. \]  

(3.12)

Note that the Bayes distance is similar to the Mahalanobis distance for features with Gaussian distributions except for the addition of the second term known as the covariance difference.

Table 3.4: Classification results with four distance functions

<table>
<thead>
<tr>
<th>Texture ID</th>
<th>Correct Classification Rate(%)</th>
<th>ED</th>
<th>BD</th>
<th>MD</th>
<th>SMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D3</td>
<td>44</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>D4</td>
<td>29</td>
<td>48</td>
<td>48.5</td>
<td>47.5</td>
<td></td>
</tr>
<tr>
<td>D6</td>
<td>49</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>D9</td>
<td>48</td>
<td>48.5</td>
<td>48</td>
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Table 3.4 summarizes the classification rate obtained for the various textures, for the various distance measures of Euclidean distance (ED), Bayes distance (BD), Mahalanobis distance (MD) and Simplified Mahalanobis distance (SMD) respectively. It extends from 30 to 50% only. The
average classification rates of the various tested textures is called classification rate and listed in the last row of the table. The results clearly indicate the comparatively low performance obtained in the experiments. This is attributable to the inability of identifying the appropriate texture patterns due to the insufficient feature description. The comparatively low classification rates obtained for the textures D52, D53, D65, D78, D79 and D82 indicate that the features generated for these textures contain insufficient orientation related information. This is because the DWT decompositions suffer from the disadvantage of poor directional selectivity. The Euclidean metric performs comparatively poor but provides a baseline to compare the other metrics. The other metrics improves considerably on the Euclidean result by considering the feature distribution within the training textures.

3.6 Summary
This chapter uses statistics obtained from the analysis of the images using DWT to represent texture characterization. The result of all the experiments gives an accuracy of around 50% and is considered to be inferior to the results pointed out in similar texture classification experiments within the present subject literature. This is mainly because good frequency and spatial localization is important for texture analysis in addition to possessing the properties of scalability and directional selectivity. The directional selectivity of the DWT is poor because its separable filters cannot distinguish between edge features on opposing diagonals. However this is taken as a preliminary step for performance assessment in our experiments.

Summarizing, the contributions of chapter 3 are:

- Identification of wavelets as a tool for multi-resolution analysis
- Exposing the methodology of applying wavelet filters on images
- Using the DWT for texture characterization of images
- Generation of texture features from the energy of the wavelet sub-bands
- Implementation of classification algorithm with 4 distance measures indicating the suitability of using the texture features for segmentation.