Abstract

The motive behind the present study is to investigate theoretically some reaction-diffusion problems of practical importance and to model the soil temperature phenomena in civil engineering and lightning phenomena in electrical engineering. In order to understand the mechanism in terms of governing parameters of the system. In order to achieve this goal discrete wavelet transform is studied first and then we discuss their properties, convergence and computational complexity. The governing equation of soil temperature is derived. Assumptions under which the equations are valid have been elaborated in detail. A brief literature survey in the area of

(i) Fundamental aspects of Reaction-diffusion equations
(ii) Wavelet schemes for solving partial differential equations
(iii) Haar wavelet schemes for solving partial differential equations
(iv) Nonlinear partial differential equations and their applications
(v) Other wavelet schemes for solving reaction and diffusion problems is conducted. Other mathematical methods that provide appropriate and accurate solutions for linear reaction-diffusion equations have been summarised. Ten major problems addressing various issues related to various engineering problems have been undertaken for study and the reaction-diffusion equations corresponding to them have been solved using Haar wavelet schemes.
The solutions obtained from Haar scheme have been compared with some of the recently applied mathematical techniques. Invariably in all problems Haar scheme is found to be more efficient than others.

Initially, a Haar wavelet based method for the estimation of soil temperature at different depths is described. Diurnal variation in the hourly soil temperature is estimated at different depths varying from 0 to 45 cms. This estimation is compared with observed data available for Trombay site at the depths 5, 10, 20 cms. The estimated values show and excellent agreement with the observed values. The Haar based estimated values are found to be more accurate than the values obtained by FDM approach. More accurate solutions can be obtained by changing the time scale in Haar wavelet; At the same time main features of the solution are preserved. Sensitivity analysis has shown that soil temperature was not so sensitive to changes of soil thermal parameters.

Next, an accurate and efficient Haar wavelet solution of Fisher’s equation, that is, a prototypical reaction-diffusion equation has been developed. The solutions of Fisher’s equation are characterized by propagating fronts that can be very steep for large values of the reaction rate coefficient. The use of Haar wavelets is found to be accurate, simple, fast, flexible, convenient with less computation costs and computationally attractive.

The Haar wavelet method for well-known Cahn-Allen and FitzHugh-Nagumo equations has been proposed and the proposed scheme can be used to a wider class of nonlinear equations. The power of this manageable method is confirmed in this thesis. Haar wavelet method for well-known one-dimensional convection-diffusion (CD) equations has been developed and their solutions are compared with restrictive Taylor (RT) approximation. The fundamental idea of Haar wavelet method
is to convert the differential equations into a group of algebraic equations, which involves a finite number of variables. The stability region is discussed here. The numerical results are found in good agreement with the exact solutions.

Haar wavelet method for some of the well-known nonlinear parabolic partial differential equations has been established. The equations include the Nowell-whitehead equation, Cahn-Allen equation, FitzHugh-Nagumo equation, Fisher’s equation, Burger’s equation and the Burgers-Fisher equation. The power of this manageable method is confirmed.

Reaction-diffusion equations are fundamental in modeling several natural phenomena. We develop an accurate and efficient Haar wavelet scheme for solving well-known one-dimensional reaction-diffusion equation. The power of this manageable method is confirmed in this thesis. An attempt is made to combine the advantages of the ADM and Haar wavelets. Good agreement with the exact solution has been observed.

An efficient method for solution of one-dimensional Burgers’ equations based on the Haar wavelets approach is proposed. For various values of viscosity at different time steps, the numerical solutions obtained were compared with the exact solutions and the Haar solutions. The accuracy and effectiveness of the proposed method is demonstrated by 6 test problems.

Haar wavelet method to find the traveling-wave solution for a set of two coupled reaction-diffusion equations has been discussed. Haar solution for traveling-waves of the reaction-diffusion equation with a general non-linearity is found. Application to partial differential equations is exemplified by solving the reaction-diffusion equation. The boundary value, the boundedness and the stability of the solution are also discussed.
Finally, a traveling wave model for describing the lightning stroke by Haar wavelet method is proposed. The calculations demonstrate that the accuracy of the Haar wavelet solutions is quite high even in the case of small number of grid points.