Chapter – II

MF radar and

Data Analysis Techniques
Chapter II

MF radar and Data Analysis Technique

2.1 Introduction

In this thesis, horizontal wind measurements collected by MF radars from Kolhapur (16.8°N, 74.2°E, 10.6° N dip lat.) and Tirunelveli (8.7°N, 77.8°E, 0.3° N dip lat) form the basic data source. The main objectives of collecting data from these radars are to study and understand the dynamics and coupling processes of winds in MLT region at low latitudes.

This chapter can be conveniently divided into two sections. The first section of this chapter is devoted to the details of MF radar systems installed at Kolhapur and Tirunelveli, mode of operation and the method of wind estimation, beginning with the basics of scattering mechanisms. Both the radars have been installed and operated by the Indian Institute of Geomagnetism. They have been providing nearly continuous wind information of the mesosphere and lower thermosphere region using spaced antenna method. The radar technique utilizes partial reflection from weakly ionized atmospheric layers. A full correlation analysis (FCA) is used to extract wind information from the data [1] and a brief account on this is given in this chapter.

The interpretation of radar echoes from the higher heights sampled by the radar, in terms of neutral winds is complicated, as the measured drifts at highest (above 92 km) probed by the radar are influenced by the electrodynamical processes unique to the equatorial region. There is another problem due to total reflection, as there will be a difference in the true and virtual heights as the transmitted electromagnetic ray approaches the total reflection heights, which normally occurs above 94 km. A brief discussion on these aspects is presented in this chapter.
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The second section of this chapter describes a few mathematical techniques namely standard Fourier technique, time-domain filtering and singular and bispectral analysis used for examining the MF radar data.

2.2 Observational Techniques

Many different techniques are currently used to study the atmosphere including radars [2], lidars [3], Satellites [4], Balloons [5] and Rockets [6], each with its own advantages and disadvantages. Ground based remote sensing techniques such as radars (RAdio Detection And Ranging) and Lidars (LIght Detection And Ranging) have the advantages of continuous measurements with good time and height resolution.

The instruments we use for the measuring the earth’s atmosphere decides much of our knowledge of the state of the earth’s atmosphere. The parameters of primary interest are temperature, pressure, humidity, precipitation and wind. Since the nineteenth century, the routine surface measurements of these parameters have been made through out the world. We began to get a systematic look at the space and time variability of the atmosphere as a function of height above the ground in the last century. With the development of the rocket and radiosonde, a significant advance in this area was made. These devices carried by a balloon upwards, measures temperature, pressure and humidity and transmit these data back to a ground station. Also the winds could be determined by tracking the motion of the balloon. Rocket observations are categorized according to the pay load of the measuring system. Among them the falling sphere and datasonde have often been launched because these payloads are small and less expensive [7]. Their observational height ranges from 30-90 km and 20-65 km respectively. Rocket and radiosonde observations are limited to 2-4 soundings per day, so that these observations are only appropriate for defining the large scale planetary waves [8].

Advances in our understanding of the atmosphere are dependent upon the measurements with a higher resolution in both time and space. Many
applications require more frequent and more closely-spaced upper air measurements. Though the radiosondes are inexpensive, the relative high cost of maintaining balloon launching and tracking facilities has prevented more extensive use. Development of remote sensing technology offers a solution to this problem which gives more successive data in height and time using radio waves and optical rays.

The temperature and wind perturbations of the global scale atmospheric waves can be observed in selected height region by choosing specific frequency bands. The instruments on board the Upper Atmospheric Research Satellite such as WIND imaging interferometer (WINDII) and High Resolution Doppler Imager (HRDI) can estimates atmospheric winds and temperatures on a global scale. Satellite based instruments generally have good altitude resolution and excellent spatial coverage and hence can obtain global data by orbiting around the planet. Since WINDII and HRDI have an effective resolution of a few hundred kilometers, they do not readily detect gravity waves of short period, but are more suitable for planetary waves and tides \[9, 10\]. These instruments have to be validated by comparisons with ground based instruments like radar, lidar etc. The first form of the ground based radar was ionosonde which sweeps the entire high frequency (HF) region of the electromagnetic spectrum. The total reflection occurs for the ionospheric electron densities in the HF region and the ionosondes obtain reflections from different altitudes at different frequencies, as atmospheric electron density changes. The ionograms are then used to determine the vertical profile of electron densities.

MST radars can detect weak backscattering arising by refractive index fluctuations in the mesosphere, stratosphere and lower thermosphere in the very high frequency (VHF and UHF) region \[11\]. The MST radar technique was developed from the incoherent radar scatter, which is used to obtain the weak partial reflections due to incoherent scatter from free electrons in the ionosphere. The MST radar technique is capable of obtaining strong and useful echo returns over the height range of 1-100 km. Figure 2.1 illustrates a good example of relative echo powers for various MST radar facilities \[12\]. As can be seen from
the figure below ~ 50 km, echoes arise primarily from the fluctuations in the refractive index due to small scale turbulence. Water vapour becomes the most important factor in the lower troposphere and atmospheric density dominates up to the stratopause. In the mesosphere, the echoes arise from the neutral turbulence fluctuations which are enhanced by free electrons. At heights above ~ 80 km, HF and VHF radars can obtain reflections from incoming meteors and hence the MST radar operating in this mode is called Meteor radar. When the meteors traverse the atmosphere, intense ionization columns are formed at the heights of the mesosphere and lower thermosphere. These ionized meteor trails scatter radio waves, before they diffuse. Observations of meteors allow estimation of neutral wind velocities by their bulk motion.

The MF radars can be used to study the partial reflections in the D region. In this case the radio waves are coherently backscattered from refractive index variations caused by a mixture of turbulence and wave motion. As the ionization is strongly coupled to the neutral air, measurements of scatter motions can be associated with motion of the neutrals. These neutral winds can be estimated and in this thesis the neutral winds are collected by using such partial reflections.
Figure 2.1: Relative echo power profiles for various MST radar facilities
[after Gage and Balsley (1984)] [12]

Radars, including MF and meteor systems, have been widely used for the
determination of mean winds, tides and the creation of seasonal and global
climatology. Both types of systems provided mean winds for CIRA 1986 and
results were inherently consistent in magnitude and directions for heights of 80-
100 km at many latitudes. Comparison with the satellite data in CIRA 1986
demonstrates good agreement below 80 km, but there was inadequacy in the
satellite data above 80 km [13]. This has been addressed by Hedin et al. (1996)
[14] in the semi-empirical horizontal wind model (HWM-93). Regarding tides, a
summary of climatology from meteor and MF radars with model comparisons was the focus of the special issue of the Journal of Atmospheric and Terrestrial Physics in July/August 1989. Such observations [15, 16] demonstrated good agreement between radar types at similar latitudes. Models, particularly of the semidiurnal tide [17] were in good agreement at mid-latitudes. Various radar measurements of the middle atmosphere have clarified the characteristics of planetary wave [18, 19] and gravity waves [2, 20]. For these reasons the MF radar measurements being used by the community have been highly regarded and widely used.

Several different techniques are applied in radar measurements of atmospheric winds in the middle atmosphere. The two ground based techniques which can directly measure the neutral winds on the continuous basis include the Doppler method [21] and the spaced antenna method [22, 1, 23 and 24]. Here, first the Doppler method will be briefly described and then some time is spent on describing the spaced antenna method as this technique was applied to the MF radars for collecting the wind data in the present work.

### 2.3 Radar scattering and reflection mechanisms

Radio waves can be backscattered from the atmosphere due to variations of the refractive index of the air [25]. The refractive index depends on the amount of bound electrons as well as the amount of free electrons in the air. The free electrons are produced along with positive ions by the ionization of a number of atmospheric constituents (including molecular oxygen, molecular nitrogen and nitrogen monoxide) by high energy solar radiation and cosmic rays.

The ionosphere is the region of the atmosphere that contains sufficient electron densities that the propagation of radio waves will be affected. A common way of classifying the ionosphere is by the peaks in the vertical profile of electron density. This results in regions, with ascending height termed the D, E, F₁ and F₂ regions [26]. The D region corresponds approximately with the mesosphere and lies between about 60-90 km. The ionization in this region is
formed mostly by the ionization of nitric oxide by the solar Lyman-α hydrogen emission line.

The concentration of electrons varies spatially throughout the ionosphere depending on height, season, time of day and other factors such as the amount of solar activity. During the polar winter, when solar ionization is absent, electron precipitation allows radio echoes to occur. The electron concentrations are significantly higher during the day than at night because of the absence of solar radiation at night combined with the recombination of the electrons and positive ions.

The ionosonde was the first form of radar used for atmospheric research. Ionosondes operate by sweeping through a range of frequencies in the high frequency (HF) range of the electromagnetic spectrum. Since different frequencies reflect from different altitudes as the atmospheric electron density changes, a vertical profile of electron densities, known as an ionogram, can be obtained.

Medium frequency (MF) radars operate in the frequency range 1-3 MHz. Radio waves of this frequency are coherently backscattered from sharp gradients in the refractive index caused by turbulence and small-scale wave motion or thin sheets of laminar flow in the D-region of the ionosphere.

Above about 50 km, solar radiation leads to an increase in free electron density with height. Below about 100 km there is a high collision frequency between the ionized particles and the neutral atmosphere causing the ionized particles to be strongly coupled to the motion of the neutral atmosphere. This allows the neutral wind to be estimated from the partial reflections of the MF radio waves, and is the essence of how MF radars can measure wind speeds.

There are other varieties of atmospheric radars which operates in different frequency ranges. Radars that operate in the very high frequency (VHF) range and the ultra high frequency (UHF) range derive their measurements from weak backscattered signals arising from refractive index fluctuations [11]. Radars of this variety are commonly known as MST radars because they have height coverage in the mesosphere, stratosphere and troposphere.
2.3.1 Refractive index

Gage and Balsley [1980] [25] has given a formula on refractive index for radio waves propagating through the lower and middle atmosphere as

\[ n - 1 = 3.73 \times 10^{-1} \frac{e}{T^2} + 77.6 \times 10^{-6} \frac{P}{T} - \frac{N_e}{2N_c} \]  

(2.1)

where \( e \) is the partial pressure of water vapour in mb, \( T \) is the absolute temperature, \( P \) is the atmospheric pressure in mb, \( N_e \) is the number density of free electrons and \( N_c \) is the critical plasma density. The first term is the contribution due to water vapour, and is dominant in the troposphere due to the presence of relatively large amount of water vapour. The second term is due to dry air and thus dominates above the tropopause. The third term represents neutral turbulence induced electron density fluctuations and is dominant at ionospheric heights (above 50 km).

The scattering/reflection mechanisms responsible for the radar signal return from the atmosphere is broadly classified into coherent scatter and incoherent scatter.

Coherent scatter results from macroscopic fluctuations in refractive index associated with clear air turbulence. In this type of scatter, the echo power depends on the density gradients associated with the coherent scatters. There are three types of coherent scatter, namely, Bragg scatter, Fresnel reflection and Fresnel scatter that are depicted in figure 2.2. We will briefly consider each of these in turn.
Figure 2.2: Artist’s conception of atmospheric refractivity structure pertinent to Bragg scatter (A), Fresnel scatter (B), and Fresnel reflection (C) [27].

1. Bragg scatter

Bragg scatter or turbulent scatter is generally a coherent scattering phenomenon that results from variations in the refractive index with a scale projected along the line of sight of the radar equal to one-half the radar wavelength. Bragg scatter can be isotropic i.e. without causing a radar aspect sensitivity. The term aspect sensitivity is typically used to describe the decrease of radar return signal power with increasing beam pointing zenith angle; there can also be a smaller variation of radar return signal power as a function of beam
azimuth angle. Small aspect sensitivity means that the scatterers are specular with the backscattered power falling rapidly with zenith angle [28]. This occurs when the turbulent irregularities of refractive index are randomly distributed in the vertical direction but exhibit coherency in the horizontal direction. Bragg scatter can be anisotropic, causing the aspect sensitivity, if the statistical properties of the irregularities, namely their correlation distances, are dependent on direction. Although the aspect sensitivity is different for these two processes, the temporal variations of the radar echoes should be similar because of the randomly fluctuating irregularities. Their Doppler spectrum approximately reveals a Gaussian shape.

2. Fresnel scatter

Fresnel scattering occurs from many closely spaced layers randomly distributed in height. In the case Bragg scatter, the scattering scale in the horizontal are less than the size of the first Fresnel zone and many scatterers are involved. Therefore, the scatter produces tends to be vary randomly in amplitude and phase and is relatively weak. Fresnel scatter produces much stronger and more coherent scattered signals.

3. Fresnel reflection

Fresnel reflection occurs from a sharp vertical gradient in the refractive index that is horizontally coherent over a scale greater that a Fresnel zone. It is same as that of the Fresnel scattering model except that the backscattered signals are from a single extended layer of sheet. In this case the returned signal may be strong and will be quite steady in amplitude and phase over relatively long periods. Fresnel reflections are responsible for “slow fading” of radio signals backscattered from the mesosphere at medium frequencies. Radar returns from the mesosphere, particularly at long wavelengths, are generally refered to as partial reflections. However, there is observational evidence to suggest that the
radar returns are due to both Fresnel reflections from coherent electron density gradients and scattering from turbulent irregularities.

Incoherent scatter is also known as Thomason scatter, or thermal scatter. This type of scatter occurs when the incident electromagnetic field interacts directly with free electrons, which are forced to oscillate by the electric field component. These oscillating electrons re-radiate at a frequency similar to the incident radiation. If the wavelength of the incident radiation is small enough, then the reradiated signal is phase-incoherent. Since the back scattered radar signal arises from small electron density fluctuations due to random thermal motions of the ions and electrons, it has to rely on the Thomson scattering cross section of the electron and hence it is extremely weak. As a consequence of this, incoherent scatter radars must possess relatively large power-aperture products in order to observe these backscattered signals.

2.4 Doppler method

The Doppler method uses a large (often steerable) antenna to form a narrow beam, and then directs a large fraction of the available transmitter power along this “pencil beam”. The radio waves scatter from irregularities within the beam, and are scattered to a receiving array which collects the signal. In many cases the receiver array is the same as the transmitter array and this arrangement is called monostatic radar. Figure 2.3 demonstrates schematically this technique. The returning signal is Doppler shifted relative to those transmitted, due to the motion of the scatterer, and the radar then measures this Doppler shift. Typical values for the Doppler shift are in range 0.01-10 Hz. The value of the Doppler shift is then is used to determine the radial velocity of the scatterer within the beam, and by using combinations of the radial velocities obtained with the different beam pointing directions, it is possible to make measurements of the nett motion of the atmosphere. For a constant zenith angle and uniform wind field, the radial velocities vary sinusoidally with the azimuth angle. The Fourier analysis of the radial velocities yields the eastward wind and the northward wind from the Fourier coefficients.
Figure 2.3: Schematic diagram demonstrating the main principles of the Doppler method. The Doppler shift of the returning radiation relative to that transmitted is indicated, and this shift is the basis of this method [29].

2.5 Spaced Antenna method

Earlier, a crude method was designed to calculate the wind vector from time series sampled from spaced antennae [22]. This method was known as “method of similar fades”. Mitra [1949] [22] used this technique to measure the
ionospheric drifts from the amplitude fadings of the echoes reflected from the ionosphere. This method is based on the assumption that the features of the fading recorded at the three antennas are similar but displaced in time with respect to each other. From the time delays, one can easily determine the velocity of the fading patterns over ground, which is twice the velocity of the irregularities of the ionosphere. However, the pattern may changes as it moves and add to the signal variation and the method described above is not applicable.

Briggs et al. [1950] [30] developed a method called ‘full correlation analysis’ (FCA) that take into account the random change in the diffraction pattern. The analysis calculated the auto- and cross-correlation functions for the three fading records, obtained from the three receiving antennas, at different time lags to extract the background wind vector. This wind vector obtained without considering random changes in the pattern is called apparent velocity and is, in general, an over-estimate of the background wind velocity. It is corrected for random changes in the diffraction pattern to obtain true velocity.

Originally developed in 1950’s, the space antenna (SA) method, used in conjunction with MF radars, has proved to be an important and relatively inexpensive technique for making measurements of atmospheric wind velocities and other parameters. Phillips and Spenser [1955] [31] extended the analysis to anisotropic case to find the size and shape of the pattern as well as the velocity parameters corrected for the anisotropy effects. It was originally used for total reflection experiments, subsequently, modified for D-region work using partial reflections [32, 33] and still later used for tropospheric and stratospheric measurements with VHF radars [34, 35]. The SA method generally utilizes different antenna for transmission and reception and determines the drift velocities of the ionised irregularities that partially reflects the radar signal in the D-region depending on time and season. Measurements are limited to the middle atmosphere and lower thermosphere where weakly ionized turbulent structures can be expected to move with neutral motions due to the high collision frequency. Many discussions of the full correlation analysis in its current form
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...and some of the computational difficulties involved in its implementation are available in the literature [36, 1, 23].

In this analysis, a moving pattern over spatially arranged three sensors at the vortices of an equilateral triangle is generally represented by a family of ellipsoids of the forms

\[
\rho(\xi, \eta, \tau) = \rho(A(\xi - V_x \tau)^2 + B(\eta - V_y \tau)^2 + 2H(\xi - V_x \tau)(\eta - V_y \tau)) + K \tau^2
\] (2.2)

\[
\rho(\xi, \eta, \tau) = \rho(A\xi^2 + B\eta^2 + C\tau^2 + 2F\xi\eta + 2G\eta\tau + 2H\xi\eta)
\] (2.3)

where \( \rho(\xi, \eta, \tau) \) is the cross-correlation function, which is a function of \( \xi \), \( \eta \), spatial separation between any two receivers in x and y-directions respectively and \( \tau \), the time shift with the two receivers. \( V_x \) and \( V_y \) are the component velocities of the moving pattern with respect to a stationary observer. The constants \( A, B, C, F, G \) and \( H \) fully describe the size, shape of the pattern and the velocities. Hence the evaluation of these constants provides the velocity and anisotropy parameters.

Maximum correlation occurs when \( \frac{\partial \rho}{\partial \tau} = 0 \) and the corresponding time shift

\[
\tau = -(F/C)\xi - (G/C)\eta
\] (2.4)

It is assumed that the cross-correlation between two receivers at zero time shift is equal to the auto-correlation function of the receivers. Mathematically,

\[
\rho(0, 0, \tau) = \rho(\xi, \eta, 0)
\] (2.5)

Applying the above equation in equation (2.2), we find

\[
\rho(C\tau^2) = \rho(A\xi^2 + B\eta^2 + 2H\xi\eta),
\] (2.6)

and thus

\[
\tau^2 = (A/C)\xi^2 + (B/C)\eta^2 + (2H/C)\xi\eta
\] (2.7)
The above equation (2.7) is for a pair of sensors and it gives the time shift at which the mean auto correlation function (averaged over all sensors) has the value equal to the value of cross-correlation between the pair of sensors at zero time shift. Likewise, we get three equations for the three pair of sensors. The coefficients $A/C$, $B/C$ and $H/C$ can be estimated simultaneously and will be in general be over-estimated and by using least squares technique the optimum values and errors can be determined.

By equating the coefficients of $\xi\eta$ and $\eta\tau$ in equations (2.1) and (2.2), we obtain,

$$AV_x + HV_y = -F, \quad BV_y + HV_x = -G. \quad (2.8)$$

These equations can be solved simultaneously to obtain the components of true velocity $V_x$ and $V_y$. Then the magnitude and direction of the true velocity are given by

$$|V|^2 = V_x^2 + V_y^2, \quad (2.9)$$

$$\tan \phi = \frac{V_x}{V_y} \quad (2.10)$$

The spatial properties of the pattern itself can be determined using spatial correlation function,

$$\rho(\xi, \eta, 0) = \rho(A\xi^2 + B\eta^2 + 2H\xi \eta), \quad (2.11)$$

They are usually specified by giving the values of the minor axis of the characteristics ellipse (A particular ellipse for which, $\rho(\xi, \eta, 0) = 0.5$), the axial ratio, and the orientation of the major axis, measured clockwise from north.

The orientation $\theta$ of the major axis is given by

$$\tan 2\theta = 2H/(B-A), \text{ clockwise from the } y\text{-axis} \quad (2.12)$$
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The major and minor axes are respectively,

\[ a = \frac{2C\tau_{0.5}^2}{[(A+B) - \sqrt{(4H^2 + (A-B)^2)}]} \]  \hspace{1cm} (2.13)

\[ b = \frac{2C\tau_{0.5}^2}{[(A+B) + \sqrt{(4H^2 + (A-B)^2)}]} \]  \hspace{1cm} (2.14)

The axial ratio \( r \) is given by the ratio of the major to the minor axes.

The random changes of the moving pattern are described by a parameter called ‘mean lifetime’ or ‘time-scale’. This is the mean lifetime of the irregularities which cause the diffraction pattern to change randomly and is often taken to be the time lag at which the auto-correlation function falls to 0.5. Mathematically,

\[ T_{0.5}^2 = \tau_{0.5}^2 \frac{C}{K} \]  \hspace{1cm} (2.15)

where \( \tau_{0.5} \) is the time lag for which the directly observed auto-correlation function for a fixed observer falls to 0.5. To find the ratio \( C/K \), the coefficient of \( \tau^2 \) in equation (2.2) and (2.3) can be equated to obtain

\[ C = AV_x^2 + BV_y^2 + K + 2HV_xV_y \]  \hspace{1cm} (2.16)

Since \( V_x \) and \( V_y \), and the ratios \( A/C, B/C, H/C \), the above equation (2.16) can be used to find \( C/K \).

Some data are rejected, when the assumptions of the analysis are not valid [1]. The following criteria are found satisfactory for the spaced antenna wind analysis.

1. The mean signal is either very weak or so strong that the receivers are saturated most of the time.
2. The fading is very shallow (signal almost constant). The standard deviation should be at least 2% of the mean signal.
3. The signal to noise ratio is less than 6 dB.
4. The mean auto-correlation function has not fallen to at least 0.5 for the maximum number of lags, which have been computed.
5. The cross-correlation functions have no maxima within the number of lags which have been computed.

6. The cross-correlation functions are oscillatory, so that it is impossible to identify the correct maxima.

7. The sum of the time displacements $\tau_{ij}'$ does not sum to zero, as it ideally should, for the data taken in pairs around the triangle of antenna. This condition may be relaxed somewhat to give a more practical criterion: if $|\sum \tau_{ij}'|/|\sum |\tau_{ij}'| > 0.2$ the results are rejected. This quantity has been called the “normalized time discrepancy”.

8. The computed value of $V_c^2$ is negative, so that $V_c$ is imaginary. This criterion should not be applied too rigidly, because if the computed $V_c^2$ is only slightly negative, this probably means that the true value is zero, and the small negative value has arisen from statistical fluctuations. Such data may therefore be particularly good, indicating pure drift with negligible random changes.

9. The computed coefficients indicate hyperbolic rather than elliptical contours, i.e. data will be rejected, if $H^2 < AB$.

10. The polynomial interpolation procedures break down at any stage.

11. It is advisable to reject any results for which the correlations of the full correlation analysis are very large, i.e. those for which the apparent or true velocities differ greatly in magnitude and or direction. Suitable criteria are: the data will be rejected if $0.5 V_t \geq V_a \geq 3V_t$ or $|\phi_t - \phi_a| \geq 40^\circ$.

### 2.6 Checking the Polarization

Here brief description is given about which kind of polarization is to be used for the radar transmission and reception. Since for this work we have used data from two radar sites, one being equatorial site and another from non-equatorial, it is important to know about the polarization method used for the transmission and reception of the signal.
In normal running O (Ordinary) mode transmission and reception will be used during the day and E (Extraordinary) mode transmission and reception will be used during the night.

When the radar system is first installed it is necessary first to empirically determine O and E mode transmission and then to empirically determine O and E mode reception. The procedure which follows must be performed during daytime. During the day, at heights above 70 km, absorption of the transmitted signal is greatest for the E mode of propagation. This is caused by the D-region of the ionosphere extending to lower heights during the day. The received signals must be examined carefully to check which polarization is being transmitted and received by the equipment. If necessary, the polarization controls for transmission and/or reception must be adjusted.

2.6.1 Non-Equatorial Sites

Non-equatorial sites use a circularly-polarised system. For transmission, each pair of parallel antenna elements is driven in-phase. A $90^\circ$ phase delay is introduced to the signal driving the orthogonal pair of transmitter antenna elements. To change the sense of the transmitted circularly-polarised signal, the $90^\circ$ phase delay is applied to the other pair of parallel antenna elements by means of a relay.

In reception, two orthogonal antenna elements are attached to each receiver. Once again, a $90^\circ$ phase delay is introduced to the signal from one antenna. To change reception mode, a model switch relay is used to switch the phase delay from one antenna element to the other.

2.6.2 Equatorial Sites

Equatorial sites use a linealy-polarised system. During transmission only one parallel pair of antenna elements is used. To
change transmission mode, a model switch relay switches transmission from one pair of parallel antennas to the other.

During reception only antenna of each of the orthogonal antenna pairs feed each receiver. To change reception mode, a mode switch relay is used to switch from one antenna to the other.

### 2.7 About the radar

Both the radar systems are similar to the system installed at Christmas Island [37]. The schematic diagram of the MF radar system is shown in **figure 2.4**. Both the radar systems have been installed by Indian Institute of Geomagnetism. The MF radar at Tirunelveli has been installed in the year 1992 and MF radar at Kolhapur have been installed in the year 1999. It consists of three sections.

#### 2.7.1 The transmitting and receiving antennas

The system details, mode of operations and method of wind estimation for both radar systems are the same as that installed at Christmas Island [37]. The transmitting antenna array is arranged in a square, and consists of four centre-fed half-wave dipoles, approximately 75 m in length. As discussed in above section, for the transmission and reception of the signal, radar system at Tirunelveli being an equatorial site use linear polarization while radar system at Kolhapur which is non-equatorial site use circular polarization. The O-mode dipoles are used for daytime transmission while the E-mode dipoles are used at nighttime. This geometrical arrangement also applies to the three receiving antennas. The ideal height of the transmitting antennas is one quarter of the transmitting wavelength above the ground plane, which for MF radar operating at 1.98 MHz is about 30 m.

The receiving antennas are of the inverted-V type, and are situated at the vertices of an equilateral triangle whose basic spacing is 180 m.
The centroids of both the transmitting and receiving arrays coincide, making the radar system a monostatic. The impedance of each receiving antenna is 50 Ω to match the coaxial cable that takes the signal to the receiving system.

![Figure 2.4: Schematic diagram of the MF radar system at Tirunelveli showing the geometry of the transmitting and receiving antennas.](image)

**2.7.2 The transmitter, receiving and data acquisition system, and computer controller**

The transmitter, the receiving and data acquisition system (RDAS) and the computer are the main components of the radar system. The transmitting system uses totally solid-state transmitters. It consists of a
combiner unit, 10 power amplifier modules, a fan unit, a driver unit and a power supply unit. The transmitter operates at a frequency of 1.98 MHz and is fully phase coherent. The transmitter power is 25 kW, with a pulse length of 30 µs which corresponds to a 4.5 km height resolution. The pulse repetition frequency is 80 Hz with 32 point coherent integration during daytime. This is reduced to 40 Hz during nighttime with 16 point integration, to avoid problems arising from multiple reflections from equatorial spread F. Table 2.1 lists some of the characteristics of the transmitting system.

The specifications of RDAS operating at Tirunelveli are listed in table 2.2. The receiving system is controlled by a pc-based microprocessor, which both controls the transmitter and acquires data. The data are subsequently transferred to a host personal computer for analysis and subsequent storage. The 1.98 MHz transmitter pulses are derived from the master oscillator and frequency synthesis module. The transmitted signal is linearly polarized. The three receivers are of superheterodyne type. In each receiver, the received 1.98 MHz signal is mixed with a 2.475 MHz local oscillator to produce a 495 kHz intermediate frequency (IF), which is then fed to the signal processor modules. The signal processors are phase sensitive: the 495 kHz IF signals are heterodyned with in-phase and quadrature local oscillators to produce in-phase and quadrature components, which are then digitized to 8-bit resolution.
### Table 2.1: Characteristics of transmitting system of MF radar

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Transmit power</td>
<td>25 kW RMS</td>
</tr>
<tr>
<td>Maximum duty cycle</td>
<td>0.25 %</td>
</tr>
<tr>
<td>Operating frequency</td>
<td>1.98 MHz</td>
</tr>
<tr>
<td>Half power pulse width</td>
<td>30 µs</td>
</tr>
<tr>
<td>Pulse rise and fall times</td>
<td>15 µs</td>
</tr>
<tr>
<td>Output impedance</td>
<td>50 Ω unbalanced 30 kHz</td>
</tr>
<tr>
<td>Half power bandwidth</td>
<td>30 kHz</td>
</tr>
<tr>
<td>Harmonics (third, fifth and even harmonics)</td>
<td>-63 dBc, &gt;-70dBc, &gt;-70dBc</td>
</tr>
<tr>
<td>Load</td>
<td>An array of four 75 Ω dipoles</td>
</tr>
<tr>
<td>Overall efficiency</td>
<td>~ 70 %</td>
</tr>
<tr>
<td>Polarisation</td>
<td>Linear/ circular</td>
</tr>
<tr>
<td>Height of transmitter towers</td>
<td>30 m</td>
</tr>
</tbody>
</table>

### Table 2.2: Characteristics of receiving system of MF radar

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of receivers</td>
<td>3</td>
</tr>
<tr>
<td>Sampling starting height</td>
<td>60-98 km (Day), 70-98 km (Night)</td>
</tr>
<tr>
<td>Sampling interval</td>
<td>2 km</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>80 Hz (Day), 40 Hz (Night)</td>
</tr>
<tr>
<td>Coherent integrations</td>
<td>32 (Day), 16 (Night)</td>
</tr>
<tr>
<td>Samples per data set</td>
<td>256</td>
</tr>
<tr>
<td>Sampling period</td>
<td>2 minutes</td>
</tr>
<tr>
<td>Centre frequency</td>
<td>1.98 MHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>~ 39 kHz</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>~ 0.3 µV</td>
</tr>
<tr>
<td>Maximum gain</td>
<td>~ 10^5</td>
</tr>
<tr>
<td>Gain control</td>
<td>Programmable in steps of 1 dB</td>
</tr>
<tr>
<td>Gain control range</td>
<td>60 dB</td>
</tr>
<tr>
<td>Input impedance</td>
<td>50 Ω</td>
</tr>
</tbody>
</table>
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The RDAS acquires and stores in its memory a complete data set. A data set comprises 256 points of 20 consecutive height samples at 2 km intervals, thus providing a 40 km range coverage. Though the radar samples at every 2 km, the radar pulse length of about 30 µs means, however, that the actual height of resolution is about 4-5 km. Each data sample is the result of integrating the digitized data over a number consecutive transmitter pulses. The transmitter pulses are coherently averaged to produce a mean data point for every 0.4 seconds. A complete data set is therefore acquired in 102.4 seconds. It is then transferred to the host computer. The coherent integration is done to improve signal-to-noise ratio, which improves the performance of the system particularly during nighttime, when the ionization is low. The other radar system details are given in the tables 2.1 and 2.2.

The recording information is programmable by the host computer. It includes the number of heights per sample, transmitter pulse repetition frequency, integrations \((T_x \text{ pulses})\) per sample point, sample points per data set and receiver gain. Different recording configurations are used for daytime and nighttime. The receiver gains are dynamically adjusted by the analysis program before the start of accumulation of each data set.

2.7.3 Computer and analysis software

After each data acquisition run, the computer performs a full correlation analysis on the 256 point complex data set to determine mean winds and various other parameters, namely, pattern decay time, pattern axial ratio, signal-to-noise ratio and the received power antenna. The system runs continuously for all the days. However the data acquisition is interrupted while taking data backup and during power failure. The data backup is normally done for every fortnight.

2.8 Data Analysis Techniques

The remainder of this chapter relies on obtaining meaningful and statistical significant information of coherent oscillations present in the time
series of the radar wind measurements. To accomplish this, a background study of mathematical techniques coupled with basic derivations, necessary for understanding and investigating the time series, is presented. The results from this analysis provides information on the probability of the existence of coherent oscillations in a given data sample. This information is then used in interpreting the source, properties and behaviour of these oscillations.

2.8.1 Harmonic Analysis

This method uses harmonic analysis, which is generally applied to a discrete time series. Usually a function containing a constant term plus a number of sinusoidal periodic terms at the frequencies where the main variance is thought to be contained is fitted to that data using least square method, which is described below.

In general, it is possible to fit a continuous physical process \( y(t) \) with a function of the form (James, 1995),

\[
T = N
\]  

More formally:

\[
y(t_n) = y_n = \sum_{k=1}^{k=N} [a_k \cos(2\pi k T^{-1}t_n) + b_k \sin(2\pi k T^{-1}t_n)], k = 0, 1, 2... \tag{2.18}
\]

where, \( y(t_n) = y_n \) represents the discrete time samples of \( y(t) \).

By least square methods, the general best fit (for any of the \( k \) coefficients) is given when \( \epsilon^2 \) is a minimum [38], where

\[
\langle \epsilon^2 \rangle = \sum_{n=1}^{n=N} [y_n - a_k \cos(2\pi k T^{-1}t_n) - b_k \sin(2\pi k T^{-1}t_n)]^2 \tag{2.19}
\]

and

\[
\frac{\partial \langle \epsilon^2 \rangle}{\partial a_k} = \frac{\partial \langle \epsilon^2 \rangle}{\partial b_k} = 0 \tag{2.20}
\]
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The solution is straightforward, the results being:

\[ f(t) = a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_2 \cos(2\omega t) + b_2 \sin(2\omega t) + a_3 \cos(3\omega t) + b_3 \sin(3\omega t) \]  

\[ (2.21) \]

\[
\begin{align*}
b_k &= \frac{\sum_{n=1}^{N} y_n \sin(2\pi k T^{-1} t_n)}{\sum_{n=1}^{N} \sin^2(2\pi k T^{-1} t_n)} - \frac{ak}{\sum_{n=1}^{N} \sin^2(2\pi k T^{-1} t_n)} \\
ak &= \frac{\sum_{n=1}^{N} y_n \cos(2\pi k T^{-1} t_n)}{\sum_{n=1}^{N} \cos^2(2\pi k T^{-1} t_n)} = 2T^{-1} \sum_{n=1}^{N} y_n \cos(2\pi k T^{-1} t_n) \\
b_k &= \frac{\sum_{n=1}^{N} y_n \sin(2\pi k T^{-1} t_n)}{\sum_{n=1}^{N} \sin^2(2\pi k T^{-1} t_n)} = 2T^{-1} \sum_{n=1}^{N} y_n \sin(2\pi k T^{-1} t_n)
\end{align*}
\]

\[ (2.22) \]

When the length of the function \( y(t) \) is an integer multiple of the period \( T/k \), for \( k = 1,2,3,\ldots \), and the \( t_n \) are equally spaced, i.e. when \( t_n = \Delta t \times n \) where \( \Delta t = TM^{-1} \), \( M = 1,2,\ldots, \frac{N}{2}, n = 1,2,\ldots, N \), and \( T = N \), then the orthogonality rules apply, and the equations 2.21 and 2.22 become

\[
\begin{align*}
ak &= \frac{\sum_{n=1}^{N} y_n \cos(2\pi k T^{-1} t_n)}{\sum_{n=1}^{N} \cos^2(2\pi k T^{-1} t_n)} = 2T^{-1} \sum_{n=1}^{N} y_n \cos(2\pi k T^{-1} t_n) \\
b_k &= \frac{\sum_{n=1}^{N} y_n \sin(2\pi k T^{-1} t_n)}{\sum_{n=1}^{N} \sin^2(2\pi k T^{-1} t_n)} = 2T^{-1} \sum_{n=1}^{N} y_n \sin(2\pi k T^{-1} t_n)
\end{align*}
\]

\[ (2.23) \]

\[ (2.24) \]

Note however, that \( a_0 \) and \( b_0 \) are special cases, where

\[
a_0 = T - 1 \sum_{n=1}^{N} y_n ; \quad b_0 = 0.
\]

\[ (2.25) \]
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The least square fits were made for a constant wind $a_0$ and solar tidal term with periods of 48, 24, 12, hours according to the equation

$$f(t) = a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_2 \cos(2\omega t) + b_2 \sin(2\omega t) + a_3 \cos(3\omega t) + b_3 \sin(3\omega t)$$

(2.26)

with $k = 1, 2, 3$ and $\omega = \frac{2\pi}{T} = 2\pi f$ where $f = 1$ cycle per day (cpd).

2.8.2 Spectral analysis

A physical process can be described either in the time domain or in the frequency domain. One representation can be written into other by means of Fourier transform.

The Fourier transform may be written in discrete form as

$$C_k = \sum c_j \exp(2\pi ikj / N), \ldots \ldots k = 0 \text{ to } N - 1.$$  

(2.27)

The discrete Fourier transform maps $N$ complex numbers (the $c_j$’s) into $N$ complex numbers (the $c_k$’s). The computation of DFT described above involves $N^2$ complex computations. It can be computed in $N \log_2 N$, which is much less than $N^2$ operations with an algorithm called the fast Fourier transform [39].

The Fourier coefficients obtained by using FFT/DFT are less then used to estimate the power spectrum, which is the energy content of the signal at particular frequency. The power spectrum curve shows how the variance of the process is distributed with frequency. The variance contributed by frequencies in the range $f$ to $f + \delta f$ is given by the area under the power spectrum curve between the two ordinates $f$ and $f + \delta f$. The periodogram estimate of the power spectrum is given by

$$P(f_k) = 1/N^2 \{C_k^2 + C_{N-k}^2\}, \ldots \ldots k = 1, 2, \ldots, (N/2-1)$$

(2.28)

Where $f_k = \frac{k}{N\Delta}$.
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The power spectral density (PSD) is given by

\[ P(k) = N \Delta P(f_k) = (\Delta / N) \{ C_k^2 + C_{N-k}^2 \} \]  (2.29)

Amplitude \( R_k \) = \( \sqrt{P(k)} = \sqrt{\{(\Delta / N) \{ C_k^2 + C_{N-k}^2 \} \}} \)  (2.30)

To test significance of the amplitude \( R_k \), Nowroozi [1967] [40] has given a method in which the probability \( P \) that the ratio \( R_k^2 / \sum R_i^2 \) (summation runs from \( k=1 \) to \( m \)) exceeds a parameter ‘g’ is given by

\[ P = m(1-g)^{m-1} - m(m-1)(1-2g)^{m-1} / 2 + \ldots + (-1)^m m! (1-Lg)^{m-1} / (L!(m-L)!) \]  (2.31)

It can be shown that \( \sum R_i^2 = 2/(2m+1) \{ \sum (X_i - X_{\text{mean}})^2 \} \).

It is, therefore, not necessary to calculate all the harmonics. The error introduced in neglecting the higher order term is only 0.1% for \( p=0.05 \) (95% confidence level). Therefore, the parameter \( g \) can be calculated only from \( P=m(1-g)^{m-1} \). The value of \( g \) for different values of \( p \) and \( m \) is determined. The parameter \( g_k \) is given by

\[ g_k = R_k^2 / \{(2 / N) \sum (X_i - X_{\text{mean}})^2 \} \]  (2.32)

If \( g_k > g_{p=0.05} \) (for 95% confidence level), the amplitude is 95% significant.

2.8.3 Digital filtering

To extract the signal lying only in a certain frequency band, the band-pass filter is used. Filtering is more convenient in the frequency domain. The whole data record is subjected to FFT. The FFT output is multiplied by a filter function \( H(f) \). Inverse FFT is taken to get the filtered data set in time domain.
The nonrecursive or finite impulse response (FIR) filter is given by

\[
H(f) = \sum_{k=0}^{M} c_k \exp(-2\pi ikf) \tag{2.33}
\]

Here, the filter response function is just a discrete Fourier transform. The transform is easily invertible, giving the desired small number of \( c_k \) coefficients in terms of the same small number of values of \( H(f) \) at some discrete frequencies \( f \). However, this fact is not very useful, as \( H(f) \) will tend to oscillate wildly between the discrete frequencies where it is pinned down to specific values. Hence, the following procedure of [41] is adopted.

I. A relatively large value of \( M \) is chosen.

II. The \( M \) coefficients \( c_k, k=0,\ldots, M-1 \) can be found by an FFT.

III. Most of the \( c_k \)'s are set to zero except only the first \( k(c_0, c_1, \ldots, c_{K-1}) \) and last \( K-1(c_{M-K}, \ldots, c_{M-1}) \).

IV. As the last few \( c_k \)'s are filter coefficients at negative lag, because of the wraparound property of the FFT, the array of \( c_k \)'s are cyclically shifted to bring everything to positive lag. The coefficients will be in the following order \( (c_{M-K+1}, \ldots, c_{M-1}, c_0, c_1, \ldots, c_{K-1}, 0, 0, \ldots, 0) \) \( \tag{2.34} \)

V. The FFT of the array will give an approximation to the original \( H(f) \). If the new filter function is acceptable, then we will have a set of 2\( K-1 \) filter coefficients. If it is not acceptable, either \( K \) is increased and the same procedure is repeated or the magnitudes of the unacceptable \( H(f) \) are modified to bring it more in line with the original \( H(f) \), and then FFT is taken to get new \( c_k \)'s.

VI. Now, all coefficients, except the first 2\( K-1 \) values are set to zero. Inverse transform is taken to get a new \( H(f) \).
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Estimation of the characteristics of planetary-scale waves

Unlike atmospheric tides, planetary-scale waves do not have fixed period. They exhibit a range of periods and these periods may vary with height as well as time. To compute the characteristics of these waves (the amplitude, phase and the period), the following procedure is adopted. The hourly averaged wind data are subjected to the time-domain filtering with finite impulse response (FIR) filters [41] to get filtered data consisting of the particular planetary wave periods only. The filtered data are then subjected to harmonic analysis with period varied in the range of expected periods of concerned planetary wave. The wave parameters are determined in least square sense.

2.8.4 Bispectral analysis

Power spectrum gives only the power of each spectral component and the information about the phase relation between different spectral components is suppressed. But, the bispectrum which is a higher order spectrum, is an ensemble average of the product of three spectral components and can be used to determine whether there is any phase relation between the three spectral components. If each spectral component is independent of other components, then the amplitude and phase of the component will be different from those of other components and the bispectrum will give a nearly zero value. If the phase associated with the three components sum to the same constant each time they occur, then the bispectrum will give a non-zero value [42].

In the present analysis, the monthly data (720 points) are divided into ‘K’ segments containing ‘M’ samples (256 data points). 75% overlap between the segments is ensured to get a large value for K. The mean of each segment is calculated and removed. The DFT of each segment is computed. The bispectrum of $i^{th}$ segment is given by

\[ B_i(f_1, f_2) = X_i(f_1)X_i(f_2)X_i(f_1 + f_2) \]  

(2.35)
where $X_i$ is the Fourier Transform of the $i^{th}$ segment.

The bispectral estimates are averaged across all segments. Then, they are normalized by dividing each value by the maximum value, resulting in a relative amplitude ranging from zero to one.

The bispectrum graphs are generally plotted with smaller frequencies along y-axis and next higher frequencies along x-axis. It will only be non-zero at locations $(f_1, f_2)$ where $f_3 = f_1 \pm f_2$ and $\phi_3 = \phi_1 \pm \phi_2$, where $\phi_i$ denotes the phase of the $i^{th}$ component. Nonlinear interactions not only yield sum and difference frequencies, but also phase relations, which are of the same form as frequency relations. Such a phase relation is termed as quadratic phase coupling. The non-zero value of bispectrum is the result of quadratic phase coupling indicating non-linear interaction. If $\phi_3$ is random and independent of $\phi_1$ and $\phi_2$, then the bispectrum will be zero revealing that the phases are not related.

The interpretation of peaks in bispectral plots is given in [43]. As a simple case, the presence of two frequencies and their sum denotes existence of quadratic phase coupling (i.e., non-linear mixing). If the bispectrum shows large value (peak) at any x-y point, it can be interpreted as a response to quadratic phase coupling between x-frequency, y-frequency and the sum frequency. The x- and y-coordinated (bifrequencies) account for two of the frequencies and the sum frequency is represented by a diagonal through the peak, which intersects x- and y-axes at the same value. The initial two mixing components could be at any two of the three frequencies.

2.8.5 Singular spectral analysis

The power spectrum obtained using FFT gives only an average contribution from a specific oscillation to the total variance and the phase information is lost in deriving the power spectral density. Singular spectral analysis extracts principal components of the variability even when the system is non-stationary. This method generates data adaptive filters, whose transfer functions highlight regions where sharp spectral peaks occur and thus helps
reconstruction of the original time series with just a few principal components close to the spectral peaks. In contrast to the Fourier components, the principal components need not be sinusoidal in nature. Briefly SSA decomposes the original time series into its significant signal components with least noise [44].

Assume a finite time series \( y(t) \) of length \( N \).

\[
y(t) = y(Kt_s), K = 1, 2, 3, \ldots, N
\]

and \( t_s \) is sampling interval. The series is normalized using the mean \( \bar{y} \) and standard deviation \( \sigma_y \). The new series will be

\[
x(t) = (y(t) - \bar{y}) / \sigma_y, t = 1, 2, 3, \ldots, N
\]

The sampled time series is then embedded in an \( M \)-dimensional space, the consecutive sequences of \( X(t) \)

\[
Z = \begin{bmatrix}
X_1 & X_2 & \ldots & X_M \\
X_2 & X_3 & \ldots & X_{M+1} \\
& \ddots & \ddots & \ddots \\
& & \ddots & \ddots \\
& & & X_{N-M+1} & \ldots & X_N
\end{bmatrix}
\]

The matrix so derived is called the Trajectory matrix. For different choices of \( M \), different trajectory matrices can be obtained. However, \( M \) should be larger than the autocorrelation time (the lag at which the first zero occurs). The eigen values of this matrix are then evaluated in descending order of magnitude together with the corresponding eigen vectors. As the matrix is positive symmetric Toeplitz (whose all diagonal elements are equal), the eigen values will always be positive. Ideally, the number of non-zero eigen values will correspond to the number of independent variables in the system. When quasi-periodic fluctuations are present in the time series, the eigen vectors appear as
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even/odd pair in phase quadrature, with corresponding eigen values nearly in magnitude. As the PCs are filters versions of the original series with the M elements of the eigen vectors serving as appropriate filter weights, the resulting series would be of length (N-M+1). Vautard et al. [1992] [45] has given a method to extract a series of length N corresponding to a given set of eigen elements which have been called the reconstructed components (RC). The formulae for the $k^{th}$ component

$$R(X_i)^k = \frac{1}{i} \sum a_{i-j} E_j^k, \quad \text{for } 1 \leq i \leq (M-1) \text{ and } j = 1 \text{ to } M$$

$$= \frac{1}{M} \sum a_{i-j} E_j^k, \quad \text{for } M \leq i \leq (N-M+1) \text{ and } j = 1 \text{ to } M \quad (2.39)$$

$$= \frac{1}{(N-1+1)} \sum a_{i-j} E_j^k, \quad \text{for } (N-M+2) \leq i \leq N \text{ and } j = 1 \text{ to } M$$

where $E_{jk}$ are the M eigen elements of the $k^{th}$ component and

$$A_k = \sum X_{i+j} E_j^k \quad 1 \leq i \leq (N-M) \text{ and } j = 1 \text{ to } M \quad (2.40)$$

The percentage of the total variance accounted by each of the reconstructed component can be computed from the ratio of the individual eigen value to the sum of all the M eigen values which then gives rise to an immediate idea of the relative importance of a particular component to the time series.

2.8.6 Auto-and-Cross-correlation functions

The correlation between two continuous functions $g(t)$ and $h(t)$, which is denoted by corr($g,h$), and is a function of lag $t$. The correlation will be large at some value of $t$ if the first function $g(t)$ is a close copy of the second $h(t)$ but lags in time by $t$, i.e., the first function is shifted to the right of the second. Likewise, the correlation will be large for some negative value of the first function leads.
the second, i.e., is shifted to the left of the second. The relation that holds when
the two functions are interchanged is

$$\text{Corr}(g, h)(t) = \text{Corr}(h, g)(-t)$$  \hspace{1cm} (2.41)

The discrete correlation of the two sampled functions $g_k$ and $h_k$, each periodic
with time $N$, is defined by

$$\text{Corr}(g, h)_j = \sum g_{j+k} h_k$$  \hspace{1cm} (2.42)

According to the discrete correlation theorem, this discrete correlation of two
real functions $g$ and $h$ is one member of the discrete Fourier transform pair

$$\text{Corr}(g, h) \leftrightarrow G_k H_k^*$$  \hspace{1cm} (2.43)

where $G_k$ and $H_k$ are the discrete Fourier transforms of $g_j$ and $h_{j+}$, and the
asterisk denotes complex conjugate.

The practical procedure adopted to find the correlation of two data sets
using FFT is given below.

I. FFT is taken for the two data sets

II. One resulting transform is multiplied by the complex conjugate of the
other.

III. Inverse transform is taken for the product

If the input data are real, the result ($r_k$) will normally be a real vector of
length $N$. The components $r_k$ are the correlation at different lags (both positive
and negative). The correlation at zero lag is in $r_0$, the first component; the
correlation at lag 1 is in $r_1$, the second component; the correlation at lag -1 is in
$r_{N-1}$, the last component; etc
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References


