CHAPTER 4

FOUR WAVE MIXING

4.1 INTRODUCTION

FWM (Four Wave Mixing) or Four Photon Mixing (FPM) is the process whereby optical power from one channel in a multi-channel system is spilled over into an adjacent channels. Three waves mix together to produce the fourth wave which may coincide with the original channel or may not be coinciding.

\[ f_{ijk} = f_i \pm f_j \pm f_k \] (4.1)

Total FWM products = \( N^2 (N-1)/2 \). (4.2)

where \( N \) is the no. of wavelengths.

Equation (4.1) shows the formula for the generation of FWM interfering term. Equation (4.2) shows the the total no. of interfering terms.

Newly formed FWM products

\[ \downarrow \]

May fall on the original signal \hspace{1cm} May not fall on the original signal

(cannot be filtered out) \hspace{1cm} (can be filtered out)

Figure 4.1 Location of Interfering FWM Terms
Spurious components are created, causing

- Interference
- Degradation of signals
- Cross talk

If $f_1$, $f_2$, $f_3$ are the input frequencies, the nine combinations of FWM products formed are shown in the following figure. Gurmeet and Singh (2009) reviewed the FWM and developed an algorithm for studying its effect in the total system noise. For example, here the FWM product $f_{223}$ gets overlapped with the original frequency $f_1$ and it is formed by the combination shown in equation (4.3).

$$f_{223} = f_2 + f_2 - f_3 \quad (4.3)$$

**Figure 4.2. Spectrum at the output of the fiber**

FWM can be substantially reduced or perhaps completely eliminated through the following steps.
(a) Individual channel power reduction
(b) Increased dispersion (Phase Mismatch)
(c) Increased channel spacing.

In a multiwavelength system like DWDM, three waves mix together and produces the fourth wave given by Equation (4.4)

\[ P = \frac{(D\chi_{\text{eff}})^2}{(A_{\text{eff}}^2)} \frac{1024\pi^6}{n^4c^2} \frac{1}{\lambda^2} P_i P_j P_k e^{-\alpha L} \eta \]  

(4.4)

Where

\( n \) is the refractive index
\( \lambda \) & \( c \) are the wavelength and speed of the light respectively
\( P_i, P_j, P_k \) are the input powers of the three channels
\( X \) is the electric susceptibility
\( D \) is the degeneracy factor which is 3 for Two tone and 6 for Three tone mixing.
\( \eta \) is the Four wave mixing efficiency (inversely proportional to Dispersion)

The excess pulse broadening introduced by the dispersion added for reducing the FWM is compensated by the Dispersion Compensating Fiber (DCF).

**4.2 FWM FOR EQUAL AND UNEQUAL CHANNEL SPACING**

From Figure 4.3, it is inferred that when the inter channel spacing is equal means then the FWM power falls on the original signals such that it will induce crosstalk. To avoid this, unequally spaced channels are used. Fabrizio (1995) has analysed the FWM for different bandwidth expansion factors and demonstrated the reduction in overlapping of interfering FWM terms with the original channels for the unequal channel spacing. The OSNR for different Inter channel separations has been observed by Gurmeet et al (2010).
Figure 4.3 Spectrum for Equally spaced channels.


Figure 4.4 Spectrum for unequally spaced channels

The main objective is to remove the nonlinearities like FWM and SRS.

EQUAL CHANNEL SPACING

Let \( \lambda_1 = 1550 \); \( \lambda_2 = 1551 \); \( \lambda_3 = 1552 \)

\[ \lambda_1 + \lambda_3 - \lambda_2 = 1550 + 1552 - 1551 = 1551 \text{ (Coinciding with } \lambda_2) \]

\[ \lambda_2 + \lambda_2 - \lambda_1 = 1551 + 1551 - 1550 = 1552 \text{ (Coinciding with } \lambda_3) \]

REDUCTION OF FWM BY UNEQUAL SPACING

Let \( \lambda_1 = 1550 \); \( \lambda_2 = 1552 \); \( \lambda_3 = 1553 \)

\[ \lambda_1 + \lambda_3 - \lambda_2 = 1550 + 1553 - 1552 = 1551 \text{ (not coinciding with any original channel)} \]

\[ \lambda_2 + \lambda_2 - \lambda_1 = 1552 + 1552 - 1550 = 1554 \text{ (not coinciding with any original channel)} \]

Thus it is proved that FWM is minimized for Unequal channel spacing. In this method, as shown in the Figure 4.4, the channels are separated with unequal spacing. This is a better way of avoiding the overlapping. In this method, the probability of FWM products falling on the original frequencies is almost zero. Here, the FWM products fall away from the original frequencies and do not interfere with them. This results in easy filtering of the FWM products, from the original channels. The effects due to the inter-modulation distortion (FWM) can be reduced by allotting large spacing between the channels of the DWDM system. This reduces the probability of interference of the FWM products on the input frequencies.
4.3 EFFECT OF DISPERSION ON FWM

The dispersion in the fiber produces the phase mismatch and hence the interfering terms may get reduced. The simulation of the layout in the optsim software gives the power of the FWM terms for various no. of channels. The dispersion in the fiber could be set from 0 ps/nm/km to any value. Table 4.1 shows the FWM power for different No. of channels.

The physical meaning of 0 ps/nm/km means that there is no pulse broadening at all. It is a desired one. As there is no broadening, bit rate could be increased to even Gbps. However, zero dispersion increases the mixing efficiency of Four Waves. As dispersion increases, the mixing efficiency decreases.

Table 4.1 FWM Vs No. of Channels for the Dispersion of 0 ps/nm/km

<table>
<thead>
<tr>
<th>No of Channels</th>
<th>Input frequency (THz)</th>
<th>Input power (mW)</th>
<th>FWM power (mW)</th>
<th>OSNR (dBm)</th>
<th>Frequency at which Max. FWM occurs (THz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>192</td>
<td>1</td>
<td>0.0000</td>
<td>$\infty$</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>192 &amp; 192.2</td>
<td>1</td>
<td>0.0400</td>
<td>13.98</td>
<td>192.2</td>
</tr>
<tr>
<td>8</td>
<td>[192 : 0.2 : 193.4]</td>
<td>1</td>
<td>0.0510</td>
<td>12.92</td>
<td>193.0</td>
</tr>
<tr>
<td>16</td>
<td>[192 : 0.2 : 195]</td>
<td>1</td>
<td>0.0511</td>
<td>12.91</td>
<td>195.0</td>
</tr>
<tr>
<td>32</td>
<td>[188 : 0.2 : 194.2]</td>
<td>1</td>
<td>0.0520</td>
<td>12.84</td>
<td>196.0</td>
</tr>
<tr>
<td>64</td>
<td>[185 : 0.2 : 198.6]</td>
<td>1</td>
<td>0.0560</td>
<td>12.51</td>
<td>203.4</td>
</tr>
</tbody>
</table>
For example, consider a two channel WDM transmitter that is operated in the frequency of $f_1 = 192$ THz and $f_2 = 192.2$ THz. When the dispersion is zero, then FWM power is 40.0 µW. When the dispersion is 5 ps/nm/km, then the FWM power is reduced to 6.8 µW.

Table 4.2  Effects of FWM Vs N for various dispersions

<table>
<thead>
<tr>
<th>No of Channels</th>
<th>Input frequency in THz</th>
<th>Input power in mW</th>
<th>FWM Power in µW for</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>D = 0 ps/nm-km</td>
</tr>
<tr>
<td>1</td>
<td>192</td>
<td>1</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>192,192.2</td>
<td>1</td>
<td>40.0</td>
</tr>
<tr>
<td>4</td>
<td>192 : 0.2 :192.6</td>
<td>1</td>
<td>49.7</td>
</tr>
<tr>
<td>8</td>
<td>192 : 0.2 :193.4</td>
<td>1</td>
<td>51.0</td>
</tr>
<tr>
<td>16</td>
<td>192 : 0.2 : 195</td>
<td>1</td>
<td>51.1</td>
</tr>
<tr>
<td>32</td>
<td>188 : 0.2 : 194.2</td>
<td>1</td>
<td>52.0</td>
</tr>
<tr>
<td>64</td>
<td>185 : 0.2 : 198.6</td>
<td>1</td>
<td>56.0</td>
</tr>
</tbody>
</table>

It also shows the FWM power and OSNR for various No. of channels. It indicates the frequency at which the dominant FWM component is present. Table 4.2 shows the FWM interference term powers for various dispersions. When the dispersion is 10 ps/nm/km then FWM power is 4.81µW. When the dispersion is 15 ps/nm/km then FWM power is 3.93 µW. When the dispersion is 17 ps/nm/km then FWM power is 3.69 µW. It can be concluded that when dispersion is increased, then FWM power is reduced and below the dispersion of 15 ps/nm/km, there is no considerable reduction in FWM power.
Figure 4.5  Variation of FWM Noise power with Dispersion

Figure 4.5 shows the variation of FWM power for different dispersion values.

Figure 4.6  Comparison of Proposed method with Kaur and Singh

\[ \alpha = 0.205 \text{ dB/km} ; \text{Degeneracy} = 6; A_e = 53 \mu m^2; \]

Dispersion  = 3 ps/nm/km (Kaur et al) ;

= 10 ps/nm/km (Proposed)
Normally, dispersion is deliberately set in an fiber so as to tackle the nonlinear effect such as FWM. Too much dispersion, of course minimize the nonlinear effect, but at the end the DCF (Dispersion Compensating Fiber) has to completely bring down this to zero, which requires more length of DCF and hence more loss of DCF. Also too low dispersion in the main fiber, only marginally nullify the nonlinear effect.

Hence the middle value of 10 ps/ nm/km has been considered. The comparison of our method with the existing graph obtained by Kaur et al (2009) is shown in Figure 4.6. It clearly indicates the reduction in FWM power than the existing one. The extra dispersion added has to be locally compensated by Dispersion Compensating Fiber (DCF).

4.4 DISPERSION COMPENSATION

The accumulated dispersion which may be deliberately introduced for reducing the effects of nonlinearities like FWM, Cross Phase Modulation etc., has to be nullified. This is done by the Dispersion Compensating Fiber (DCF), which introduces negative dispersion. The challenge here is to compensate the dispersion introduced for all the wavelengths in the WDM signal. The accumulated dispersion varies with wavelength. Hence the Relative dispersion slope (RDS) compensation is needed. RDS is the ratio of dispersion slope to dispersion in nm\(^{-1}\). That is not a single negative dispersion is needed; a range of dispersion values is needed over the operating wavelength range.
4.5 Programs for finding out the exact number of FWM interfering terms

4.5.1 Program for equal channel spacing with 16 channels

\[ L = [1550 1551 1552 1553 1554 1555 1556 1557 1558 1559 1560 1561 1562 1563 1564 1565]; \]
\[ LFWM = \text{zeros}(16); \]
for \( i = 1:16 \)
  for \( j = 1:16 \)
    for \( k = 1:16 \)
      \[ LFWM(i,j,k) = L(i) + L(j) - L(k); \]
    end
  end
end

4.5.2 Program for unequal channel spacing with 16 channels

\[ L = [1501 1505 1508 1510 1515 1516 1518 1523 1526 1530 1531 1534 1536 1541 1545 1548]; \]
\[ LFWM = \text{zeros}(16); \]
for \( i = 1:16 \)
  for \( j = 1:16 \)
    for \( k = 1:16 \)
      \[ LFWM(i,j,k) = L(i) + L(j) - L(k); \]
    end
  end
end
LFWM
4.5.3 Program for Equal channel spacing with 32 channels
clc;
clear all;
close all;
L=[1547 1548 1550 1551 1552 1553 1554 1555 1556 1557 1558 1559 1560 1561 1562 1563 1564 1565 1566 1567 1568 1569 1570 1571 1572 1573 1574 1575 1576 1577 1578 1579 ];
LFWM=zeros(32,32);
for i=1:32
    for j=1:32
        for k = 1:32
            LFWM(i,j,k) = L(i)+L(j)-L(k);
        end
    end
end
LFWM

4.5.4 Program for Unequal channel spacing with 32 channels
clc; clear all; close all;
L=[1547 1548.6 1549.8 1550 1551 1551.6 1553 1554 1554.6 1556.2 1557 1557.4 1558 1559.4 1560.4 1562 1563.2 1564 1564.4 1565 1566.4 1567.4 1569 1570.2 1571 1571.4 1572 1573.4 1574.4 1576 1577.2 1578 1578.4];
LFWM=zeros(32,32);
for i=1:32
    for j=1:32
        for k = 1:32
            LFWM(i,j,k) = L(i)+L(j)-L(k);
        end
    end
end
LFWM
4.5.5 Program for Equal channel spacing with 64 channels

```matlab
clc; clear all; close all;
L=[1547 1548 1549 1550 1551 1552 1553 1554 1555 1556 1557
  1558 1559 1560 1561 1562 1563 1564 1565 1566 1567 1568 1569
  1570 1571 1572 1573 1574 1575 1576 1577 1578 1579 1580 1581
  1582 1583 1584 1585 1586 1587 1588 1589 1590 1591 1592 1593
  1594 1595 1596 1597 1598 1599 1600 1601 1602 1603 1604 1605
  1606 1607 1608 1609 1610 1611 ];
LFWM=zeros(64,64);
for i=1:64
    for j=1:64
        for k = 1:64
            LFWM(i,j,k) = L(i)+L(j)-L(k);
        end
    end
end
LFWM
```

4.5.6 Program for Unequal channel spacing with 64 channels

```matlab
clc;
clear all;
close all;
L=[1547 1548.6 1549.8 1550.6 1551 1553 1554 1555.6 1556.2 1557
  1557.4 1558 1559.4 1560.4 1562 1563.2 1564 1564.4 1565 1566.4
  1567.4 1569 1570.2 1571 1571.4 1572 1573.4 1574.4 1576 1577.2
  1578 1578.4 1579 1580.4 1581.4 1583 1584.2 1585 1585.4 1586
  1587.4 1588.4 1590 1591.2 1592 1592.4 1593 1594.4 1595.4 1597
  1598.2 1599 1599.4 1599.8 1600.4 1601.8 1602.8 1604.4 1605.6
  1606.4 1606.8 1607.4 1608.8 1609.8]```
The Matlab programs are executed and the interfering terms are tabulated in Table 4.3.

**Table 4.3 Comparison of Interfering Terms**

<table>
<thead>
<tr>
<th>Number of Channels</th>
<th>Equal Channel Spacing</th>
<th>Unequal Channel Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bandwidth Expansion Factor = 1</td>
</tr>
<tr>
<td>4</td>
<td>104</td>
<td>92</td>
</tr>
<tr>
<td>8</td>
<td>344</td>
<td>296</td>
</tr>
<tr>
<td>16</td>
<td>11732</td>
<td>7984</td>
</tr>
<tr>
<td>32</td>
<td>21844</td>
<td>13009</td>
</tr>
<tr>
<td>64</td>
<td>39720</td>
<td>22815</td>
</tr>
</tbody>
</table>

The numbers inside the cells of the table are obtained by counting the coinciding terms for each channel from the results of execution of Matlab program. In the actual spectrum most of the terms may vanish because of the phase mismatch.
4.6 DISCUSSION OF THE RESULTS

The exhaustive number of FWM terms are obtained in Table 4.3. Comparing to Kaur and Singh (2009), the increased value of dispersion has minimized the FWM power considerably. Without altering the existing already laid lengthy fiber, at the end equipment side, unequal wavelength spacing of laser sources could be set with the help of tunable laser transmitters.

As Unequal channel spacing could simultaneously reduce the two nonlinearities [FWM & SRS], it may result in better performance like reduced BER arising out of more OSNR.

The paper titled ‘Analysis of Four Wave Mixing in Fiber Optic Communication and to devise a mechanism for combating the FWM’, has been published in an International Conference on Emerging Trends in Engineering Technologies-2010, organized by Noorul Islam University, March 25-26, 2010. The paper titled ‘Analysis and Reduction of Four Wave Mixing in DWDM Systems’ has been published in the National Conference on Communication Technologies [NCCT’10], held in Mepco Schlenk Engineering College, on March 19, 2010.