CHAPTER 3

VOLTAGE STABILITY INDICES AND
PROBLEM FORMULATION

3.1 INTRODUCTION

The voltage stability indices are introduced in order to evaluate the stability limit. Voltage stability indices are invaluable tools for gauging the proximity of a given operating point to voltage instability. The objective of the voltage stability indices is to quantify how close a particular point is to the steady state voltage stability margin. These indices can be used on-line or off-line to help operators in real time operation of power system or in designing and planning operations. These indices will be presented to demonstrate how close to voltage instability a system can be operated and which could lead to blackout in large parts of the interconnected power system.

Elements such as reactive power generating devices, tap changing transformers are optimally adjusted at each operating point to reach the objective of minimizing voltage stability index at each individual bus as well as minimizing the global voltage stability indices. The system can be operated in the stable region by minimizing voltage stability index of buses and lines.

3.2 LOADING MARGIN

Loading Margin is the most basic and widely accepted method to approximate the voltage collapse in the power system. For a current operating point, the total increment of load in a specified pattern of load increase that
would cause a voltage collapse is called the loading margin to voltage collapse.

The P-V and Q-V curves are mostly used to determine the loading margin of a power system in an individual load bus. A typical P-V curve of a load bus in the power system is shown in Figure 3.1. To build the P-V curve, at a base case, the power system load is gradually increased. For each incremental load, it is necessary to recalculate power flows so that the bus voltage corresponding to the load is determined. The increment of load is stopped when the voltage collapse point or the nose of the P-V curve is reached. The power margin between the current operating point and the voltage collapse operating point is used as a voltage stability criterion.

![Figure 3.1 P-V Curve of a Load Bus in the Power System](image)

In Figure 3.1, \( P_0 \) is the load power at the current operating point and \( P_m \) is the maximum active power that the load can consume from the system. With Q-V curve, it is possible to know the maximum reactive power that can be achieved or added to a bus before reaching the minimum voltage limit. A
typical Q-V curve is presented in Figure 3.2. The curve can be produced by varying the reactive power demand (or injection) at the load bus while maintaining the active power constant, corresponding load voltage is determined through load flow recalculation. The reactive power margin is the MVAR distance from the operating point to the bottom of the Q-V curve. The Q-V curve can be used as an index for voltage instability. The point where dQ/dV is zero is the point of voltage instability.

![Figure 3.2 Q-V Curve of a Load Bus in the Power System](image)

**Figure 3.2 Q-V Curve of a Load Bus in the Power System**

In Figure 3.2, \( V_0 \) is the voltage at the load bus at the current operating point. \( V_c \) is the voltage at the bottom of Q-V curve, which is the minimum voltage within the stability limit of the system. Generally such curves are developed by load flow analysis, using conventional, direct and continuation power flow methods. The advantage of the loading margin
approach is simple and easy to understand. The procedure of building these curves can be automated. However, the curves must be generated at each bus. Furthermore, it needs information of the system which is beyond the operating point and hence the cost of calculations will be very high.

### 3.3 TYPES OF INDICES

Different types of indices are used in literature for optimization of power system. Some of the available indices are:

1. Line Stability Index (LSI)
2. Voltage Collapse Prediction Index (VCPI)
3. Power Transfer Stability Index (PTSI)
4. Line Voltage Stability Index (LVSI)
5. Equivalent Node Voltage Collapse Index (ENVCI)
6. L- Index (LI)
7. Fast Voltage Stability Index (FVSI)

Among the different indices for voltage stability and voltage collapse prediction, the L-index gives fairly consistent results (Durairaj et al 2005). The reason behind choosing FVSI index is that FVSI index is capable to identify critical areas in a large power system, capable to determine the point of voltage collapse, maximum permissible load, weak bus in the system and the most critical line in an interconnected system (Ismail Musirin and Titik Khawa Abdul Rahman 2002; Ramasamy et al 2009). Due to the above mentioned reasons, in this proposed work, FVSI and L-index approaches are used to assess the power system stability and vulnerable line and bus are identified for the placement of FACTS devices and it will be explained in further sections.
3.3.1 **Fast Voltage Stability Index (FVSI)**

Ismail Musirin and Titik Khawana Abdul Rahman (2002) proposed a new voltage stability index, which is called Fast Voltage Stability Index (FVSI). This index can either be referred to a bus or a line. The voltage stability index developed in this work is referred to a line. Generally, it is started with the current equation to form the power or voltage quadratic equations. The criterion employed in this work was to set the discriminant of the roots of voltage or power quadratic equation to be greater than zero. When the discriminant is less than zero, it causes the roots of the quadratic equations to be imaginary which in turn cause voltage instability in the system. The line index that is evaluated close to 1 will indicate the limit of voltage instability.

![Two Bus Power System Model](image)

**Figure 3.3 Two Bus Power System Model**

FVSI is derived using a two bus power system model as shown in Figure 3.3. The symbols are explained as follows:

- \( V_1, V_2 \) = voltage on sending and receiving end buses.
- \( P_1, Q_1 \) = active and reactive power on the sending end bus
- \( P_2, Q_2 \) = active and reactive power on the receiving end bus
S_1, S_2 = apparent power on the sending and receiving end buses

\( \delta = \delta_1 - \delta_2 \) (angle difference between sending and receiving end buses)

The line impedance is noted as \( Z = R+jX \) with the current (I) that flows in the line given by;

\[
I = \frac{V_1 \angle \theta - V_2 \angle \delta}{R + jX} \quad (3.1)
\]

\( V_1 \) is taken as the reference and therefore the angle is shifted to 0.

The apparent power at bus 2 can be written as;

\[
S_2 = V_2 I^* \quad (3.2)
\]

Rearranging the Equation (3.2) yields;

\[
I = \left( \frac{S_2}{V_2} \right)^* \quad (3.3)
\]

\[
\frac{P_2 - jQ_2}{V_2 \angle - \delta} \quad (3.4)
\]

Equating Equations (3.1) and (3.4),

\[
\frac{V_1 \angle 0 - V_2 \angle \delta}{R + jX} = \frac{P_2 - jQ_2}{V_2 \angle - \delta}
\]

\[
V_1 V_2 \angle \delta - V_2^2 \angle 0 = (R + jX)(P_2 - jQ_2) \quad (3.5)
\]

Separating the real and imaginary parts yields,

\[
V_1 V_2 \cos \delta - V_2^2 = RP_2 + XQ_2 \quad (3.6)
\]

and,
\[-V_1V_2 \sin \delta = XP_2 - RQ_2 \quad (3.7)\]

Rearranging Equation (3.6) for \(P_2\) and substituting into Equation (3.5) yields a quadratic equation of \(V_2\):

\[V_2^2 - \left( \frac{R}{X} \sin \delta + \cos \delta \right) V_1 V_2 + \left( X + \frac{R^2}{X} \right) Q_2 = 0 \quad (3.8)\]

The roots for \(V_2\) will be;

\[V_2 = \frac{\frac{R}{X} \sin \delta + \cos \delta \frac{V_1}{2} \pm \sqrt{\left( \frac{R}{X} \sin \delta + \cos \delta \right) V_1}^2 - 4 \left( X + \frac{R^2}{X} \right) Q_2}{2} \quad (3.9)\]

To obtain real roots for \(V_2\), the discriminant is set greater than or equal to ‘0’; i.e.,

\[\left( \frac{R}{X} \sin \delta + \cos \delta \right) V_1 ^2 - 4 \left( X + \frac{R^2}{X} \right) Q_2 \geq 0\]

\[
\frac{4Z^2Q_2X}{(V_1^2(\sin \delta + X \cos \delta))^2} \leq 1 \quad (3.10)
\]

Since \(\delta\) is normally very small then,

\[\delta \approx 0, R \sin \delta \approx 0 \text{ and } X \cos \delta \approx X\]

Taking the symbols ‘i’ as the sending bus and ‘j’ as the receiving bus, the Fast Voltage Stability Index (FVSI) can be written as follows;

\[FVSI_i = \frac{4Z^2Q_j}{V_i^2X_j} \quad (3.11)\]

where,

\[Z = \text{ line impedance}\]
\[ X_{ij} = \text{line reactance} \]
\[ Q_j = \text{reactive power at the receiving end} \]
\[ V_i = \text{sending end voltage} \]

The value of FVSI that is evaluated close to 1 indicates that the particular line is close to its instability point which may lead to voltage collapse in the entire system. To maintain a secure condition the value of FVSI should be maintained well less than 1.

### 3.3.2 L- Index (LI)

Huang G.M. and Nair N.K.C. (2001) proposed a new voltage stability index, which is called L- index. A simple power system is considered, through which the useful index of the voltage stability is derived. The line model is conceived as the simplest power system and can also be treated analytically. It is given by Figure 3.4, where node 1 is assumed to supply the load whose voltage behavior is of interest and node 2 is a generator.

![Figure 3.4 Single Generator and Load System](image-url)
The properties of node 1 can be described in terms of admittance matrix of the system as,

\[ Y_{11}V_1 + Y_{12}V_2 = I_1 = \frac{S_1}{V_1} \]  
(3.12)

The element \( Y_{11}, Y_{12}, Y_{21} \) and \( Y_{22} \) form the admittance matrix \([Y]\) where as \( S_1 \) is the complex power.

\[ S_1 = V_1I_1^* \]

Equation (3.12) can be written as,

\[ Y_{11}V_1 + Y_{12}V_2 = \frac{S_1^*}{V_1} \]

\[ Y_{11}V_1^2 + Y_{12}V_2V_1^* = S_1^* \]

\[ V_1^2 + \left( \frac{Y_{12}}{Y_{11}} \right)V_2V_1^* = \frac{S_1^*}{Y_{11}} \]

\[ V_1^2 + V_0V_1^* = \frac{S_1^*}{Y_{11}} = a + jb \]  
(3.13)

Where \( a \) and \( b \) are real and imaginary parts of the Equation (3.13) and \( V_0 \) is an equivalent voltage which is given as,

\[ V_0 = \left( \frac{Y_{12}}{Y_{11}} \right)V_2 \]  
(3.14)

Taking complex conjugate of Equation (3.13),

\[ V_1^2 + V_0^*V_1 = \frac{S_1}{Y_{11}}^* = a - jb \]  
(3.15)
Multiplying Equation (3.13) and Equation (3.15),

\[ V_1^4 + V_1^2 (V_0 V_1^* + V_0^* V_1) + V_0^2 V_1^2 = a^2 + b^2 \]  
(3.16)

Adding Equations (3.13) and (3.15),

\[ V_0 V_1^* + V_0^* V_1 = 2a - 2V_1^2 \]  
(3.17)

Now, substituting Equation (3.17) in Equation (3.16),

\[ V_1^4 + V_1^2 (2a - 2V_1^2) + V_0^2 V_1^2 = a^2 + b^2 \]

\[ V_1^4 - V_1^2 (2a + V_1^2) + (a^2 + b^2) = 0 \]  
(3.18)

Equation 3.18 is a quadratic equation in terms of \( V_1^2 \). The solution of this equation is,

\[ V_1^2 = \frac{V_0^2}{2} + a \pm \sqrt{\frac{V_0^4}{4} + aV_0^2 - b^2} \]  
(3.19)

Taking roots on both sides of the above equation, the solution for \( V_1 \) is obtained.

\[ V_1 = \sqrt{\frac{V_0^2}{2} + a \pm \sqrt{\frac{V_0^4}{4} + aV_0^2 - b^2}} \]  
(3.20)

The voltage collapse occurs in the load bus i.e., bus 1, when,

\[ \sqrt{\frac{V_0^4}{4} + aV_0^2 - b^2} = 0 \]  
(3.21)

Substituting Equation (3.21) in Equation (3.20),
\[ V_1 = \sqrt{\frac{V_0^2}{2} + a} \]

\[ \text{ie.}, \quad V_1^2 = \frac{V_0^2}{2} + a \]  \hspace{1cm} (3.22)

From Equation (3.13),

\[ a = \text{Re}[V_1^2 + V_0^* V_1] \]  \hspace{1cm} (3.23)

Substituting Equation (3.23) in Equation (3.22),

\[ V_1^2 = \frac{V_0^2}{2} + \text{Re}[V_1^2 + V_0^* V_1] \]

\[ V_1^2 = \frac{V_0^2}{2} + V_1^2 + \text{Re}[V_0^* V_1] \]

\[ -\frac{V_0^2}{2} = \text{Re}[V_0^* V_1] \]

On simplification,

\[ \text{Re}[V_1/V_0] = -0.5 \]  \hspace{1cm} (3.24)

Now, from Equation (3.15),

\[ V_1^2 + V_0^* V_1 = \frac{S_1}{Y_{11}} \]

\[ Y_{11}^* V_1^2 + V_0^* V_1 Y_{11}^* = S_1 \]

\[ 1 + \left( \frac{V_0^* V_1}{Y_{11}^* V_1^2} \right) = \frac{S_1}{Y_{11}^* V_1^2} \]
Substituting Equation (3.24) in Equation (3.25), the result at the time of voltage collapse.

\[ |1 + (V_0/V_i)| = \frac{S_i^*}{Y_{ii}V_i^2} \]  

(3.25)

This relation is used to define the indicator L for the assessment of voltage stability.

\[ L = |1 + (V_0/V_i)| = \frac{S_i^*}{Y_{ii}V_i^2} \]  

(3.27)

The range of this L-index is given as,

\[ R = \{ L/0 \leq L \leq 1 \} \]

### 3.4 PROBLEM FORMULATION

In this work, the problem is formulated with three different approaches for optimizing the power system and it will be explained in further sections.

#### 3.4.1 Contingency Analysis

Linear estimation method are used to approximate the values when more than one type of errors are encountered which lead to erroneous estimate in power system. The estimation errors can be decreased using computational numerical method and the values can be optimized to improve the result based
on genetic algorithm. Linear estimation method accommodates any small changes in occurrence of a contingency (K).

The objective function of this approach is to identify the accuracy of the results and the speed of calculation that depends on the power flow technique and the optimization algorithm used. This can be represented as:

Minimize $F(x)$ \hspace{1cm} (3.28)

Where, $F(x)$ is the objective function (minimization of errors) adapted to the problem under study.

3.4.2 Fast Voltage Stability Index (FVSI) Approach

The objective function of this approach is to find the optimal rating of FACTS devices, which minimizes the real power loss, voltage deviation and FVSI. This is mathematically stated as:

Minimize

$$F = [f_1, f_2, f_3]$$ \hspace{1cm} (3.29)

The first term $f_1$ represents real power loss

$$f_1 = \sum_{k \in NL} g_k (V_i^2 + V_j^2 - 2V_iV_j \cos \theta_{ij}) = P_{\text{loss}}$$ \hspace{1cm} (3.30)

The second term $f_2$ represents the total voltage deviation (VD) of all load buses from desired value of 1 p.u.

$$f_2 = \text{VD} = \sum_{k=1}^{N_{pq}} (V_k - V_{ref})^2$$ \hspace{1cm} (3.31)
The last term $f_3$ is the Fast Voltage Stability Index (FVSI) of the line $ij$

$$f_3 = \text{FVSI}_{ij} = \frac{4Z^2 Q_j}{V_i^2 X_{ij}}$$  \hfill (3.32)

Where,

$Z = \text{Line impedance}$

$X_i = \text{Line reactance}$

$Q_j = \text{Reactive power at the receiving end}$

$V_i = \text{Sending end voltage}$

### 3.4.3 L-Index (L1) Approach

The objective function of this approach is to find the optimal rating of FACTS devices that minimizes real power loss, voltage deviation, total cost of the system and L-index. This is mathematically stated as:

Minimize

$$F = [f_1, f_2, f_3, f_4]$$  \hfill (3.33)

The first term $f_1$ represents real power loss as

$$f_1 = \sum_{k \in N_i} g_k \left(V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}\right) = P_{\text{loss}}$$  \hfill (3.34)

The second term $f_2$ represents the total voltage deviation (VD) of all load buses from desired value of 1 p.u.

$$f_2 = \text{VD} = \sum_{k=1}^{N_{pq}} (V_k - V_{\text{ref}_k})^2$$  \hfill (3.35)
The third term $f_3$ represents total cost minimization

$$f_3 = W_c + I_c$$  \hspace{1cm} (3.36)

Where ‘$W_c$’ represents the total cost of energy loss as follows

$$W_c = h \sum_{i \in N} d_i P_i^{loss} + h \sum_{k \in N} d_i g_k (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij})$$  \hspace{1cm} (3.37)

$I_c$ represents the cost of reactive power source, installation cost and purchase cost

$$I_c = \sum_{i \in N_c} (e_i + C_{ci} |Q_{ci}|)$$  \hspace{1cm} (3.38)

Where $Q_{ci}$ can either be positive or negative, $C_{ci}$ is the capacitance or reactance installation and $e_i$ is the fixed installation cost. The absolute variable is used to compute the cost.

The last term $f_4$ is the L-index of the $j^{th}$ bus and is given by

$$f_4 = L_j = \left| 1 + \frac{V_{0j}}{V_j} \right| = \frac{S_j^*}{Y_{jj} V_j^2}$$  \hspace{1cm} (3.39)

$S_j^*$ = Complex power at $j^{th}$ bus

$V_j$ = Voltage magnitude at $j^{th}$ bus

$Y_{jj}$ = Admittance from bus admittance matrix

$V_{0j}$ = An equivalent generator voltage comprising the contribution from all generators
3.4.4 Constraints

The minimization problem (contingency analysis, FVSI and L-index) is subjected to the following equality and inequality constraints:

(i) Load Flow Constraints:

\[
P_i - V_i \sum_{j=1}^{N_g} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0, i = 1, 2, ..., N_b - 1
\]

\[
(3.40)
\]

\[
Q_i - V_i \sum_{j=1}^{N_g} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0, i = 1, 2, ..., N_b - 1
\]

\[
(3.41)
\]

(ii) Voltage Constraints:

\[
V^\text{min}_i \leq V_i \leq V^\text{max}_i; i \in N_b
\]

\[
(3.42)
\]

(iii) Real Power Limit:

\[
P^\text{min}_{pq} \leq P_{pq} \leq P^\text{max}_{pq}
\]

\[
(3.43)
\]

(iv) Reactive Power Generation Limit:

\[
Q^\text{min}_{gi} \leq Q_{gi} \leq Q^\text{max}_{gi}; i \in N_g
\]

\[
(3.44)
\]

(v) Reactive Power Generation Limit of capacitor banks:

\[
Q^\text{min}_{ci} \leq Q_{ci} \leq Q^\text{max}_{ci}; i \in N_c
\]

\[
(3.45)
\]

(vi) Transformer tap setting limit:

\[
t^\text{min}_k \leq t_k \leq t^\text{max}_k; k \in N_t
\]

\[
(3.46)
\]
(vii) Transmission line flow limit:

\[ S_i \leq S_i^{\text{max}} ; i \in N_i \]  \hspace{1cm} (3.47)

(viii) FACTS devices constraints:

\[-0.8X_L \leq X_{\text{TCSC}} \leq 0.2X_L \]  \hspace{1cm} (3.48)

\[-100 \text{MVAR} \leq Q_{\text{SVC}} \leq 100 \text{MVAR} \]  \hspace{1cm} (3.49)

Equation (3.48) is for TCSC, (3.49) is for SVC and Equations (3.48) and (3.49) are for UPFC.

(ix) Contingency value constraint:

\[ K^0 < K < K^c \]  \hspace{1cm} (3.50)

(x) Active and Reactive Power equality constraints:

\[ \varphi_p = \sum_{i=1}^{G} P_{G_i} - \sum_{i=1}^{N} P_{D_j} - P_L = 0 \]  \hspace{1cm} (3.51)

\[ \varphi_q = \sum_{i=1}^{G} Q_{G_i} - \sum_{i=1}^{N} Q_{D_j} - Q_L = 0 \]  \hspace{1cm} (3.52)

3.5 CONCLUSION

In this chapter a brief discussion about the loading margin is done. The P-Q and Q-V curves which are used to determine the loading margin of a power system in an individual load bus is explained. The method of finding out the FVSI and L-index are derived with two bus power system model. From these indices, depending upon the value, the vulnerability of the line and bus is determined for the placement of FACTS devices in a power system. In order to optimize the power system, the problem was formulated using contingency analysis, FVSI and L-index approaches.