CHAPTER 2

MODELING OF VOLTAGE SOURCE INVERTER FED INDUCTION MOTOR AND MODULATION STRATEGIES

2.1 INTRODUCTION

An inverter is an electrical device that converts direct current to alternating current; the converted AC can be at any required voltage and frequency with the use of appropriate transformers, switching and control circuits. Static inverters have no moving parts and are used in a wide range of applications, from small switching power supplies in computers, to large electric utility high voltage direct current applications that transport bulk power (Zhang and Fahmi 2003). Inverters are commonly used to supply AC power from DC sources such as solar panels or batteries. The electrical inverter is a high power electronic oscillator. It is named because early mechanical AC to DC converter was made to work in reverse mode and thus was inverted, to convert DC to AC.

A variable frequency drive controls the operating speed of an AC motor by controlling the frequency and voltage supplied to the motor. An inverter provides the controlled power. DC voltage for an inverter is obtained by using rectifier from AC power supply. Since an inverter is the key component, variable frequency drives are sometimes called inverter drives. DC motors have been used during the last century in industries for variable speed control applications, because its flux and torque can be controlled easily by changing the field and armature currents respectively. Furthermore, four quadrant operation of induction motor was also achieved.
At present, induction motor is popularly used in industries due to ruggedness and robustness. The induction motors were mainly used for essentially constant speed applications because of the unavailability of the variable frequency voltage supply. The advancement of power electronics has made it possible to vary the frequency of the voltage. Thus, it has extended the use of induction motor in variable speed drive applications. The concept of multilevel inverter control has opened a new possibility that induction motors can be controlled to achieve dynamic performance equally as that of DC motors. DC motors have several limitations on construction as well as in operation. AC induction motor has many advantages over DC motor. Hence the induction motor is preferred for drive applications.

2.2 MODELING OF INDUCTION MOTOR

The modeling of induction motor includes steady state model and dynamic model. The equivalent circuit of steady state model and dynamic model helps to study the performance of the machine (Krishnan 2006). This implies all the electrical transients are neglected, during load changes and stator frequency variations. The key variables in the machines are the air gap power, mechanical and shaft output power and electromagnetic torque. These are derived from the equivalent circuit of the induction machine as follows. The variables are real power transmitted from the stator \( P_i \), to the air gap \( P_a \), is the difference between total input power to the stator windings and copper losses in the stator \( I_s^2 R_s \), and is given as

\[
P_a = P_i - 3I_s^2 R_s \quad \text{(Watts)} \tag{2.1}
\]

where

\[ I_s \quad \text{Stator current (A)} \]
\[ R_s \quad \text{Stator resistance (Ω)} \]
Neglecting the core losses, the air gap power is equal to the total power dissipated in $R_r/s$ in the three phases of the machine; there is no other element to consume power in the rotor equivalent circuit. It is given as

$$P_a = 3I_r^2 \frac{R_r}{s} \text{ (Watts)} \quad (2.2)$$

which could be written alternatively as

$$P_a = 3I_r^2 R_r + 3I_r^2 R_r \frac{(1-s)}{s} \text{ (Watts)} \quad (2.3)$$

where

$I_r$ - Rotor current (A)

$R_r$ - Rotor resistance (Ω)

$s$ - Slip

The common term of above three equations and accounts for the number of phases in the machine which throughout taken as the mechanical power output, $P_m$ is obtained as,

$$P_m = 3I_r^2 R_r \frac{(1-s)}{s} \text{ (Watts)} \quad (2.4)$$

Alternately in terms of the electromagnetic torque and rotor speed, the mechanical power output is equal to the product

$$P_m = T_e \omega_m \text{ (Watts)} \quad (2.5)$$

where $T_e$ is the internal or electromagnetic torque, derived from Equations (2.4) and (2.5) as

$$T_e = \frac{3I_r^2 R_r (1-s)}{s \omega_m} \text{ (Nm)} \quad (2.6)$$
Substituting for the rotor speed in terms of the slip and stator frequency given by

$$\omega_m = \frac{\omega_r}{p/2} = \frac{\omega_s(1-S)}{p/2} \quad (2.7)$$

The electromagnetic or air gap torque is obtained as

$$T_e = 3 \left( \frac{p}{2} \right) \frac{l_s^2 R_r}{s_{0g}} \quad (Nm) \quad (2.8)$$

where

- $\omega_m$ - Rotor speed
- $P$ - Number of poles
- $\omega_s$ - Synchronous speed

The dynamic model considers the instantaneous effects of varying voltages/currents, stator frequency, and torque disturbances the dynamic model of the induction motor is derived, using a two phase motor in direct and quadrature axis. This approach is desirable because of conceptual simplicity obtained with two sets of windings, one on the stator and the other on the rotor. The equivalence between the three phase and two phase machine models are derived for simple observation, and this approach is suitable for extending it to model and n-phase machine by means of a two-phase machine. Derivation for electromagnetic torque involving the currents and flux linkages are specified and also induction motor model in arbitrary reference frames is given below for use in subsequent sections,

$$\begin{bmatrix} v_{qs}^c \\ v_{dq}^c \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s \\ -\omega_c L_s & R_s + L_s p \end{bmatrix} \begin{bmatrix} i_{qs}^c \\ i_{ds}^c \end{bmatrix} + \begin{bmatrix} L_m p & \omega_c L_m \\ -\omega_c L_m & L_m p \end{bmatrix} \begin{bmatrix} i_{q}^c \\ i_{d}^c \end{bmatrix} + \begin{bmatrix} \omega_c L_r \\ R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qr}^c \\ i_{dr}^c \end{bmatrix}$$

$$\quad (2.9)$$
The Equation (2.9) can be written as

\[ V = [R] i + [L] i + [G] \omega_r i + [F] \omega_c i \]  

(2.10)

where the vectors and matrices are identified from the Equations (2.9) and (2.10). Premultiplying the Equation (2.10) by the transpose of the current vector gives the instantaneous input power as,

\[ p_i = i^T V = i^T [R] i + i^T [L] i + i^T [G] \omega_r i + i^T [F] \omega_c i \]  

(2.11)

where,

\[ [R] = \text{matrix consists of resistive elements}, \]
\[ [L] = \text{matrix consists of the coefficient of the derivative operator } p, \]
\[ [G] = \text{matrix consists of the coefficients of the electrical rotor speed } \omega_r, \text{ and} \]
\[ [F] = \text{frame matrix consists of the coefficient of the reference frame speed } \omega_c. \]

\[ i^T [R] i \text{ gives the stator and rotor resistive losses} \]
\[ i^T [L] i \text{ denotes the rate of change stored magnetic energy} \]
\[ i^T [G] \omega_r i \text{ air gap power as to be associate with the rotor speed} \]
\[ i^T [F] \omega_c i \text{ is the reference frame power} \]

The air gap power is the product of mechanical rotor speed and electromagnetic torque. Hence, \( T_e \) is derived from the term rotor speed, \( \omega_m \) in mechanical radian per second, as

\[ \omega_m T_e = P_a = i^T [G] i \omega_r \]  

(2.12)
Substituting for $\omega_r$ in terms of $\omega_m$ leads to electromagnetic torque as

$$T_e = \frac{P}{2} i^t [G] i$$  \hfill (2.13)

By substituting for $[G]$ in Equation (2.13) by observation from (2.9) the electromagnetic torque is obtained as

$$T_e = \frac{3P}{2} L_m (i^c_{qs} i^c_{dr} - i^c_{ds} i^c_{qr})$$  \hfill (2.14)

The factor 3/2 is introduced into the right hand side of the Equation (2.14) from the power equivalence condition between the three-phase and two-phase induction motors. The frequently used model in various reference frames and their derivations from the generalized induction motor in arbitrary reference frames. The relationship between the stationary reference frames denoted by $d$ and $q$ axes and the arbitrary reference frames denoted by $d^c$ and $q^c$ axes as shown in Figure (2.1).
Three particular cases of the generalized model of the induction motor in arbitrary reference frames are

1. Stator reference frames model \((\omega_c = 0)\)
2. Rotor reference frames model \((\omega_c = \omega_r)\)
3. Synchronously rotating reference frames model \((\omega_c = \omega_s)\)

Here,

Synchronously rotating reference frame model is considered to derive the torque equation

\[ \omega_c = \omega_s \] \hspace{1cm} (2.15)

where

\[ \omega_c = \text{speed of reference frames}, \]
\[ \omega_s = \text{stator supply angular frequency / rad / sec} \]

The instantaneous angular position is

\[ \theta_c = \theta_s = \omega_s t \] \hspace{1cm} (2.16)

By substituting the Equation (2.16) into (2.9), the induction motor model in the synchronous reference frames is obtained. By using the superscript \(e\) to denote this electrical synchronous reference frame, the model is obtained as
The electromagnetic torque is

\[ T_e = \frac{3P}{2} L_m (i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e) \]  

(Nm) \hspace{1cm} (2.18)

The transformation from \( abc \) to \( dqu \) variables is obtained by Clarke’s transformation and it is determined as

\[
[T_{abc}^e] = \frac{2}{3} \begin{bmatrix}
\cos \theta_s & \cos (\theta_s - \frac{2\pi}{3}) & \cos (\theta_s + \frac{2\pi}{3}) \\
\sin \theta_s & \sin (\theta_s - \frac{2\pi}{3}) & \sin (\theta_s + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]  

(2.19)

It may be seen that the synchronous reference frames transform the sinusoidal input into dc signals. This model is useful where the variables in steady state need to be dc quantities, as in the development small signal equations. Some high performance control schemes use this model to estimate the control inputs; this leads to a major breakthrough in induction motor control, by decoupling the torque and flux channels for control in a manner similar to that for separately excited dc motor drives.
2.3 HARMONICS

Power quality is a more important issue to both industrial and domestic customers. The commercial equipments are not designed to cope up with the increasing levels of power pollution. With the growing concern about the levels of harmonics present in electrical systems, a number of regional standards have been introduced in various countries throughout the world. These standards are imposed on electrical items, electricity customers, electricity utilities and drive applications. There are many standards available and the most widely recognized being the IEEE Std 519-1992. ("IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems").

This chapter deals with harmonics on an electrical system. Limitations on the levels of harmonics are constrained and methods to limit the generation of harmonics are examined. With a power frequency (fundamental frequency) of 50Hz, the equation representing a harmonic frequency is given by,

\[ f_h = h \times 50 \text{ Hz} \quad (2.20) \]

where, \( h \) is the order of the harmonics. In power system, these harmonics interact with the fundamental frequency waveform and produce a distorted waveform. The magnitudes of the harmonics and their phase shifts determine the shape of the resulting waveform.
Figure 2.2 The Formation of Harmonic Distortion

The distorted waveform in Figure 2.2, from a perfect sinusoidal is generally expressed in terms of harmonic components in the frequency spectrum as shown in Figure 2.3
2.3.1 Fourier Series Representation

Using the Fourier series, any periodic waveform will be recomposed into its fundamental frequency component and the sum of other harmonic components.

\[ v(t) = a_o + \sum_{h=1}^{\infty} V_h \sin(h2\pi ft + \theta_h) \]  

(2.21)

where, \( a_o \) is the DC component, \( V_h \) is the peak voltage level, \( f \) is the fundamental frequency, \( \theta_h \) is the phase angle and \( h \) is the harmonic order.

2.3.2 Total Harmonic Distortion

In order to determine the relative distortion due to harmonics on a power system, the term Total Harmonic Distortion (THD) has emerged. The THD is a measure of the amount of distorted harmonics that are impeded on the system voltage, expressed as a percentage of the fundamental. Both voltage and current waveform distortion are represented by THD.

\[ \text{THD} = \sqrt{\frac{\text{sum of squares of amplitude of all harmonics}}{\text{square of amplitude of fundamental}}} \times 100\% \]  

(2.22)

(or)

\[ \text{THD} = \frac{\sqrt{\sum_{h=2}^{\infty} V_h^2}}{V_1} \times 100\% \]  

(2.23)

where, \( V_h \) is the RMS value of harmonic component \( h \), \( V_1 \) is the RMS value of the fundamental component of voltage.
2.4 CAUSES OF HARMONICS IN VARIABLE SPEED DRIVES

Variable speed drives utilize both rectifying and inverting circuitry in their operation as seen in Figure 2.4. Variable speed drives will take two forms of circuits which are Voltage Source Inverter (VSI) and Current Source Inverter (CSI). The VSI drives which use large capacitors to regulate the DC voltage to the inverter.

Variable frequency is provided when the inverter chops the DC voltage up into the required waveform. The CSI, which use a similar principle, but a large inductance is used instead of a capacitor to regulate the current input to the inverter. The current that is shaped to provide the variable frequency component. Harmonic currents produced by variable speed drives depend heavily on the type of drive, the loading upon the drives and the characteristics of the system supplying the drives.

![Figure 2.4 Typical Variable Speed Drive Structure](image)

Variable speed induction motor drives are widely spread in electro mechanical systems for a large spectrum of industrial applications (Hammond 2000). The recent advancement in power electronics has been initiated to improve the level of inverter rather than increasing the filter size. Using multilevel inverter better harmonics reduction is achieved. The performance of the multilevel inverter is better than a conventional inverter. THD of the conventional inverter is very high, whereas THD for multilevel inverter is
In this thesis, pulse width modulation technique is employed for multilevel inverter operation.

2.5 PULSE WIDTH MODULATION

Pulse Width Modulation (PWM) of a signal or power source involves the modulation of its duty cycle, to either convey information over a communication channel or control the amount of power sent to a load. In general, PWM techniques use two signals i.e., one is a reference signal and other is a carrier signal. In this section, the reference signal is going to modify, whereas carrier is a triangular wave (Espinoza 2001).

Some of the different modified reference signals are as follows. In sinusoidal PWM, instead of maintaining the width of all pulses as in the case of multiple PWM, the width of each is varied in proportion to the amplitude of a sine wave evaluated at the same pulse. The distortion is reduced significantly compared to multiple PWM technique. Voltage source inverter with pulse width modulation is frequently used in industrial applications.

Traditional PWM schemes are tailored to control the fundamental frequency of the output voltage. But practical limitations in VSI implementation produce unexpected output voltage distortion which leads to harmonics. More recently, the application of VSI in active power filters requires accurate control of low frequency output harmonics.

2.5.1 Advantages of PWM

Pulse width modulation has several advantages.

- The output voltage control can be obtained without any additional components.
With this method, lower order harmonics can be eliminated or minimized along with its output voltage control.

The higher order harmonics can be filtered easily.

The PWM strategies, which are used for a conventional inverter, can be modified to be used for multilevel inverters as well (Pital et al 1980). According to the switching frequency, the modulation methods for multilevel inverters are classified as illustrated in Figure 2.5.

A very popular method in industrial applications is the classic carrier based sinusoidal PWM that uses the phase shifting technique to reduce the harmonics in the load voltage. Another interesting alternative is the Space Vector strategy, which has been used in three level inverters. Methods that works with low switching frequencies are the multilevel selective harmonic elimination and the space vector control (Mohamed and Abdul Kadir 2006 Mohamed et al 2006, Mohamed and Agelidis 2008a,b).

![Figure 2.5 Classifications of Multilevel Modulation Method](image-url)
2.5.2 Sinusoidal Pulse Width Modulation

Based on the classical Sinusoidal Pulse Width Modulation (SPWM) with triangular carriers, several multicarrier techniques have been developed to reduce the distortion in multilevel inverters (Mohamed et al 2008). Three of the most common techniques are the following:

- The waveform of a carrier is phase shifted by $180^\circ$ from the waveform of the next carrier (Alternate Phase Opposition Disposition, APOD).

- All positive carrier waveforms are in phase, but have the opposite phase from the negative ones. Positive and negative carrier waveforms are defined based on the reference signal values (Phase Opposition Disposition, POD).

- All the carrier waveforms are in phase (Phase Disposition, PD), as shown in Figure 2.6.

Figure 2.6 Phase Disposition SPWM
Figure 2.7 Carrier Shifted SPWM

For multilevel inverters which are implemented using cascaded cells, the carrier shifted SPWM technique is proven to be producing a load voltage with the smallest distortion. In this modulation technique, the carriers of a number of cascaded cells \(N_C\) are phase shifted by an angle of \(\theta_c = \frac{360}{N_C}\). Figure 2.7 outlines the principle of the carrier shifted PWM modulation technique.

2.6 SUMMARY

In this chapter, the induction motor model in synchronously rotating reference frame was considered. This chapter describes the causes of harmonics in variable speed drives and also the concepts of PWM techniques were analyzed.