CHAPTER 6

CONCLUSIONS AND SCOPE OF FUTURE WORK

In the present work, we have observed different interesting facts regarding higher order nonclassical states in general and possibilities of observing higher order nonclassicality in different physical systems in particular. To be precise, we have extensively studied the possibilities of observing HOA, HOS and HOSPS in different intermediate states. Although study of higher order nonclassicality in intermediate states constitutes the major part of the present thesis, the thesis work is not limited to that. For example, we have studied the possibilities of observing reduction of quantum phase fluctuations in the intermediate states. The criterion of reduction of quantum phase fluctuations is a kind of stronger nonclassical criterion but does not provide signature of higher order nonclassicality. Further we have studied the possibilities of observing HOA in some optical processes too. All the results obtained through these studies are reported in the previous chapters. The conclusions of the previous chapters have been summarized in the present chapter. Possibilities of experimental verification of the theoretical results of the present thesis, the limitations of the present work and the scope of the future work are also discussed in the present chapter.
6.1 Conclusions

The significant results and conclusions obtained through the studies reported in Chapter 1 to 5 are as following.

1. Normally criterion of a particular kind of nonclassicality is presented through an inequality and once an inequality is established several technical reports appears with the observation that a particular physical system satisfies this criterion. Here we have shown that arbitrarily large number of such higher order inequalities can be established and consequently reports of satisfaction of particular nonclassical criterion in particular system does not provide much new Physics unless the criterion itself has a very specific physical meaning.

2. Such meanings are associated with the condition of HOA, HOS and HOSPS. For example, potential SPSs are expected to satisfy the condition of HOA. In Chapter 2, HOA is observed by us in several physical systems (e.g. Intermediate states; BS, GBS, NBS, RBS, PACS, HS, NLESS and NLVSS ) [39, 68, 75, 85]. Their quality as potential SPS is numerically compared by using a quantitative measure of quality of SPS (defined in 2.48). The comparison among the different intermediate states shows that Hypergeometric state is better than BS and GBS as a source of single photon.

3. Most of the intermediate states studied in the present thesis show HOA. But all the intermediate states are not higher order antibunched (for example, Photon subtracted coherent state $|\alpha, -m\rangle$ is always higher order bunched).

4. An intermediate state which shows HOA may also show higher order bunching or higher order coherence for particular values of the control parameters (for example, negative binomial state and generalized binomial
state show both higher order bunching and higher order antibunching for different parametric values).

5. A clear definition of higher order nonclassicality is provided in Chapter 3 and it is shown that Hillery type squeezing are not higher order according to this definition. It is also shown that the criteria of HOSPS and Hong Mandel type of HOS can be derived from a single framework. Criteria of HOS and HOSPS have been considerably simplified by using the tricks of operator ordering algebra. Now if a quantum state is available in its Fock state representation then we can directly use MATHEMATICA or MAPLE or any other analytic mathematical software to study the possibilities of observing HOS and HOSPS in that quantum state.

6. It is also shown that the lower order antibunching, HOA and HOSPS appear in novel regimes (i.e. they may or may not appear simultaneously as shown in Table 3.1). But in literature HOA and HOSPS have been used as synonymous [17]. Our observations establish that it is incorrect to use the condition of HOA as a test of HOSPS.

7. In Chapter 4, Binomial state always shows HOA and HOSPS but it does not show HOS for all values of \( p \). So we conclude that existence of HOSPS does not guarantee the existence of HOS. This is consistent with the corresponding observations in lower order. It is also observed by us that the NLVSS which shows higher order squeezing does not show HOSPS and HOA. The opposite is observed in NLESS and consequently it is established that the HOSPS and HOS are two independent signatures of higher order nonclassicality. Other intermediate states also confirm higher order nonclassicality in different regimes. Therefore appropriate choice of parameters is important to observe higher order nonclassicality.

8. All the intermediate states studied in Chapter 5 show reduction of \( U \) with
respect to its coherent state value\(^1\). This establishes that the intermediate states can satisfy the stronger criterion of nonclassicality compared to the criterion of usual antibunched state.

9. The Binomial state can show reduction of fluctuation of quantum phase with respect to its coherent state counterpart and thus it can satisfy the stronger criterion of nonclassicality. But it does not satisfy the criterion for all values of \(p\) close to 1 (see Fig. 5.1). In Chapter 2 we have shown that Binomial state is always antibunched up to any order. Thus for higher values of \(p\) (i.e. when \(p\) is close to 1) it is higher order antibunched but do not satisfy the criterion laid down on the basis of quantum phase fluctuations. Similar phenomenon is also observed in HS. Earlier Gupta and Pathak reported [71] that reduction of quantum phase fluctuation means antibunching but the converse is not true. This is the first time when an example of such a state which is antibunched but does not show reduction of quantum phase fluctuation with respect to coherent state, is found.

10. Further from the study of phase properties of Roy Roy GBS and HS we have learnt that the reduction of quantum phase fluctuation means antibunching but does not essentially mean higher order antibunching and therefore, it is not essential that these two stronger conditions of nonclassicality appear simultaneously. In connection to PACS we have observed that the more photon are added to coherent state the more nonclassical the PACS, is as far as the depth of nonclassicality associated with quantum phase fluctuation is concerned. This particular characteristic has also been reflected in higher order antibunching (see Chapter 2).

11. The present work provides an insight into the mutual relations between the well known nonclassical states and opens up a possibility of similar

\(^1\)In reciprocal binomial state, we have not observed this phenomenon.
work in broader class of nonclassical states. The simpler framework provided in Chapter 3 for the study of possibilities of observing Hong Mandel squeezing is also expected to be useful in the future work. We have shown that the depth of nonclassicality of a higher order nonclassical state varies with different control parameters (e.g. $\alpha$, $N$, $m$ etc.). These parameters represent some physical quantity and their value may be controlled and consequently by controlling these parameters we can control the depth of nonclassicality.

11. In the next section we will show that the theoretical predictions of the present work can be verified experimentally.

6.2 Possibility of the experimental verification of the results reported in the present work

We have mentioned that the results reported in the present work are experimentally verifiable. In Chapter 2, we have already discussed the possibility of detection of signature of HOA. But to establish that all the results of the present thesis are in principle experimentally verifiable we need to mention how intermediate state can be generated and once it is generated, how can we verify the existence of theoretical predictions of HOA, HOS and HOSPS. We briefly describe the possibilities and proposals in the next two subsections. The discussion below clearly indicates the scope of experimental verification of the present work in near future.

6.2.1 Existing proposals for generation of intermediate states

In Chapter 1, we have briefly mentioned that several proposals for the generation of different intermediate states exist. All these proposals are specific to
particular context and specific description. It would be of great use if we can obtain a general procedure which may generate intermediate states in general. Several attempts \cite{44, 81, 111} in this direction have resulted in two main approaches. One approach is based on time evolution generated by a generic and controlling Hamiltonian which drives an initial state to the final target state (pure state). Another approach is realized in following two steps: 1) The quantum state of interest is correlated (entangled) with another auxiliary system; 2) A measurement is performed on the auxiliary system, reducing the state of the system of interest to the desired target state. In principle, all these experiments (or experimental proposals) combine parametric amplification and state reduction techniques. In next few paragraphs we have briefly described the existing proposals for generation of intermediate states. We have not tried to go in to the details of the experiment as the present thesis is a theoretical one.

In 2004, two successful and encouraging experimental observations appeared. To be precise, coherent state was prepared by Zavatta et al \cite{67} and a single photon subtracted coherent state was generated by Wenger et al \cite{112}. There exist other proposals for production of PACS, These proposals use special state reduction and feedback method \cite{82, 83}. For example, consider the process of parametric amplification in which the signal (a) mode and idler (b) mode are generated with strong correlation. If we assume that initially the signal field is in the state $|\alpha\rangle$ and the idler is measured in the Fock state $|m\rangle$, then the state of the (a) mode is reduced to $|\alpha,m\rangle$.

A scheme for the generation of the RBS as well for BS, has been proposed in \cite{62}. The experimental proposal setup consists Ramsey zone, cavity (high quality) and ionization detector. For the generation of RBS, N atoms are prepared as a superposition of ground state in Ramsey zone. The atoms are then injected one by one in the cavity. The on resonant interaction of each atom with the cavity field Hamiltonian provides ground state at the output of the cavity as a result RBS is generated. Generation of NLESS and NLVSS with arbitrary non-
linearity functions can be engineered using a number of laser fields in trapped ions experiments for the quantized states of the atomic centre of mass motion [34]. Various proposals for experimental generation of binomial states are also present in the literature [44, 15, 81]. The binomial state may be generated when excited molecule undergoes M level vibrational relaxation under certain conditions [44]. Roy and Roy introduced GBS on the basis of Hahn polynomial which contains all other classical orthogonal polynomials as limiting cases [48]. There does not exist any proposal in the literature for experimental generation of Roy and Roy GBS, but generation of their limiting cases (e.g. Binomial state, Number state and coherent state) are reported in the literature. NBS can be generated in two ways. The first is, using displacement operator formalism (theoretical generation) and unitary time evolution operator in the interaction picture [72] and the second by producing geometric state (limiting case) which can be prepared in the non-degenerate three wave interaction system [57] or by the non-degenerate parametric amplifier described by two mode Hamiltonian [80]. Hypergeometric state can also be generated by following the same procedure as it is simply an one parameter generalized binomial state. In brief among the intermediate states studied in the present chapter PACS has already been produced experimentally and feasible experimental proposals exist for the other quantum states. The discussion above clearly establishes that a considerable scope of future work exists in the field of generation of intermediate states. Scope of future work in general is discussed in next section.

6.2.2 Possibility of experimental detection of higher order nonclassicalities of light

In the present thesis we have discussed mainly three type of higher order non-classicality (HOA, HOSPS and HOS) and have shown that these type of non-classicalities may be observed in different intermediate states. Now, it is natural to ask following questions:
1. Is it possible to experimentally detect the signature of higher order non-classicalities?

2. If yes, then how to detect signatures of different type of higher order non-classicalities?

Probably these questions arose in Mandel and Hong’s mind too and they have nicely answered these questions in [113]. They have shown that the signature of higher order nonclassicality can be obtained by homodyning the nonclassical state with an intense coherent light beam (local oscillator) of complex amplitude $v = |v|e^{i\theta}$. The homodyning is done by using an almost fully transmitting beam splitter (i.e. $t \simeq 1$ and $R \ll 1$). The schematic of experimental setup is shown in Fig. 6.1. In the experiment, the incoming single mode of nonclassical light (described by annihilation operator $a$) is mixed by the beam splitter with the coherent light of a strong local oscillator of the same optical frequency, whose phase $\theta$ can be varied. The combined light having amplitude $\hat{a} + v$ then falls on the photo-detector that counts the number $N$ of photons detected in some time interval $T$ short compared to the coherence time. The moments of $N$ can be determined by repeating the measurement many times.

The intrinsic beauty of this excellent correlation experiment lies in the fact that it can be used for experimental detection of HOS, HOSPS, HOA and other
moment based criterion of nonclassicality [35 and references there in]. To be precise, Mandel and Hong had shown that the existence of HOS may be observed through measurement of appropriate HOSPS [113]. In other words if we can experimentally measure \( d_h \) then we can obtain \( S_{HM}^2 \). It is straightforward to observe that we can experimentally measure \( d_h \) and \( d \) if we can measure the moments of number operator (i.e. quantities like \( \langle N \rangle, \langle N^2 \rangle, \langle N^3 \rangle \ldots \)). As the experimental values of these moments can be obtained from repeated measurement of photon count (in the counter shown in Fig. 6.1), so this particular correlation experiment is sufficient for experimental detection of all kind of higher order nonclassicalities discussed here.

The Mandel-Hong experimental scheme [113] depicted in Fig. 6.1 is already shown to be useful for study of HOS [13, 29, 113], HOSPS [29] and HOA [26, 39, 88]. Keeping these facts in mind we find it tempting to describe the relation between HOS and HOSPS\(^3\) in little more detail as it may interest others to experimentally verify the theoretical predictions of the present work.

The relation between the generating function for the factorial moments of the photoelectric counts \( n \) in a short time \( T \) and the generator of normal moments of the light intensity \( I \) is [113]

\[
\langle (1 - \xi)^N \rangle = \langle e^{-\alpha T I \xi} : \rangle,
\]

where \( \xi \) is generating function and the intensity operator after homodyning (see Fig. 6.1) is

\[
I = (a^\dagger + v^*)(a + v).
\]

\(^2\)Prakash and Mishra [29] have also shown that \( d_h(2n - 1) \approx \alpha^{2n} |v|^{2n} \langle : (\Delta X)^{2n} \rangle \), where \( \alpha \) is the efficiency of the detector.

\(^3\)The approach followed to derive the relation is available in more detail at [113, 13].
Expanding both sides of (6.1) and comparing the coefficients of $\xi$ one can easily obtain,
\[
\langle N \rangle = \alpha T \langle I \rangle.
\] (6.3)

Now, if we multiply both sides of (6.1) by $e^{\alpha T \langle I \rangle \xi}$ and use (6.3), then we obtain,
\[
\left\langle (1 - \xi)^N \right\rangle e^{\xi \langle N \rangle} = \left\langle e^{-\alpha T \xi (I - \langle I \rangle)} \right\rangle = \left\langle e^{-\alpha T \xi \Delta I} \right\rangle.
\] (6.4)

Expansion of both sides of (6.4) as power series in $\xi$ and comparison of the coefficients of $\xi^n$ yields
\[
\sum_{r=0}^{n} \binom{n}{r} (-1)^{n-r} \langle N^{(r)} \rangle \langle N \rangle^{N-r} = (\alpha T)^n \left\langle (\Delta I)^n \right\rangle.
\] (6.5)

As HOS is well defined for even orders only, we can consider $n$ as even in (6.5). For even values of $n$, (6.5) reduces to
\[
\left\langle (\Delta N)^n \right\rangle = (\alpha T)^n \left\langle (\Delta I)^n \right\rangle.
\] (6.6)

From (6.2) we obtain
\[
\Delta I = v \Delta a^\dagger + v^* \Delta a + \Delta (a^\dagger a).
\] (6.7)

Since the local field used for homodyning is assumed to be strong hence the first two term in (6.7) will dominate and we can neglect the third term. Using definitions of field quadratures $X = \frac{1}{\sqrt{2}}(a + a^\dagger)$ or $P = \frac{i}{\sqrt{2}}(a - a^\dagger)$ and neglecting the third term in (6.7) we obtain
\[
\Delta I = \frac{|v|}{\sqrt{2}} [\Delta X \cos(\theta) + \Delta P \sin(\theta)].
\] (6.8)

Substituting (6.8) in (6.6) and choosing $\theta = n\pi$, we finally get
\[
\left\langle (\Delta N)^n \right\rangle = \frac{(\alpha T)^n |v|^n}{2^n} \left\langle (\Delta X)^n \right\rangle.
\] (6.9)
We can use (6.9) to rewrite the condition of $n$th order HOS (3.12) as

$$\sum_{i=0}^{n-1} t_{2i}^n C_{2i} \langle (\Delta X)^{n-2i} : \rangle = \frac{2^n}{(\alpha T)^n |\mu|^n} \sum_{i=0}^{n-1} t_{2i}^n C_{2i} \langle (\Delta N)^{n-2i} \rangle < 0. \quad (6.10)$$

Now from (6.10) we can easily conclude the following:

1. The signature of HOS can be experimentally observed through several HOSPSs.

2. HOA only reflects the existence of lowest order squeezing it does not provide any information about HOS.

3. Even the existence of $n$th order HOSPS does not establish the existence of $n$th order HOS. This is so because HOSPS of higher order can exist in absence of lower order (as shown in Table 4.1).

### 6.3 Limitations of the present work and scope of future work

The major limitation of the present work lies in the fact that apart from PACS no other intermediate states have been experimentally realized so far. But still the situation is quite optimistic because there exist several strong theoretical proposals [57, 80, 82] for experimental generation of the other intermediate states. We believe that the theoretical predictions of the present work, recent experimental generation of PACS [67], Fock states [114] and the possibility of using intermediate state for quantum communication will motivate experimentalist to create these quantum states in the laboratory. Here we would like to note that the present study is the first one of his kind in which rigorous attempts have been made to understand the mutual relationship between different higher order nonclassical states. The effort is successful to provide an insight into the mutual relations between the well known nonclassical states and opens up a
possibility of similar work in broader class of nonclassical states. The simpler framework provided for the study of possibilities of observing Hong Mandel squeezing is also expected to be useful in the future work. The scope of future work is many fold. At present strong possibilities of future work appears in the following:

1. Applications of higher order antibunched light in quantum cryptography. Specially in the realization of single photon sources and quantum random number generator. The quantitative measure used here to compare the potential single photon sources may be used to compare the other existing proposals of SPS.

2. Experimental verification of the theoretical results of the present work.

3. Use of higher order squeezed light in quantum teleportation, continuous variable, quantum cryptography and precise optical measurement.

4. Study of higher order nonclassicalities in other quantum states of relevance (e.g. superposition states).

In the previous section, it is shown that the higher order nonclassical effects reported in the present thesis are experimentally observable and in the present section it is stated that the higher order nonclassical states have potential applications in quantum communications, optical communications and other field of science which are related to our day to day life. Keeping these facts in mind, we finish the present thesis with the expectation that the present work will be useful in future development of higher order nonclassical states of light.