CHAPTER 8

CREEP TRANSITION OF TRANSVERSELY ISOTROPIC
THICK-WALLED ROTATING CYLINDER UNDER
INTERNAL PRESSURE

8.1 INTRODUCTION
Thick-walled circular cylinders are used commonly either as pressure vessels intended for storage in industrial gases or as media for transportation of high pressurized fluids. A literature survey indicates that many authors [134-144] have solved problems on creep of thick-walled cylinder under different conditions. Rimrott [100] considered the hollow cylinder in the fully plastic state, using large strain theory and von Mises yield condition. For an ideal plastic material without strain hardening, the stress distribution in solid rotating cylinder has been given by Nadai [5]. Hodge Jr. and Balaban M. [101] study the elastic-plastic problem of a rotating cylinder and compared results obtained with finite and infinitesimal strains. Luke [145] has obtained the creep stresses of a rotating hollow circular cylinder made of isotropic and homogeneous materials. Wahl [146] has given stress distributions under steady state creep at elevated temperature for long rotating cylinders having axial bores and subjected to external radial tension. In analyzing the problem, these authors used some simplifying assumptions. First, the deformations assumed to be small enough to make infinitesimal strain theory applicable. Second, simplifications were made in the constitutive equations like incompressibility of the material. Incompressibility of material is one of the most important assumption which simplifies the problem. In fact in most of the cases, it is not possible to find a solution in closed form without these assumptions.
Transition theory [42-44, 59, 92-95, 124-125] does not require any of the above assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out.

In this chapter, we calculated creep stresses and strain rates for a thick-walled circular rotating cylinder under internal pressure by using transition theory. Results obtained have been discussed numerically and depicted graphically.

### 8.2 GOVERNING EQUATIONS

Consider a thick-walled circular cylinder made of transversely isotropic material of internal and external radii 'a' and 'b' respectively and rotating with an angular velocity 'ω' and pressure 'p' applied at the internal surface.

The governing equations for this problem are same as given in section 3.2.

### 8.3 SOLUTION THROUGH PRINCIPAL STRESS DIFFERENCE

It has been shown that the transition function through the principal stress - difference [126-133, 147-148] at the transition point $P \to -1$ gives the creep stresses. For finding the creep stresses at the transition point $P \to -1$, we define the transition function $R$ as

$$R = T_{rr} - T_{θθ} = \frac{2C_{66}}{n} β^n \left[ 1 - (1 + P)^n \right]. \quad (8.1)$$

Taking the logarithmic differentiation of equation (8.1) with respect to 'r', we get

$$\frac{d}{dr} \left( \log R \right) = \frac{1}{rR} \left[ 2C_{66} P \beta^n \left( 1 - (1 + P)^n \right) - 2C_{66} P β^{n+1} \frac{dP}{dβ} (1 + P)^{n-1} \right]. \quad (8.2)$$

Substituting the value of $\frac{dP}{dβ}$ in equation (8.2) from equation (3.3), the equation (8.2) becomes

$$\frac{d}{dr} \left( \log R \right) = \frac{1}{rR} \left[ 2C_{66} P \beta^n \left( 1 - (1 + P)^n \right) + 2PC_{66} β^n (1 + P)^n \right. + \left. 2C_{66} (1 - C_1) P β^n - \frac{2}{n} C_{66} C_1 β^n \left( 1 - (1 + P)^n \right) \right], \quad (8.3)$$

where $C_1 = 2C_{66} / C_{11}$.

Asymptotic value of equation (8.3) as $P \to -1$ is
\[
\frac{d}{dr} \left( \log R \right) = \left[ -4C_{66} \beta^n + 2C_1C_{66} \beta^n - \frac{2C_{66}C_1}{n} \beta^n - C_1 r^2 \omega^2 \right] \left( \frac{2C_{66}}{n} \beta^n \right) \]

Integrating equation (8.4) and taking asymptotic value of \( \beta \) as \( P \rightarrow -1 \) is \( \frac{D}{r} \), \( D \) being a constant, we get

\[
R = T_{rr} - T_{\theta\theta} = A_1 r^{-c_1} \left[ \frac{2C_{66} D^n}{n r^n} \right]^2 \exp(f), \quad (8.5)
\]

where \( f = \int \frac{2C_{66} D^n}{n r^n} \frac{2C_{66} C_1 D^n}{n r^n} - C_1 r^2 \omega^2 dr \) and \( A_1 \) is a constant of integration.

From equation (8.5) and (2.6), the radial stress is calculated as

\[
T_{rr} = -A_1 \int r^{-(1+c_1)} \left[ \frac{2C_{66} D^n}{n r^n} \right]^2 \exp(f) dr - \frac{D r^2 \omega^2}{2} + A_2, \quad (8.6)
\]

where \( A_2 \) is constant of integration.

From equation (8.5), we have

\[
T_{\theta\theta} = T_{rr} - A_1 r^{-c_1} \left[ \frac{2C_{66} D^n}{n r^n} \right]^2 \exp(f). \quad (8.7)
\]

The integration constants \( A_1 \) and \( A_2 \) are obtained by using boundary conditions (3.4) in equation (8.6). Thus

\[
A_1 = \frac{\rho \omega^2}{2} \left( a^2 - b^2 \right) - p, \quad \int_a^b \left[ \frac{2C_{66} D^n}{n r^n} \right]^2 \exp(f) dr
\]

and

\[
A_2 = \frac{\rho b^2 \omega^2}{2} + \left[ \frac{\rho \omega^2}{2} \left( a^2 - b^2 \right) - p \right] \left[ \frac{2C_{66} D^n}{n r^n} \right]^2 \exp(f) \left[ \int_a^b \frac{2C_{66} D^n}{n r^n} \exp(f) dr \right].
\]

Substituting the values of \( A_1 \) and \( A_2 \) in equation (8.6) and (8.7), we get
The axial stress is obtained from equation (3.2) as
\[ T_{zz} = \frac{C_{13}}{2(C_{11} - C_{66})} (T_{rr} + T_{\theta \theta}) + \left[ \frac{C_{33}(C_{11} - C_{66}) - C_{13}^2}{C_{11} - C_{66}} \right] e_{zz}. \]  

(8.10)

Using equation (8.10) in the end condition (3.5), the axial strain is given as
\[
K e_{zz} = \frac{a^2 p}{(b^2 - a^2)} - \frac{\rho \omega^2 C_{13}}{4(C_{11} - C_{66})(b^2 - a^2)} \left[ b^4 - a^4(2b^2 - a^2) \right] \\
+ \left\{ \frac{C_{13} \rho \omega^2}{C_{11} - C_{66}} \right\} \left[ \frac{b^4 - a^4(2b^2 - a^2)}{4} \right] - \frac{2C_{13}}{(b^2 - a^2)(C_{11} - C_{66})} \int_a^b T_{rr} dr, \\
- \frac{C_{13}}{(b^2 - a^2)} \left\{ \int_a^b \left[ p - \frac{\rho \omega^2 (a^2 - b^2)}{2} \right] r^{-C_{11}} \left( \frac{2C_{66} D_n^a}{n r^a} \right)^2 \exp(f) dr \right\}. \\
\]  

(8.11)

where
\[ K = \left[ \frac{C_{33}(C_{11} - C_{66}) - C_{13}^2}{C_{11} - C_{66}} \right]. \]

Substituting equation (8.11) in equation (8.10), we get
\[ T_{zz} = \frac{C_{13}}{2(C_{11} - C_{66})} (T_{rr} + T_{\theta \theta}) + \frac{p}{\left( \frac{b}{a} \right)^2 - 1} - \frac{C_{13}}{(b^2 - a^2)(C_{11} - C_{66})} \int_a^b r(T_{rr} + T_{\theta \theta}) dr. \]  

(8.12)

Now we introduce the following non-dimensional quantities as
\[ R = \frac{r}{b}, \quad R_0 = \frac{a}{b}, \quad \sigma_r = \frac{T_{rr}}{C_{66}}, \quad \sigma_\theta = \frac{T_{\theta \theta}}{C_{66}}, \quad \sigma_z = \frac{T_{zz}}{C_{66}} \quad \text{and} \quad \Omega^2 = \frac{\rho b^2 \omega^2}{C_{66}}. \]

The transitional stresses in non-dimensional form are

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\[
\sigma_r = \frac{-p + \frac{\Omega^2}{2} (R_0^2 - 1)}{C_{66} \cdot 2} \left[ \int_{R_0}^{R^{(1+C_1)}} \left[ \frac{2C_{66}}{nR^n} \left( \frac{D}{b} \right)^n \right] \exp(f_1) dR \right] + \frac{\Omega^2}{2} (1 - R^2),
\]

(8.13)

\[
\sigma_\theta = \sigma_r - \frac{\Omega^2}{2} \left( R_0^2 - 1 \right) - \frac{p}{C_{66}} \left( R^{-C_1} - \frac{2C_{66}}{nR^n} \left( \frac{D}{b} \right)^n \right)^2 \exp(f_1),
\]

(8.14)

\[
\sigma_z = \frac{C_{13}}{C_{11} (2 - C_1)} \left[ \sigma_r + \sigma_\theta \right] + \frac{p}{C_{66}} \left( R_0^2 \right) \left( 1 - \frac{R_0^2}{1 - R_0^2} \right) - \frac{2C_{13}}{C_{11} (2 - C_1) (1 - R_0^2)} \int_{R_0}^{R} \exp(f_1) dR ,
\]

(8.15)

where \( f_1 = \int \frac{2C_{66} C_{1} \left( \frac{D}{b} \right)^n}{nR^n} dR \).

If the angular speed becomes zero, then the transitional stresses become

\[
\sigma_r = \frac{-p}{C_{66}} \left[ \int_{R_0}^{R^{(1+C_1)}} \left[ \frac{2C_{66}}{nR^n} \left( \frac{D}{b} \right)^n \right] \exp(f_2) dR \right],
\]

(8.16)

\[
\sigma_\theta = \sigma_r + \frac{p}{C_{66}} \left( R^{-C_1} - \frac{2C_{66}}{nR^n} \left( \frac{D}{b} \right)^n \right)^2 \exp(f_2),
\]

(8.17)

\[
\sigma_z = \frac{C_{13}}{C_{11} (2 - C_1)} \left[ \sigma_r + \sigma_\theta \right] + \frac{p}{C_{66}} \left( R_0^2 \right) \left( 1 - \frac{R_0^2}{1 - R_0^2} \right) - \frac{2C_{13}}{C_{11} (2 - C_1) (1 - R_0^2)} \int_{R_0}^{R} \exp(f_2) dR ,
\]

(8.18)

where \( f_2 = \int \frac{2C_{66} C_{1} \left( \frac{D}{b} \right)^n}{nR^n} dR \).

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8.4 ISOTROPIC MATERIAL

Creep stresses for isotropic materials are

\[
\sigma_r = -\frac{p}{C_{66}} + \frac{\Omega^2}{2} \left( R_0^2 - 1 \right) \int_{R_0}^{1} R^{-(1+C)} \left[ \frac{2C_{66}}{nR^n} \left( \frac{D}{b} \right)^n \right] \exp(f_2) \, dR + \frac{\Omega^2}{2} (1 - R^2), \tag{8.19}
\]

\[
\sigma_\theta = \sigma_r - \frac{\Omega^2}{2} \left( R_0^2 - 1 \right) \int_{R_0}^{1} R^{-(1+C)} \left[ \frac{2C_{66}}{nR^n} \left( \frac{D}{b} \right)^n \right] \exp(f_2) \, dR,
\]

\[
\sigma_z = \left( 1 - C \right) \left( \sigma_r + \sigma_\theta \right) + \frac{p}{C_{66}} \left( \frac{R_0^2}{1 - R_0^2} \right) - \frac{2}{\left( 1 - R_0^2 \right)^2} \int_{R_0}^{1} R \left( \sigma_r + \sigma_\theta \right) \, dR.
\tag{8.21}
\]

These equations are same as obtained by Gupta et al. [137].

8.5 STRAIN RATES

The stress-strain rate relationship can be given as

\[
\dot{\varepsilon}_{rr} = -\frac{(A - 2C_{66})}{H} \Theta + 2 \frac{(A - C_{66})}{H} \dot{T}_{rr} + \frac{T_{zz}}{H} \left( A - 2C_{66} - \frac{2C_{13} C_{66}}{C_{33}} \right), \tag{8.22}
\]

\[
\dot{\varepsilon}_{\theta\theta} = \frac{2(A - C_{66})}{H} \dot{T}_{\theta\theta} - \frac{(A - 2C_{66})}{H} \Theta + \frac{T_{zz}}{H} \left( A - 2C_{66} - \frac{2C_{13} C_{66}}{C_{33}} \right), \tag{8.23}
\]

\[
\dot{\varepsilon}_{zz} = -\frac{2C_{13} C_{66}}{HC_{33}} \Theta + \frac{T_{zz}}{C_{33}} \left[ \frac{C_{11} - C_{66} + 2C_{13}}{4(A - C_{66})} \right], \tag{8.24}
\]

where \( \dot{\varepsilon}_{rr}, \dot{\varepsilon}_{\theta\theta}, \dot{\varepsilon}_{zz} \) is the strain rate tensor with respect to flow parameter ‘\( t \)’ and

\[
\Theta = T_{rr} + T_{\theta\theta} + T_{zz}, \quad H = 4C_{66}(A - C_{66}), \quad A = C_{11} - \frac{C_{13}^2}{C_{33}}.
\]

Differentiating equation (2.3) with respect to ‘\( r \)’, we get

\[
\dot{\varepsilon}_{\theta\theta} = -\beta^{n-1} \dot{\beta}.
\tag{8.25}
\]

For SWAINGER measure \( (n = 1) \) we have from equation (8.25)

\[
\dot{\varepsilon}_{\theta\theta} = -\dot{\beta}.
\tag{8.26}
\]

The transition value of equation (8.1) as \( P \to -1 \) gives
Using equations (8.25), (8.26) and (8.27) in equations (8.22), (8.23) and (8.24), we get

\[
\dot{\sigma}_{rr} = \frac{\alpha}{\eta} \left( \sigma_r + \sigma_\theta + \sigma_z \right) + \frac{\sigma_z}{\eta} \left( \alpha - \frac{2C_{13}C_{66}}{C_{11}C_{33}} \right),
\]

\[
\dot{\sigma}_{\theta\theta} = \frac{\alpha}{2} \left( \sigma_r + \sigma_\theta + \sigma_z \right) + \frac{\sigma_z}{\eta} \left( \alpha - \frac{2C_{13}C_{66}}{C_{11}C_{33}} \right),
\]

\[
\dot{\sigma}_{zz} = \frac{2C_{13}C_{66}}{C_{11}C_{33}\eta} \left( \sigma_r + \sigma_\theta + \sigma_z \right) + \frac{\sigma_z}{C_{33}\eta} \left( 1 - \frac{C_{66}}{C_{11}} + \frac{2C_{13}}{C_{11}} \right),
\]

where \( \eta = 4 \left( 1 - \frac{C_{13}^2}{C_{11}C_{33}} - \frac{C_{66}}{C_{11}} \right) \),

\( \alpha = 1 - \frac{C_{13}^2}{C_{11}C_{33}} - \frac{2C_{66}}{C_{11}} \),

\( \beta = 1 - \frac{C_{13}^2}{C_{11}C_{33}} \),

\( \chi = \left[ \frac{n}{2} (\sigma_r - \sigma_\theta) \right]^{1-n} \).

For isotropic materials, strain rates (8.28) becomes

\[
\dot{\sigma}_{rr} = \frac{\alpha}{\eta} \left( \sigma_r + \sigma_\theta + \sigma_z \right) + \frac{\sigma_z}{\eta} \left( \alpha - C(1-2C) \right),
\]

\[
\dot{\sigma}_{\theta\theta} = \frac{\alpha}{2} \left( \sigma_r + \sigma_\theta + \sigma_z \right) + \frac{\sigma_z}{\eta} \left( \alpha - C(1-2C) \right),
\]

\[
\dot{\sigma}_{zz} = C(1-2C)(\sigma_r + \sigma_\theta + \sigma_z) + \frac{\sigma_z}{C_{33}\eta} \left( 1 - \frac{C_{66}}{C_{11}} + \frac{2C_{13}}{C_{11}} \right).\]

8.6 NUMERICAL ILLUSTRATION AND DISCUSSION

For calculating the stresses and strain rates distribution based on the above analysis, the following values of measure \( n \), constant \( D \), pressure \( P \) and angular velocity \( \Omega^2 \) have been taken as: \( \Omega^2 = 0,5 \) \( n = 1, 1/3 \) and \( 1/7 \) (i.e., \( N = 1, 3 \) and \( 7 \))

\( p = 0.5, 1.5 \) \( D = 1 \)
Elastic constants $C_{ij}$'s for transversely isotropic material (Magnesium) and isotropic material (Brass) have been given in table 2.1

Curves have been drawn in figure 8.1 and figure 8.2 between stresses and radii ratio ($R = r/b$) for transversely isotropic/isotropic material for different angular velocity.

It can be seen from figure 8.1 that without rotation, circumferential stress is maximum at the internal surface for isotropic and transversely isotropic circular cylinder under internal pressure for measure $n = 1$ and $n = 1/3$ ($N = 1, 3$) while for measure $n = 1/7$ ($N = 7$), the circumferential stress is maximum at external surface. With the increase in angular speed, it can be seen from figure 8.2, that circumferential stress is going on increasing at internal surface for transversely isotropic circular cylinder under internal pressure as compared to isotropic circular cylinder. With the increase in measure, circumferential stress is going on decreasing at internal surface for transversely isotropic and isotropic circular cylinder under internal pressure.

In figure 8.3 and figure 8.4, curves have been drawn for creep strain rates and radii ratio ($R = r/b$) for measure $n = 1, 1/3, 1/7$ and $p = 0.5, 1.5$. With the increase in angular speed, thick-walled circular cylinder made of isotropic material under internal pressure, the creep rate has large value at the internal surface as compared to cylinder made of transversely isotropic material for measure $n = 1/3$ ($i.e. N = 3$) and these values further increases at the internal surface with the increase in pressure. For measure $n = 1/7$ ($i.e. N = 7$), the creep rates have lesser value at the internal surface as compared to measure $n = 1/3$ ($i.e., N = 3$). The value of creep rates decreases with the increase in strain.

**8.7 CONCLUSION**

Circumferential stress is maximum at the internal surface for transversely isotropic material as compared to isotropic material. With the increase in angular speed, it can be seen from figure 8.2, that circumferential stress is going on increasing at internal surface for transversely isotropic circular cylinder under internal pressure as compared to isotropic circular cylinder. Therefore, rotating circular cylinder under internal pressure made of isotropic material is on the safer side of the design as compared to rotating circular cylinder under internal pressure made of transversely isotropic material.
Figure 8.1: Creep stresses in a thick-walled rotating cylinder along the radius ($R$) for different measure of $N (= 1/n)$ and angular speed ($\Omega^2 = 0$).

Figure 8.2: Creep stresses in a thick-walled rotating cylinder along the radius ($R$) for different measure of $N (= 1/n)$ and angular speed ($\Omega^2 = 5$).
Figure 8.3: Strain rates distribution in a thick-walled rotating cylinder along the radius \((R)\) for different measure of \(N (=1/n)\) and angular speed \((\Omega^2 = 0)\).

Figure 8.4: Strain rates distribution in a thick-walled rotating cylinder along the radius \((R)\) for different measure of \(N (=1/n)\) and angular speed \((\Omega^2 = 5)\).