Chapter 5

MHD Non-Darcy Mixed Convective Flow with Non-uniform Heat Source/Sink and Variable Viscosity *

5.1 Introduction

Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially for fluid viscosity. To accurately predict the flow and heat transfer rates, it is necessary to take into account this variation of viscosity. The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions Chamkha (2000) investigated thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink. Seddeek (2000) studied the effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation. Hossain and Munir (2000) analyzed a two-dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate. Hossain et al. (2001) discussed the effect of radiation on free convection flow of a fluid with variable viscosity from a porous vertical plate. Seddeek (2005) analyzed the effects of non-Darcian on forced convection

heat transfer over a flat plate in a porous medium with temperature dependent viscosity. Prasad et al. (2006) analysed the radiation effects on an unsteady two-dimensional hydromagnetic free convective boundary layer flow of a viscous incompressible fluid past a semi-infinite vertical plate with mass transfer in the presence of heat source or sink. Seddeek and Salem (2006) have analyzed the effects of variable viscosity with magnetic field on the flow and heat transfer. Ali (2006) analyzed the effect of variable viscosity on mixed convection heat transfer along a vertical surface in a saturated porous medium. Salem (2007) studied the problem of flow and heat transfer of all electrically conducting viscoelastic fluid having temperature dependent viscosity as well as thermal conductivity fluid over a continuously stretching sheet in the presence of a uniform magnetic field for the case of power-law variation in the sheet temperature. Ahmad et al. (2010) studied the boundary layer flow and heat transfer past a stretching plate with variable thermal conductivity.

In view of the above discussions, an investigate has been made to study the effect of variable viscosity on MHD non-Darcy flow and heat transfer over a continuous stretching sheet with electric field in presence of Ohmic dissipation and non-uniform source/sink of heat. The flow is subjected to a transverse magnetic field normal to the plate. The Forchheimer’s extension is used to describe the fluid flow in the porous medium. Highly non-linear momentum and heat transfer equations are solved numerically using fifth-order Runge-Kutta Fehlberg method with shooting technique. The effects of various parameters on the velocity and temperature profiles as well as on local skin-friction co-efficient and local Nusselt number are presented in graphical and in tabular form. It is hoped that the results obtained from the present investigation will provide useful information for application and also serve as a complement to the previous studies.

5.2 Mathematical Formulations

5.2.1 Flow Analysis

Consider two-dimensional study incompressible electrically conducting fluid flow over a continuous stretching sheet embedded in a porous medium. The fluid properties are assumed to be isotropic and constant, except for the fluid viscosity $\mu$ which is assumed to vary as be an inverse linear function of temperature $T$, in the form (see Lai and Kulacki
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\[
\frac{1}{\mu} = \frac{1}{\mu_\infty} \left[ 1 + \gamma(T - T_\infty) \right] \tag{5.1}
\]

\[
\frac{1}{\mu} = a(T - T_r) \tag{5.2}
\]

where

\[
a = \frac{\gamma}{\mu_\infty} \quad \text{and} \quad T_r = T_\infty - \frac{1}{\gamma} \tag{5.3}
\]

Both \(a\) and \(T_\infty\) are constant and their values depend on the reference state and the thermal property of the fluid, i.e. \(\gamma\). In general, \(a \geq 0\) for liquids and \(a < 0\) for gases. Consider the uniform flow of velocity \(U_\infty\) and temperature \(T_\infty\) through a highly porous medium bounded by a semi-infinite flat plate parallel to the flow. Also, \(\theta_r\) is a constant which is defined by

\[
\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)} \tag{5.4}
\]

and primes denote differentiation with respect to \(\eta\). It is worth mentioning here that for \(\gamma \to 0\) i.e. \(\mu = \mu_\infty\) (constant) then \(\theta_r \to \infty\). It is also important to note that \(\theta_r\) is negative for liquids and positive for gases. The flow model is based on the following assumption that the flow is steady, incompressible, laminar and the fluid viscosity which is assumed to be an inverse linear function of temperature.

The flow region is exposed under uniform transverse magnetic fields \(\vec{B}_0 = (0, B_0, 0)\) and uniform electric field \(\vec{E}_0 = (0, 0, -E_0)\) (see Fig.5.1). Since such imposition of electric and magnetic fields stabilizes the boundary layer flow (Dandapat and Mukhopadhyay, (2003)). It is assumed that the flow is generated by stretching of an elastic boundary sheet from a slit by imposing two equal and opposite forces in such a way that velocity of the boundary sheet is of linear order of the flow direction. We know from Maxwell’s equation that \(\nabla \cdot \vec{B} = 0\) and \(\nabla \times \vec{E} = 0\). When magnetic field is not so strong then electric field and magnetic field obey Ohm’s law \(\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})\), where \(\vec{J}\) is the Joule current. The viscous dissipation and velocity of the fluid far away from the plate are assumed to be negligible.

We assumed that magnetic Reynolds number of the fluid is small so that induced magnetic field and Hall effect may be neglected. We take into account of magnetic field effect as well as electric field in momentum. Under the above stated physical situation, the governing boundary layer equations for momentum and energy for mixed convection under Boussinesq’s approximation are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5.5}
\]
\[
\frac{1}{\varepsilon^2} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{1}{\rho_\infty \varepsilon} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\sigma}{\rho_\infty} (E_0 B_0 - B_0^2 u) - \nu \frac{C_b}{\sqrt{k}} u^2 + g \beta_r (T - T_\infty) \] (5.6)

where \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions respectively; \( \nu \) is the kinematic viscosity; \( g \) is the acceleration due to gravity; \( \rho_\infty \) is the density of the fluid; \( \beta \) is the coefficient of thermal expansion; \( T, T_w \) and \( T_\infty \) are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively. \( k \) is the permeability of the porous medium, \( \epsilon \) is the porosity of the porous medium, \( C_b \) is the form of drag coefficient which is independent of viscosity and other physical properties of the fluid but is dependent on the geometry of the medium. The third and fourth terms on the right hand side of Eq. (5.6) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively. It is assumed that the normal stress is of the same order of magnitude as that of the shear stress in addition to usual boundary layer approximations for deriving the momentum boundary layer Eq. (5.6). The following appropriate boundary conditions on velocity are appropriate in order to employ the effect of stretching of the boundary surface causing flow in \( x \)-direction as

\[
u = U_w(x) = bx, \quad v = 0 \quad \text{at} \quad y = 0 \quad (5.7)
\]

\[
u = 0 \quad \text{as} \quad y \to \infty. \quad (5.8)
\]

To solve the governing boundary layer Eq. (5.6), the following similarity transformations are introduced:

\[
u = b x f'(\eta), \quad v = -\sqrt{\nu_\infty} f(\eta), \quad \eta = \sqrt{\frac{b}{\nu_\infty}} y \quad (5.9)
\]

Here \( f(\eta) \) is the dimensionless stream function and \( \eta \) is the similarity variable. Substitution of Eq. (5.9) in the Eq. (5.6) results in a third-order non-linear ordinary differential equation of the following form.

\[
\frac{f'''}{\epsilon} + \left( 1 - \frac{\theta}{\theta_r} \right) \frac{f'}{\epsilon^2} + \frac{1}{\theta - \theta_r} \frac{\theta f''}{\epsilon} + \left( 1 - \frac{\theta}{\theta_r} \right) H a^2 (E_1 - f') = \left( 1 - \frac{\theta}{\theta_r} \right) \left( \frac{f'^2}{\epsilon^2} + F^* f'^2 - \lambda \theta \right) + k_1 f' \quad (5.10)
\]

where \( k_1 = \frac{\nu_\infty}{k} \) is the porous parameter, \( H a = \sqrt{\frac{F_*}{F}} B_0 \) is Hartmann number, \( E_1 = \frac{E_0}{T_\infty b x} \) is the local electric parameter, \( F^* = \frac{C_b}{\sqrt{k}} x \) is the local inertia-coefficient, \( \lambda = \frac{g \beta_r (T_w - T_\infty)}{b x^2} \) is the buoyancy or mixed convection parameter.

In view of the transformations, the boundary conditions (5.7)-(5.8) take the following non-dimensional form on stream function \( f \) as

\[
\frac{f}{0} = 0, \quad \frac{f'(0)}{1} = 1, \quad \frac{f'(\infty)}{0} = 0 \quad (5.11)
\]
The physical quantities of interest are the skin-friction coefficient \( C_f \), which is defined as
\[
C_f = \frac{\tau_w}{\rho_\infty U^2/2}, \tag{5.12}
\]
where wall sharing stress \( \tau_w \) is given by
\[
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \tag{5.13}
\]
Using the non-dimensional variables (5.9), we get from Eqs. (5.12) and (5.13) as
\[
\frac{1}{2} C_f Re_x^{1/2} = \frac{\theta_r}{\theta_r - \theta^f}(0) \tag{5.14}
\]
where \( Re_x = \frac{U_x}{\nu} \) is the local Reynolds number.

### 5.2.2 Similarity Solution of the Heat Transfer Equation

The governing boundary layer heat transfer with viscous and Ohmic dissipations is given by:
\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho_\infty C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma}{\rho_\infty C_p} (u B_0 - E_0)^2 + \frac{1}{\rho_\infty C_p} q''' \tag{5.15}
\]
where \( C_p \) is the specific heat at constant pressure and \( \kappa \) is the thermal conductivity.

The non-uniform heat source/sink, \( q''' \), is modeled as
\[
q''' = \frac{k u_w(x)}{x \nu_\infty} \left[ A^*(T_w - T_\infty) f' + (T - T_\infty) B^* \right], \tag{5.16}
\]
where \( A^* \) and \( B^* \) are the coefficient of space and temperature dependent heat source/sink respectively. Here we make a note that the case \( A^* > 0, \ B^* > 0 \) corresponds to internal heat generation and that \( A^* < 0, \ B^* < 0 \) corresponds to internal heat absorption.

We consider non-isothermal temperature boundary condition as follows:
\[
T = T_w = T_\infty + A_0 \left( \frac{x}{l} \right)^2 \quad \text{at} \quad y = 0 \tag{5.17}
\]
\[
T \to T_\infty \quad \text{as} \quad y \to \infty \tag{5.18}
\]
where \( A_0 \) is the parameters of temperature distribution on the stretching surface, \( T_w \) stands for stretching sheet temperature and \( T_\infty \) is the temperature far away from the stretching sheet.

We introduce a dimensionless temperature variable \( \theta(\eta) \) of the form:
\[
\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{5.19}
\]
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where expression for $T_w - T_\infty$ is given by Eq. (5.17)-(5.18). Making use of the Eq. (5.19) in Eq. (5.15) we obtain non-dimensional thermal boundary layer equation as

$$\theta'' - \left(1 - \frac{\theta}{\theta_r}\right) Pr \left[2f'\theta - f\theta' \right] - Ha^2 E_c(E_1 - f')^2 = -E_c Pr(f'')^2 - (A^* f' + B^* \theta) \quad (5.20)$$

where $Pr = \left(1 - \frac{\theta}{\theta_r}\right)^{-1} Pr_\infty$ is the Prandtl number and $Pr_\infty = \frac{\rho \infty \infty C_p}{\kappa}$ is the ambient Prandtl number, and $E_c = \frac{\nu}{4 l^2 A_0 c_p}$ is the Eckert number. Temperature boundary conditions of the Eq. (5.17)-(5.18) take the following non-dimensional form

$$\theta(0) = 1, \quad \theta(\infty) = 0 \quad (5.21)$$

The local Nusselt number which are defined as

$$Nu_x = \frac{x q_w}{\kappa(T_w - T_\infty)} \quad (5.22)$$

where $q_w$ is the heat transfer from the sheet is given by

$$q_w = -\kappa \left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (5.23)$$

Using the non-dimensional variables (5.19), we get from Eqs. (5.22) and (5.23) as

$$Nu_x/Re_x^{1/2} = -\theta'(0) \quad (5.24)$$

5.2.3 Similarity Solution of the Concentration Equation

To study the effects of various physical parameters on concentration profiles as well as on local Sherwood number which are presented graphically. Conservation of species:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (5.25)$$

The boundary conditions for Eq. (5.25) as

$$u = U_w(x) = bx, \quad v = 0, \quad T = T_w = T_\infty + A_0 \left(\frac{x}{l}\right)^2, \quad (5.26)$$

$$C = C_w = C_\infty + A_1 \left(\frac{x}{l}\right)^2 \quad \text{at} \quad y = 0, \quad (5.27)$$

$$u = 0, \quad T \to T_\infty, \quad C \to C_\infty, \quad \text{as} \quad y \to \infty. \quad (5.28)$$

where $u$ and $v$ are the velocity components in the $x$ and $y$-directions respectively; $\nu$ is the kinematic viscosity; $C_\infty$ is the concentration of the species in the free stream, $T_w$ stands for stretching sheet temperature and $T_\infty$ is the temperature far away from the stretching...
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sheet, \( C_w \) stands for concentration at the wall and \( C_\infty \) is the concentration far away from the stretching sheet. To solve the governing boundary layer Eq. (5.25), the following similarity transformations are introduced

\[
u = bx f'(\eta), \quad v = -\sqrt{b\nu_\infty} f(\eta), \quad \eta = \sqrt{\frac{b}{\nu_\infty}} y. \tag{5.29}
\]

\[
\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{5.30}
\]

Substitution of Eqs. (5.29)-(5.30) into the governing Eq. (5.25) and using the above relations we finally obtain a system of non-linear ordinary differential equations with appropriate boundary conditions

\[
\phi'' + Sc (f \phi' - 2f' \phi) = 0. \tag{5.31}
\]

The boundary conditions (5.26)-(5.28) becomes

\[
f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad \text{at} \quad \eta \to 0 \tag{5.32}
\]

\[
f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \quad \text{as} \quad \eta \to \infty, \tag{5.33}
\]

where, \( Sc = \frac{\nu_\infty}{D} \) is the Schmidt number. The physical quantity of interest is the local Sherwood number which are defined as

\[
Sh_x = \frac{x q_m}{D(C_w - C_\infty)} \tag{5.34}
\]

where \( q_m \) is the mass transfer which is defined by

\[
q_m = -D \left( \frac{\partial C}{\partial y} \right)_{y=0}. \tag{5.35}
\]

Using the non-dimensional variables (5.29) and (5.30), we get from Eq. (5.34)-(5.35) as

\[
Sh_x/Re_x^{1/2} = -\phi'(0). \tag{5.36}
\]

5.3 Results and Discussion

Numerical solutions for effects of Non-Darcy mixed convection heat transfer over a stretching sheet in presence of magnetic field are reported. The results are presented graphically from Figs. 5.2-5.16 and conclusions is drawn that the flow field and other quantities of physical interest have significant effects. Comparisons with previously works are performed and excellent agreement between the results is obtained. Non-linear ordinary
differential equations are integrated by Runge-Kutta Fehlberg method with shooting technique.

Comparison of our results of $-\theta'(0)$ with those obtained by Chen (1998), Grubka and Bobba (1985) and Ishak et al. (2008), see Table 5.1 in absence of buoyancy force and magnetic field show a very good agreement. The values of skin-friction co-efficient $f''(0)$ for different values of Hartmann number $Ha$, Eckert number $Ec$, local electric field $E_1$ and Prandtl number $Pr$ are presented in Table 5.2. Physically, positive sign of $f''(0)$ implies that the fluid exerts a drag force on the sheet and negative sign implies the opposite. It is seen from this table that the skin-friction co-efficient $f''(0)$ decreases with an increase in the electric parameters $E_1$ and Prandtl number $Pr$ in absence of Hartmann number $Ha$. It is also observed that the skin-friction co-efficient increases with increasing the value of Hartmann number $Ha$ and Eckert number $Ec$.

Table 5.3 gives the values of wall temperature gradient $-\theta'(0)$ for different values of Hartmann number ($Ha$), Eckert number ($Ec$), local electric parameter ($E_1$) and Prandtl number ($Pr$). Analysis of the tabular data shows that magnetic field enhance the rate of heat transfer across the stretching sheet to the fluid. However, the effect of Prandtl number ($Pr$), in absence of local electric field parameter ($E_1$), is to reduce the rate of heat transfer from boundary stretching sheet to the fluid whereas in the presence of local electric parameter ($E_1$), the effect of Prandtl number ($Pr$) is to increase the rate of heat transfer. Thus application of electric field may change the limitation of heat transfer.

Fig. 5.2 shows the effect of Hartmann number ($Ha$) on velocity profiles by keeping other physical parameter fixed. Fig. 5.2 depicts that the effect of Hartmann number is to reduce the velocity distribution in the boundary layer which results in thinning of the boundary layer thickness. The decrease in the velocity profile is due to the fact that the transverse magnetic field has a tendency to retard the motion of the fluid as Hartmann number increases the Lorentz force.

Fig. 5.3 is the plot of velocity profile for various values of electric field parameter $E_1$. It is clearly observed from this figure that the effect of electric parameter $E_1$ is to increase velocity throughout the boundary layer but more significantly far away from the stretching sheet. Analysis of the graph reveals that the effect of local electric field parameter $E_1$ is to shift the streamlines away from the stretching boundary. This shifting of streamlines is seen little away from the stretching sheet. This is because Lorentz force arising due to electric field acts as an accelerating force in reducing the frictional resistance.

Fig. 5.4 is the plot of velocity profile for various values of porosity $\epsilon$. It is clearly
observed from this figure that the effect of porosity $\epsilon$ on velocity is to increase its value throughout the boundary layer but more significantly little away from the stretching sheet. This is due to fact that the obstruction in the motion of the fluid reduces as the porosity increases (pore size increases) hence the velocity increases is the boundary layer.

Fig. 5.5 represents the graph of temperature profile for different values of Prandtl number $Pr$. It is seen that the effect of Prandtl number $Pr$ is to decrease, temperature throughout the boundary layer, which results in decrease of the thermal boundary layer thickness with the increase of values of Prandtl number $Pr$. The increase of Prandtl number means slow rate of thermal diffusion. Figs. 5.6 represent the variations of velocity distribution in the boundary layer profiles for various values of mixed convection parameter or buoyancy parameter $\lambda$. It is observed from these figures that the velocity distribution increases with increasing the buoyancy parameter $\lambda$, this is due to the fact the boundary layer thickness increases with $\lambda$.

Fig. 5.7 depicts the effect of space dependent heat source/sink parameter $A^*$. It is observed that the boundary layer generates the energy, which causes the temperature profiles to increase with the increasing values of $A^* > 0$ (heat source) where as in the case of $A^* < 0$ (absorption) boundary layer absorbs energy resulting in the temperature to fall considerably with decreasing in the value of $A^* < 0$. The effect of temperature dependent heat source/sink parameter $B^*$ on heat transfer is demonstrated in Fig.5.8. This graph illustrates that energy is released when $B^* > 0$ which causes the temperature to increase, whereas energy is absorbed by decreasing the values of $B^* < 0$ resulting in the temperature to drop significantly near the boundary layer.

Figs. 5.9-5.10 displays results for the velocity and temperature distribution, respectively for different values of fluid viscosity parameter, $\theta_r$. The figures indicate that as $-\theta_r \to 0$, the boundary layer thickness decreases and the velocity distribution becomes shallower whereas the temperature distribution approaches a linear shape. Figs. 5.9 and 5.10 shows the variation of the dimensionless temperature parameter $\theta(\eta)$ for various values of $\theta_r$ for both air and liquid. It is seen that for air, the temperature decreases very rapidly with $\eta$ and its value decreases with increase in $\theta_r$ whereas in the case of liquid, the temperature decreases very steadily with $\eta$. Further it is observed that the decrease in the temperature with $\theta_r$ is not very remarkable within the boundary in this case. This effect is much noticeable little away from the stretching sheet.

Fig. 5.11 depicts the effects of non-uniform heat generation ($A^*, B^* > 0$) or absorption ($A^*, B^* < 0$) parameter on the concentration distribution $\phi(\eta)$. It is observed that there is
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generation of energy in the solutal boundary layer by increasing the values of $A^*, B^* > 0$ (heat source) which causes the temperature of the fluid to increase, which in turn results in increase of the flow field due to the buoyancy effect. This is the main reason behind the concentration profiles to decrease, whereas in the case of $A^*, B^* < 0$ (absorption) the boundary layer absorbs energy resulting in the concentration profiles to increase with increasing in the value of $A^*, B^* < 0$ in the boundary layer. Fig. 5.12 shows the variations in concentration profile with electric field parameter $E_1$ along $\eta$. It can be seen from this figure that an increase in the value of electric field results in reduced concentration of the species in the solutal boundary layer.

The effect of porous parameter $k_1$ on the local Sherwood number in terms of $-\phi'(0)$, is displayed in Fig. 5.13. It is seen from this figure that the local Sherwood number increases with increase in the value of $k_1$ for all the values of Schmidt number $Sc$ but the effect is not very appreciable. Fig. 5.14 shows the local Sherwood number along $Sc$ for different values of local inertia co-efficient $F^*$. It is observed that the local Sherwood number increases with increasing the value of local inertia co-efficient $F^*$ of the porous medium.

Figs.5.15 and 5.16 displays the variation of the concentration gradient $\phi'(0)$ with $Sc$ for various values of $\theta_r$. It is observed that the values of concentration gradient increases with increase in the value of $\theta_r > 0$ as seen from Fig.5.15, whereas from the plot of local Sherwood number against $Sc$ for different values of $\theta_r < 0$, it is observed that as the value of $\theta_r$ increases the concentration gradient increases everywhere within the boundary layer. Further, it is observed from Figs. 5.15 and 5.16 that the effect of increasing the values of $Sc$ is to decrease the Sherwood number for both $\theta_r > 0$ and $\theta_r < 0$.

5.4 Conclusions

Mathematical analysis has been carried out to study the effect of variable viscosity on MHD non-Darcy boundary layer flow and heat transfer characteristics in an incompressible electrically conducting fluid over a linear stretching sheet in the presence of Ohmic dissipation and non-uniform heat source/sink. Highly non-linear third-order momentum boundary layer equation is converted into a ordinary differential equation using similarity transformations. Fifth-order Runge-Kutta-Fehlberg method with shooting is used to solve momentum and heat transfer equations numerically. The effects of various physical parameters like Prandtl number, Eckert number, Hartmann number and local electric
parameter on velocity and temperature profiles are obtained.

The following main conclusions can be drawn from the present study:

(i) Boundary layer flow attain minimum velocity for higher values of Hartmann number ($Ha$).

(ii) The effect of the local electric field is to increase velocity distribution and decrease temperature in the boundary layer more significant little away from the stretching sheet.

(iii) The effect of increasing the values of Prandtl number ($Pr$) is to decrease temperature largely near the stretching sheet and the thermal boundary layer thickness decreases with Prandtl number.

(iv) The effect of porous permeability parameter is to increase velocity profile throughout the boundary layer.

(v) Buoyancy parameter is to increase the velocity distribution in the momentum boundary layer.

(vi) The effect of non-uniform and temperature dependent heat source/sink parameters is to generate temperature for heat source and absorb temperature for heat sink values. Hence non-uniform heat sink is better for cooling purposes.

(vii) The effect of increase in the space-dependent heat source/sink parameter $A^*$ and $B^*$ is to decrease concentration profile throughout the boundary layer.

(viii) The effect of increase in the value of electric field parameter is to decrease concentration profiles.

(ix) The effect of increase in the value of porous parameter and local inertia parameter is to increase the local Sherwood number.

(x) The effect of increase in the variable viscosity parameter is to increase the Sherwood number when both $\theta_r$ are positive or negative for $Pr = 0.7$ and $Pr = 2.0$.  

### Table 5.1: Comparison of Local Nusselt number $-\theta'(0)$ for $Ha = 0$, $\lambda = 0$ and various values of $Pr$ with Chen (1998), Grubka and Bobba (1985) and Ishak et al.(2008).

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### Table 5.2: The skin-friction Co-efficient $f''(0)$ for different values of Hartmann number $Ha$, Eckert number $E_c$, local electric parameter $E_1$ and Prandtl number $Pr$.

$| Ha  | E_c | E_1 | Pr  | $f''(0)$$ |$ (k_1 = 0, \lambda = 0, F^* = 0)$
|-----|-----|-----|-----|--------|----------------|
| 0.0 | 0.0 | 0.0 | 3.0 | -1.000000
| 0.0 | 1.0 | 1.0 | 5.0 | -1.000000
| 1.0 | 1.0 | 0.0 | 3.0 | -1.414214
| 1.0 | 1.0 | 1.0 | 3.0 | -0.6561953
Table 5.3: The Wall temperature gradient $-\theta'(0)$ for different values of Hartmann number $Ha$, Eckert number $E_c$, local electric parameter $E_1$ and Prandtl number $Pr$ with Abel et al. (2008).

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<th>$E_c$</th>
<th>$E_1$</th>
<th>$Pr$</th>
<th>$-\theta'(0)$ $(k_1 = 0, \lambda = 0, F^* = 0)$</th>
</tr>
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Figure 5.1: Schematic diagram of the problem under consideration

Figure 5.2: Influence of the Hartmann number $Ha$, on the dimensionless velocity profile $f'(\eta)$. 
Figure 5.3: Variation of velocity profile for different values of Electric parameter $E_1$.

Figure 5.4: Variation of velocity profile for different values of porosity $\epsilon$ of the porous medium.
Figure 5.5: Variation of temperature profile for different values of Prandtl number $Pr$.

Figure 5.6: Effects of buoyancy or mixed convection parameter $\lambda$ on the velocity profile in the boundary layer.
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Figure 5.7: Effect of non-uniform heat source/sink parameter $A^*$ on temperature profiles.

Figure 5.8: Effect of temperature dependent heat source/sink parameter $B^*$ on temperature profiles.
Figure 5.9: Effect of variable viscosity parameter $\theta_r$ on the velocity profile $f'(\eta)$.

Figure 5.10: Effect of variable viscosity parameter $\theta_r$ on the temperature profile $\theta(\eta)$. 
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Figure 5.11: Concentration profile for different values of $A^*$ and $B^*$.

Figure 5.12: Concentration profile for different values of $E_1$.
Figure 5.13: Effect of $Sc$ on Sherwood number for various values of $k_1$.

Figure 5.14: Effect of $Sc$ on Sherwood number for various values of $F^\ast$. 

Ha=0.5, $A^\ast$ = -0.4, $B^\ast$ = -0.4, $F^\ast$ = 0.2, $E_i$ = 0.5
$\lambda$=0.5, Pr=0.7, Ec=0.1, $\varepsilon$ = 0.6, $\theta_\infty$=2.0
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Figure 5.15: Effect of $Sc$ on Sherwood number for various positive values of $\theta_r$.

Figure 5.16: Effect of $Sc$ on Sherwood number for various negative values of $\theta_r$. 

Ha=0.5, $A^*= -0.4$, $B^*= -0.4$, $F^*= -0.2$, $E_1=0.5$
$\lambda=0.5$, Pr=0.7, Ec=0.1, $\varepsilon=0.6$, $k_1=0.2$