CHAPTER – 1
INTRODUCTION TO RELIABILITY
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INTRODUCTION

1.1 Introduction:

In the present scenario of global competition and liberalization, it is imperative that Indian industries become fully conscious of the need to produce reliable products meeting international standards. Even though the “Reliability Engineering” has taken birth during World War II with a significant contribution by defense personnel, today it has taken a new shape by blending itself in all phases of the product life cycle from proposal to manufacturing.

It is a known fact that reliability program increases the initial cost of every device, instrument or system and also it is true that the reliability decreases when the complexity of the system increases. In this type of complex situation, reliability of a product or service is best assured when it is designed by the design engineer and built in by production engineer, rather than conducting externally an experiment by a reliability engineer.

Once the product is accepted by the buyer and put in operation, either by itself or as a part of a larger assembly, the quality of performance would be judged by how long the product gives useful service; this is indicated by the word “Reliability”. One can find many definitions for reliability engineering, according to E.E.Lewis, “Reliability is probability that a component, device, equipment or a system will perform its intended function adequately for a specific period of time under a given set of conditions”. According to the definition, the basic
elements of reliability are probability, adequate performance, duration of adequate performance and operating conditions.

The above definition covers all four aspects of product, unlike quality, which speaks only conformance to specifications. In other words reliability is quality over time, which is under the influence of time and environment unlike quality, which is a degree of confirmation alone not considering the time length and environment of operation.

Another important difference between quality and reliability is that one can manufacture reliable systems using less reliable components by altering product configuration, whereas it is not possible to manufacture high quality systems with less quality components. Adding one or more similar components in parallel can increase the reliability of the system.

Reliability can only be meaningful, if it is related to time. It can be explained with reliability of a motor car. If any car fails, even after following manufacturer’s instructions specified, the car will be considered as unreliable. If it does not fail before its stated life, it would be considered as reliable.

It is difficult to predict at the start whether a particular car will be reliable because no two are ever absolutely identical, even though they are of the same make. There are always small manufacturing differences and a few may contain defects. The car designers try to make the car insensitive to the likely variations. The factory quality engineer tries to reduce variations and eliminate defects. If both the designer and the quality engineer had been completely successful and every car had been
used identically, then every car would have the same reliability. In practice this is impossible.

Suppose that out of every 100 cars of a particular type, 99 prove to be trouble free, if used and maintained correctly and one fails to work as intended, then it can be said that the reliability of each car is 99 percent. Since reliability is a probability, it is expressed in decimals of 1.00 as given below.

Reliability = 1.00 means certain to work as intended.
Reliability = 0.99 means 99 percent likely to work as intended.
Reliability = 0.50 means 50 percent likely to work as intended.
Reliability = 0.00 means absolutely certain not to work as intended.

Reliability at time 't' can be defined as.

\[ R(t) = \frac{\text{Number surviving at instant 't'} }{\text{Number at start (when 't'= 0)}} \]

1.2 Modes of Failure and Causes:

A failure is the partial or total loss or change in the properties of a device in such a way that its functioning is seriously affected or totally stopped. The concept of failures and their details help in the evaluation of quantitative reliability of a device. In general, some components have well defined failures; others do not. In the beginning, when the item or component is installed, the item fails with high frequency, which is known as initial failure or infant mortality. These are generally due to manufacturing defects. They are very high at initial stages and gradually decreases and stabilize over a longer period of time.
Stable or constant failures due to chance can be observed on an item for a longer period. These types of failures are known as random failures and characterized by constant number of failures per unit of time. Due to wear and tear with the usage, the item gradually deteriorates and frequency of failures again increases. These types of failures are called as wear-out failures. At this stage failure rate seems to be very high due to deterioration. Therefore the whole pattern of failures could be depicted by a bathtub curve as shown in Figure 1.1

![The Bathtub Curve](image)

**Figure 1.1 Failure Rate Curve (Bathtub curve)**

**1.3 Design for Reliability:**

Reliability is now a well recognized and rapidly developing branch of engineering. Manufacturing of a perfect component is almost impossible because of inherent variations and the cost for parts improvement is very high and the approach becomes unwieldy with large and complex systems. Since reliability study is considered essential for proper utilization and maintenance of engineering systems and
equipment, it has gained much importance among the practicing engineers and manufacturers.

The system designer is encountered with several problems while planning and designing the system for a reasonable level of reliability. Therefore a thorough reliability analysis needs to be attended at the design stage itself. The various means of increasing the system reliability and the constraints associated with them must be known. Reliability of a system can be improved by any one or combination of the following two methods namely

- Improving the components

- By using redundancy technique

A number of techniques are available to enhance the system reliability. Some of the important techniques are shown in Table 1.1.

**Table 1.1 System Reliability Enhancement Techniques**

<table>
<thead>
<tr>
<th>SLNO</th>
<th>TECHNIQUE</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Parts improvement method</td>
<td>Leads to higher cost</td>
</tr>
<tr>
<td>2.</td>
<td>Effective and creative design</td>
<td>Failures cannot be completely eliminated</td>
</tr>
<tr>
<td>3.</td>
<td>System simplification</td>
<td>Leads to poor quality</td>
</tr>
<tr>
<td>4.</td>
<td>Use of over rated components</td>
<td>Leads to higher cost</td>
</tr>
<tr>
<td>5.</td>
<td>Structural redundancy</td>
<td>Effective method for higher reliability</td>
</tr>
<tr>
<td>6.</td>
<td>Maintenance and repair</td>
<td>Best for high reliability</td>
</tr>
</tbody>
</table>
Combination of structured redundancy and maintenance and repair yield maximum reliability nearing to 1.

In general it is not possible to produce components with high reliability due to number of constraints, such as cost, non-availability of production facilities etc., In such cases redundancy comes handy to the reliability engineer. In simple words, redundancy is the existence of more than one means for carrying out a given function.

The following are the methods for introducing redundancy in to a system for improving reliability.

- Element redundancy
- Unit redundancy

### 1.3.1 Element Redundancy:

Let $C_1$ and $C_2$ be the two elements with reliabilities $R_1(t)$ and $R_2(t)$ respectively connected in parallel as shown in the Figure 1.2. In this arrangement of elements, the reliability of the system will be much better due to the presence of redundant element and proper operation of one element is sufficient for the successful operation of the system.

![Figure 1.2 Element Redundancy](image)

### 1.3.2 Unit Redundancy:
To improve the reliability of the system, another similar system is connected either in series or parallel to the existing one is called the concept of unit redundancy. Consider a system with two elements C1 and C2 as shown below. For improving the reliability of the system, a similar system in parallel is added to the existing system, which is shown in Figure 1.3

![Figure 1.3 Unit Redundancy](image.png)

Unit redundancy is further classified into two types

- Active redundancy
- Stand by redundancy

1.3.2.1 Active Redundancy:

Redundant system consisting of two or more components connected in parallel and both components were operating simultaneously is called active redundancy. In active redundancy all the redundant units are operated simultaneously instead of switching on only when need arises. The schematic diagram representing the active redundancy is shown in Figure 1.3a
1.3.2.2 Stand By Redundancy:

In case of stand by redundancy the alternate means of performing the function is not operated until it is needed. The alternative means is switched on only when the primary means of performing the function fails. Standby redundancy is more appropriate for mechanical devices such as motors and pumps etc.

The schematic diagram representing the standby redundancy is shown in the Figure 1.3b

1.3b Standby Redundancy

While designing the system the redundancy is highly appropriate in almost all the cases. The advantages of the redundancy approach are:

1. Desired level of reliability can be achieved under the resource flexibility.
2. Improvement in reliability per unit of resource is optimum when compared to any other approach.
3. Redundancy requires less skill on the part of the design engineer.
4. It is a quick method of solution.
5. This method can be the best choice for improvement of reliability in case of failure of all other approaches.

1.4 Types of System Reliability Models:

The objective of system engineer is to estimate various reliability parameters of the system. The system may vary from simple to complex. The system can be analyzed by decomposing it into smaller sub systems and estimating reliability of each subsystem to assess the total system reliability. The procedure to determine the system reliability is as follows.

1. Identify the sub systems and elements of the given system.
2. Identify corresponding individual reliabilities of the sub systems and elements.
3. Draw a block diagram to represent the logical manner in which these units are connected.
4. Determine the constraints for the successful operation of the system.
5. Apply rules of probability theory to determine the system reliability.

To determine an appropriate reliability or reliability model for each component of the system by applying the rules of the probability according to the configuration of the components within the system is also known as system reliability. Several methods exist to improve the
system reliability like using large safety factors, reducing the complexity of the system, increasing the reliability of the components etc. There are several types of configurations available, such as

1. Series configuration
2. Parallel configuration
3. Mixed configuration
4. Series-parallel configuration
5. Parallel-series configuration
6. k-out-of-m- configuration
7. Non-series parallel configuration
8. Complex configuration
9. Coherent system
10. Miscellaneous and general systems

**1.4.1 Series Configuration:**

In series configuration all components must be connected in series in order to make the system to perform continuously. In this system all components are considered critical in that sense that their function must be performed in order to make the system to operate successfully. Under this concept if any one component connected serially fails, the System will fail. The reliability block diagram as shown in Figure 1.4 represents the series configuration.

![Figure 1.4 Series Configuration](image)
The characteristics of series configuration are

- The components are interconnected in such a way that the entire System will work satisfactorily if all the components work without fail.
- The entire system will fail even if one of its components fails.

System reliability \( R_S \) can be determined by using component reliabilities.

If each component has a constant failure rate of \( \lambda_i \), then the system reliability is equal to

\[
R_S(t) = \prod_{i=1}^{n} R_i(t) = \exp\left(-\lambda_S t\right) = \exp\left(-\sum_{i=1}^{n} \lambda_i t\right) = \exp\left(-\lambda_S t\right)
\]

where \( \lambda_S = \sum_{i=1}^{n} \lambda_i \)

1.4.2 Parallel Configuration:

A system can have several components to perform the same operation and the satisfactory performance of any one of these components is sufficient to ensure the successful operation of the system. The elements for such a system are also said to be connected in parallel configuration as shown in Figure 1.5
The characteristics of a system with parallel configuration are

- The system will function satisfactorily even any one of the parallel units operates satisfactorily.
- The entire system will fail only when all the units in the system fail.

The system reliability $R_S$ is given by

$$R_S(t) = 1 - \prod_{i=1}^{m} [1 - R_i(t)]$$  \hspace{1cm} (2)

If life times of the components follow exponential distribution then

$$R_S(t) = 1 - \prod_{i=1}^{m} [1 - \exp(-\lambda_i t)]$$  \hspace{1cm} (3)

If all components are identical

$$R_S(t) = 1 - [1 - \exp(-\lambda t)]^m$$  \hspace{1cm} (4)

### 1.4.3 Mixed Configuration:

In mixed configuration, the elements are connected in series and parallel arrangement to perform a required system operation. In mixed configuration, to compute the system reliability, the network is broken
into series or parallel subsystems. The reliability of each sub system is found and then the system reliability may be obtained on the basis of the relationship among the sub systems. The schematic diagram representing the mixed configuration is shown in the Figure 1.6

![Figure 1.6 Mixed Configuration](image)

1.4.4 Series - Parallel Configuration:

The System consists of different stages connected in series as shown in Figure 1.7. Each stage contains number of redundant elements. For example stage-1 consists of \( n_i \) redundant elements connected in parallel. The reliability of the system \( R_s \) is the product of reliabilities at the each stage. The reliability of the system is

\[
R_S(t) = \prod_{i=1}^{k} \prod_{j=1}^{n_i} |1 - (i - R_{ij})|
\]  

(5)

where \( R_{ij} \) is the probability of successful operation of \( j^{th} \) component in the \( i^{th} \) stage.

\( n_i \) is the no. of parallel components at each stage

\( k \) is no. of series stages
1.4.5 Parallel – Series Configuration:

A number of activities such as industrial process, economic systems and mechanical devices can be represented as multi stage systems. These multi stage systems are generally categorized according to the component configuration at each stage. The system consists of different stages connected in parallel as shown in Figure 1.8 and each path contains number of elements in series. For successful running of system, one path is sufficient. Redundant components are added in parallel-series system to enhance the reliability of the stages as well as the system. The reliability of the system $R_s$ is

$$R_s(t) = 1 - \prod_{i=1}^{n_1} \left[ 1 - \prod_{j=1}^{m_i} (1 - R_{ij}) \right]$$

(6)

$R_{ij}$ is the probability of successful operation of $j^{th}$ component in $i^{th}$ parallel path.
1.4.6 K – Out – Of – M – Configuration:

A generalization of ‘n’ parallel components occurs when a requirement exists for k out of ‘m’ identical and independent components to function for the system to operate.

If k = 1 the system will become parallel configuration. If k = m, the system will become serial configuration.

To evaluate the reliability of k – out – of – m systems, generally the binomial law can be used with identical and statistically independent components.

If P is probability of successful operation of a component then the probability that exactly k – out – of – m components are successful is given by

$$P (m, x) = B(m, x) \cdot P^x \cdot (1 - p)^{m-x}$$

(7)

Where $B(m, x)$ is binomial co-efficient.

k is the minimum number of components required for successful operation of the system.
where \( B(m,i) \) is binomial coefficient.

For a constant failure rate, the reliability for a \( k \)-out-of-\( m \) configuration is expressed as

\[
R_{S}(t) = \sum_{i=k}^{m} B(m, i) p^i (1 - p)^{m-i}
\]

(8)

1.4.7 Non Series – Parallel Configuration:

A complex system on simplification can produce a non-series parallel structure. Bridge configuration comes under this category, which is shown in the Figure 1.9

The reliability of the below configuration can be evaluated using logic diagram and appropriate probability rules.

![Figure 1.9 A Bridge Network](image-url)
1.4.8 Complex Configuration:

A structure is called a complex when the reliability block diagram either cannot be reduced to a serial or parallel structure with independent elements. For any complex system, two steps have to be followed for obtaining the reliability or unreliability expressions of the system.

STEP 1: Enumeration of minimal path sets or cut sets.

STEP 2: Reliability expression or unreliability expression is obtained from minimal path sets or sets by using disjoint techniques.

1.4.9 Coherent System:

A coherent system is defined by the following conditions:

1. If all the components in a coherent system are functioning, then the system is functioning.

2. If all the components in a coherent system are in a failed state, then the system is in failed state.

If a system fulfills above conditions then it is a coherent system.

The schematic representation of the coherent system is shown in the Figure.1.10.
1.4.10 Miscellaneous & General System:

General network system includes bridge networks, non series – non parallel structures and other complex system configurations. Unspecialized systems where the structure is not explicit and the molecules of the system are not necessarily physically connected.

1.5 Reliability Optimization through Redundancy:

Redundancy makes it possible to achieve high reliable system using less reliable elements. However redundancy increases product cost, weight and complexity of the system substantially. It is therefore, essential to optimize reliability of the system by keeping constraints like cost, weight, volume etc.

In fact, there may be specific constraints on cost, weight and volume or even an optimal allocation of redundancy itself to maximize system reliability. In series – type of systems consisting of k stages (or sub systems) function if and only if each stage function. Even with this constraint, a number of distinct variations of the basic problem exist.
Redundant units may be operating actively in parallel and thus be subject to failure. Alternately they may be serving as spares to be used in succession for replacement of failed units. The first type of redundancy is some times referred to as parallel redundancy and the second type is referred as stand by redundancy.

Fixed specific constraints may exist on cost, weight, volume, etc., which cannot be violated and a redundancy allocation, which satisfies these constraints while maximizing reliability is desirable. Alternately fixed constraint values may not be available for cost, weight, and volume etc., rather what may be desired is a family of redundancy allocation, each member of the family having reliability must be either costlier, heavier or bulkier. Such a family of optimal redundancy allocations that the decision maker needs to consider.

In making the selection among the members of the family of optimal redundancy allocations, the decision maker is assured that whatever the cost, weight and volume, the maximum reliability is attained. For clarity and ease of reference, the following models can be used to represent the different situations.

1.5.1 Parallel Redundancy, Given the Set of Constraints:

Stage i consists of $n_i + 1$ units in parallel, each of which has independent probability $q_i \ (0 < q_i < 1)$ of failing linear “cost” constraints exists as

$$n_i = (n_1, n_2... n_k)$$

and

$$\sum_{i=1}^{k} C_{ij} n_i \leq C_j \quad j = 1, 2... r$$

(10)
where each \( C_{ij} > 0 \), \( r \) is the maximum number of units in parallel. Select a vector of non-negative integers \( n_i \) to maximize system Reliability \( R_s \), where in the present model \( R_s \) is given by

\[
R_s(t) = \prod_{i=1}^{k} \left\{ 1 - q_i^{n_i+1} \right\} 
\]  

subject to the constraint.

\[
\sum_{i=1}^{k} C_{ij}n_i < C_j \quad j = 1, 2, \ldots r
\]

### 1.5.2 Parallel Redundancy, No Specific Set of Constraints:

When there is no specific set of constraints, a family of undominated allocations can be generated. Ultimate objective of the owner of the system is to make the system operative with no cost increase.

### 1.5.3 Standby Redundancy, Given Set of Constraints:

A system is required to operate for the period \([0, t_0]\). When a component fails, it is immediately replaced by a spare component of the same type, if one is available. If no spare is available, system failure result. Only the spares originally provided may be used for replacements, that is, no re-supply of spares can occur during \([0, t_0]\).

### 1.5.4 Stand By Redundancy, No Specific Set of Constraints:

The model is as in 1.5.3 except that instead of desiring to maximize \( R_s \) subject to constraints such as in 1.5.1 to generate a family \((n^*)\) of undominated allocations.
1.6 Problem under Consideration:

The objective of the present work is to maximize the reliability of a system subject to cost, weight and volume as constraints. The reliability of a system can be maximized in two different ways.

1. The component reliabilities are known to determine the number of components in each stage and maximum system reliability for the given cost, weight and volume.

2. The number of components are known to determine the component reliabilities to maximize system reliability for the given cost, weight and volume.

In literature most of the reported works have considered one of the above situations to maximize the system reliability.

1.6.1 Statement of the Problem:

The general problem considers both the unknowns i.e. the component reliabilities and the number of components in each stage for the given cost constraint to maximize the system reliability which is termed as an Integrated Reliability Model (IRM). So far, in literature the integrated reliability models are optimized using cost constraint where there is an established relationship between the cost and reliability. The consideration of weight and volume as constraints along with cost constraint is not considered in optimizing redundant system reliability.

The author in this Thesis also makes an attempt to negotiate the impact of weight and volume as constraints in optimizing the redundant systems under consideration for the selected mathematical function. Though cost has direct relation in maximizing system reliability, which
includes the influencing variables, which were not considered particularly the important variables like weight and volume in optimizing system reliability of a redundant system.

The author in this thesis wants to study indirect impact of weight and volume as constraints in optimizing the reliability of a redundant system which is a novel beginning in the mentioned area of research.

The series parallel systems are considered with cost, weight and volume as constraints to maximize the reliability of redundant system as its objective function.

1.6.2 Assumptions of the Model:

1. All the components in each stage are assumed to be identical i.e. all the components have the same reliability.

2. The components are assumed to be statistically independent i.e. failure of a component does not affect the performance of the other components in any system.

3. A component is either in working condition or non-working condition.

1.6.3 Mathematical Model:

The objective function and the constraints of the model are

Maximize \[ R_S = \prod_{j=1}^{n} R_j = \prod_{j=1}^{n} (1 - (1 - r_j)^{X_j}) \] \[ \text{subjected to the constraints} \]

\[ \sum_{j=1}^{n} c_{j \cdot X_j} \leq C_o \]
Non negative restriction $x_j$ is an integer and $r_j, R_j > 0$

where $R_j$ is reliability of stage j, $r_j$ is reliability of each component in stage j, $x_j$ is total number of units in stage j, n is the maximum number of stages, $C_j$ is cost coefficient of each units in stage j, $W_j$ is weight coefficient of each components in stage j, $V_j$ is volume coefficient of each components in stage j, $C_o$ is allowable system cost, $W_o$ is allowable system weight, $V_o$ is allowable system volume.

1.6.4 Reliability cost models and their significance:

There is always a cost associated with changing a design, use of high quality materials, retooling costs, administrative fees, or other factors. Before attempting at improving the reliability, the cost as a function of reliability for each component must be obtained. Otherwise, the design changes may result in a system that is needlessly expensive or over-designed. Development of the “cost of reliability” relationship offers the engineer an understanding of which components or subsystems to improve. The first step is to obtain a relationship between the cost of improvement and reliability. The next step is to model the cost as a function of reliability. The preferred approach would be to formulate the cost function from actual cost data. This can be done taking the past data. However, there are many cases where no such information is available. For this reason, a general behavior model of the cost versus the
component reliability can be developed for performing reliability optimization. The objective of cost functions is to model an overall cost behavior for all types of components. But, it is impossible to formulate a model that is precisely applicable to every situation. However, one of the reliability cost models available can be used depending on situation. All these models can be tried and one which is suitable to component or situation can be adopted. The four cost models used in the present work are taken from the literature.

**Function 1:**

$$r_j = \left[ \frac{c_j}{b_j}x \right]^\frac{1}{d_j}$$

where $c_j, b_j$ and $1/d_j > 0$ and $c_j$ is cost co-efficient, $b_j$, $d_j$ are constants.

**Function 2:**

$$c_j = a_j \cdot \exp \left[ \frac{b_j}{1-r_j} \right]$$

where $a_j$, $b_j$ are constants.

**Function 3:**

$$r_j = \frac{r_j,\tan^{-1} \left( \frac{c_j}{b_j} \right) \cdot \frac{1}{d_j}}{2}$$

where $b_j$ and $1/d_j$ are constants.

**Function 4:**

$$c_j = e^{(1-f_j)(r_j-r_{j,min})(r_{j,max}-r_j]}$$

where $f_j$ is Feasibility factor of cost function, $r_{j,min}$ is Minimum reliability, $r_{j,max}$ is Maximum reliability

**1.7 Organization of the Thesis:**
The introduction, types of system reliability models, reliability optimization through redundancy, Problem under consideration, of the work are presented in first chapter.

Literature survey on system reliability models by system configuration, by optimization techniques, integrated reliability models are presented in second chapter.

In third chapter of the present work the Lagrangian method for problem formulation for four functions is presented. Case problems without rounding off number of components and with rounding of number of components are presented and percentage variation of parameters with without rounding of number of components is compared.

In Chapter four and five, optimization of the integrated reliability models for redundant systems with multiple constraints using integer programming approach and heuristic approach are established for the four models, discussed in chapter three.

In sixth chapter conclusions and scope of the present work are presented.