CHAPTER 4 NUMERICAL MODELING OF THE HEAT PIPE

Mathematical modeling of a heat pipe is complex since the physical phenomenon of the heat pipe requires solving of a highly nonlinear system of partial differential equations in temporal and spatial variables, velocity, temperature and pressure. In this work, a two-dimensional numerical model is developed to simulate the transient behavior of the cylindrical heat pipe. The effect of nanofluids on the performance of heat pipe is also studied using this model. To validate the developed models, comparisons are made between the predicted and experimental results.

4.1 Model description

The physical dimensions of the cylindrical heat pipe investigated are shown in Figure 4.1. The heat pipe, which is considered for simulation, has a length of 350 mm, outer diameter of 19.5 mm and wall thickness of 1 mm. The evaporator, adiabatic and condenser lengths are 100, 100 and 150 mm respectively. Both wall and wick materials are assumed as copper. Four layers of screen mesh with a total thickness of 1 mm are assumed. The porosity and the effective thermal conductivity of the wick are 63% and 1.14 W/m-K respectively. The thermophysical properties of the wick, wall and the working fluid are presented in Table 4.1. There are two different working fluids used in the present study and they are water and Cu-water nanofluids (0.1 wt% of Cu). The wick is present on both sides of the vapor as shown in Figure 4.1

4.2 Assumptions used in the model:

The following assumptions are made to develop a model to simulate the operation of the heat pipe
I. Vapor and liquid flows are considered to be laminar
II. The compressibility effect is negligible
III. The vapor is considered to be saturated at $t = 0$
IV. The volume averaged density of the liquid in the wick is changed to conserve liquid mass and to include the changes in vapor and liquid masses appropriately during the transient operating conditions
V. All thermophysical properties are assumed as constant except for the vapor density, which is computed from the operating pressure
VI. The body forces are considered to be negligible

Table 4.1 Thermophysical properties of wall, wick and working fluids

<table>
<thead>
<tr>
<th></th>
<th>Copper wall/wick</th>
<th>Water</th>
<th>Cu nanofluid (0.1wt %)</th>
<th>Water vapor</th>
<th>Water/vapor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity</td>
<td>387.6 W/m-K</td>
<td>0.6 W/m-K</td>
<td>0.78 W/m-K</td>
<td>0.0189 W/m-K</td>
<td>2446.36 kJ/kg</td>
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<tr>
<td>Specific heat</td>
<td>381 J/kg-K</td>
<td>4200 J/kg-K</td>
<td>4200 J/kg-K</td>
<td>1861 J/kg-K</td>
<td></td>
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<tr>
<td>Density</td>
<td>8978 kg/m$^3$</td>
<td>991.08 kg/m$^3$</td>
<td>1030 kg/m$^3$</td>
<td>0.01 kg/m$^3$</td>
<td></td>
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<tr>
<td>Thermal conductivity of the wick</td>
<td>1.11 W/m-K</td>
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<tr>
<td>Specific heat</td>
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<td>Density</td>
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</tbody>
</table>
4.3 Governing equations

4.3.1 Continuity equation

The continuity equation for the wick and vapor region is written as

\[ \varepsilon \frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{V} = 0 \]  \hspace{1cm} (4.1)

where \( \frac{\partial \rho}{\partial t} \) term is the rate of change in density and \( \rho \nabla \cdot \vec{V} \) is the convective term. Here the mass addition and/or depletion due to evaporation and condensation in the vapor and liquid region are accounted in the \( \frac{\partial \rho}{\partial t} \) term. \( \vec{V} \) denotes the velocity vector.

4.3.2 Momentum equations

The two dimensional momentum equations in the wick and vapor core are given by

\[ \frac{\partial (\rho \mu)}{\partial t} + \nabla \cdot (\rho \vec{V} u) = - \frac{\partial (\rho p)}{\partial x} + \nabla \cdot (\mu \nabla u) - \frac{\mu}{K} u - \frac{c_e}{\sqrt{\epsilon}} \rho |\vec{V}| u \]  \hspace{1cm} (4.2)
\[
\frac{\partial (\rho v^2)}{\partial t} + \nabla \cdot (\rho \vec{v}) = -\frac{\partial (p) }{\partial y} + \nabla \cdot (\mu \nabla v) - \frac{\mu e}{K} v - \frac{C_{pe}}{\sqrt{K}} \rho \nabla T \cdot \vec{v} \tag{4.3}
\]

In the vapor core, \( K = \infty \) and \( \varepsilon = 1 \). The last two terms of the Equation 4.2 and 4.3 is considered as a momentum source for wick region.

### 4.3.3 Energy equation

The energy equation for the wall, wick and vapor region is

\[
\frac{\partial ((\rho C)_m T)}{\partial t} + \nabla \cdot [(\rho C)_l \vec{v} T] = \nabla \cdot (k_{eff} \nabla T) \tag{4.4}
\]

In this equation \((\rho C)_m\) assumes different values in the wall, wick and vapor region

For the wall \((\rho C)_m = (\rho C)_s\)

In the wick \((\rho C)_m = (1 - \varepsilon)(\rho C)_s + \varepsilon(\rho C)_l\)

In the vapor core \((\rho C)_m = (\rho C)_v\)

The second term of the Equation 4.4 is neglected for the solid regions. \(k_{eff}\) is the effective thermal conductivity of the region of interest. Appropriate values for the wall and vapor region are assumed. For the wick region, \(k_{eff}\) of the liquid - wick combination is calculated by the Equation 4.5

\[
k_{eff} = k_l \left( \frac{(k_l + k_s) - (1 - \varepsilon)(k_l - k_s)}{(k_l + k_s) + (1 - \varepsilon)(k_l - k_s)} \right) \tag{4.5}
\]

In the above equation, \(k_l\) is the thermal conductivity of the working fluid and \(k_s\) is thermal conductivity of the wick material. When the nanofluid is substituted instead of DI water, the effective thermal conductivity of the liquid - wick combination changes from 1.11 to 1.69 W/m-K. The porosity of the wick while using the nanofluid is calculated using the Equation 4.6 (Faghri, 1995).
The increase in wire diameter and change in porosity due to the deposition of nanoparticles are included in this model through the Equation 4.6. When the nanoparticles are deposited, wick diameter is increased. Due to this, the will change and is found to be 1.75 W/m-K. The wire diameter of the wick structure is 80 and the mesh number ($N$) is 100 mesh/inch. The increase in average wire diameter ( ) is assumed to 5 . The increase in wire diameter leads to reduction in porosity and permeability of the wick structure. The permeability of the wick is found to be $1.52 \times 10^{-10}$ and $1.43 \times 10^{-10}$ m$^2$ respectively in the case of with and without deposition.

### 4.4 Computational domain

In the present study, the computational domain (Figure 4.2) is meshed using GAMBIT software. In the x direction, 350 cells are used in the wall, wick and vapor region of the model. In the y direction, 15 and 10 cells are used in the wall and wick respectively for both side of the vapor. In the vapor region 145 cells are used in the y direction.

![Figure 4.2 Computational mesh](image)
4.5 Boundary conditions

Various boundary conditions used to solve governing equations are as follows

In the evaporator wall, the heat flux is applied as

\[ q_e = -k_w \frac{\partial T}{\partial y}, \quad u = v = 0, \quad 0 \leq x \leq L_e \]  \hspace{1cm} (4.7)

In the adiabatic section

\[ \frac{\partial T}{\partial y} = 0, \quad u = v = 0, \quad L_e \leq x \leq L_e + L_a \]  \hspace{1cm} (4.8)

In the condenser wall, convective heat transfer is assumed as

\[ q_c = -k_w \frac{\partial T}{\partial y} = h_c (T - T_e), \quad x > L_e + L_a \]  \hspace{1cm} (4.9)

In the wick-wall interface

\[ u = 0, \quad v = 0 \]  \hspace{1cm} (4.10)

In the left and right lateral walls

\[ u = v = \frac{\partial T}{\partial x} = 0 \]  \hspace{1cm} (4.11)

4.6 Wick-vapor interface

In the wick-vapor interface the change of phase from liquid to vapor is assumed to occur. The interface temperature, \( T_i \) is obtained from the energy balance equation

\[ -k_{w} A_i \frac{\partial T}{\partial y} + m_i C_i T_i = -k_v A_i k_v A_i \frac{\partial T}{\partial y} + m_i C_v T_i + m_i h_{fg} \]  \hspace{1cm} (4.12)

In the above equation \( m_i \) denotes the mass of liquid evaporated or condensed. The interfacial pressure, \( P_i \) is calculated using the Clausius-Clapyron equation

\[ \frac{R}{h_{fg}} \ln \left( \frac{P_i}{P_o} \right) = \frac{1}{T_o} - \frac{1}{T_i} \]  \hspace{1cm} (4.13)
where $T_o$ and $P_o$ are considered as a reference values.

The interface mass flux is calculated by the Equation 4.14 using kinetic theory

$$
\left( \frac{2\sigma}{2-\sigma} \right) \ln \left( \frac{M}{2\pi R^2} \right)^{1/2} \left( \frac{P_v}{\langle T_v \rangle^{1/2}} - \frac{P_i}{\langle T_i \rangle^{1/2}} \right) = m_i^{''} \tag{4.14}
$$

In the Equation 4.14, the accommodation coefficient, $\sigma$ is assumed as 0.03 since the experimental accommodation coefficient for water is between 0.02-0.04.

The initial temperature and pressure is assumed as follows

$$T(x, y, 0) = T_i, \quad P_{op}(t = 0) = P_{sat}(T_i) \tag{4.15}$$

### 4.7 Computation of operating pressure in the vapor core

To allow the system for pressurization under incompressible assumption, the system pressure term is split into two components.

$$P = \hat{P} + P_{op} \tag{4.16}$$

where $\hat{P}$ is the hydrodynamic pressure component which is calculated using the continuity equation through the pressure correction method. The component $P_{op}$ is a function of time and is computed using the ideal gas law and overall mass balance in the vapor core as follows

$$P_{op} = \frac{M_\nu^0 + \Delta t (\Sigma_{wick/vapor face} \cdot m_i)}{1 + \Sigma_{all vapor cells} \cdot V_{cell} \cdot \frac{T_P}{T_P}} \tag{4.17}$$

where $V_{cell}$ is the volume of computational cell and $T_P$ is the temperature at the center of the computational cell. $m_i$ is the interfacial evaporation/condensation mass flow rate and can be written as

$$m_i = \left( \frac{2\sigma}{2-\sigma} \right) \frac{A_i}{(2\pi R)^{1/2}} \left( \frac{\hat{P}_{pv} + P_{op}}{(T_{pv})^{1/2}} - \frac{P_i}{(T_i)^{1/2}} \right) \tag{4.18}$$

where $\hat{P}_{pv}$ and $T_{pv}$ are the hydrodynamic pressure and temperature in the vapor cell adjacent to the wick-vapor interface.
4.8 Computation of liquid and vapor densities

By keeping with the incompressible flow assumption, the vapor density at a cell is computed from the system pressure as

\[ \rho_p = \frac{p_{op}}{RT_p} \]  \hspace{1cm} (4.19)

The wick may, in general, be unsaturated during the transient state. Flow in an unsaturated wick is not resolved in this work, but instead, the mean liquid density is computed so as to conserve the liquid mass:

\[ \frac{dM_l}{dt} = \sum_{\text{wick/vapor faces}} m_i \]  \hspace{1cm} (4.20)

\[ \rho_l = \frac{M_l}{eV_i} \]  \hspace{1cm} (4.21)

The \( M_l \) is computed during every time step and the mean liquid density is computed.

4.9 Solution procedure

The governing equations are solved in their transient form using the commercial CFD code FLUENT 6.3 (Fluent user guide, 2006). Required User Defined Functions (UDFs) to compute evaporation/condensation mass flow rate, temperature and pressure at the wick-vapor interface are developed. Also, UDFs are developed to calculate the liquid and vapor densities. Steady state is assumed at the time when the heat transfer at the condenser side reaches within 2% of heat input at the evaporator. For each time step, the following are obtained from the UDFs:-

1. Wick- vapor interface temperature using the Equation 4.12
2. The wick-vapor interface pressure using Clausius - Clapeyron Equation 4.13 with the latest interface temperature
3. Interface mass flow rate using the Equation 4.14 and
4. Operating pressure using the Equation 4.17 and
5. Mean density of the liquid using the Equation 4.21
After this computation, the u momentum and v momentum equations are solved using FLUENT 6.3 solver. Subsequently, pressure correction equations are solved. At the final stage, energy equation is solved to determine the temperature field. Pressure based coupled algorithm is enabled in the fluent for pressure velocity coupling. A flow chart for the present pressure based coupled algorithm is presented in Figure 4.3. Standard discretization scheme for the pressure, second order upwind scheme for momentum and energy equations are used. For the convergence, residual value of $10^{-4}$ is used for u and v velocities and $10^{-10}$ is used for temperature.